PHYS 1444 – Section 003
Lecture #14

Wednesday, Oct. 19, 2005
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• RC circuit example
• Discharging RC circuits
• Application of RC circuits
• Magnets
• Magnetic field
• Earth’s magnetic field
• Magnetic field by electric current
• Magnetic force on electric current
Announcements

• There is a colloquium at 4pm in SH103
  – All Physics faculty will introduce their own research
  – An extra credit opportunity

• Extra credit opportunity was announced on Sept. 14th:
  – 15 point extra credit for presenting a professionally prepared
    3 page presentation on any one of the exhibits at the UC
    gallery (till 9/16) and the subsequent themed displays at the
    central library.
    • Must include what it does, how it works and where it is used.
      Possibly how it can be made to perform better.
    • Due: Oct. 19, 2005
Analysis of RC Circuits

• Since \( Q = C \varepsilon \left(1 - e^{-t/RC}\right) \) and \( V_C = \varepsilon \left(1 - e^{-t/RC}\right) \)

• What can we see from the above equations?
  – Q and \( V_C \) increase from 0 at \( t=0 \) to maximum value \( Q_{\text{max}} = C \varepsilon \) and \( V_C = \varepsilon \).

• In how much time?
  – The quantity \( RC \) is called the time constant, \( \tau \), of the circuit
    • \( \tau = RC \), What is the unit? Sec.
  – What is the physical meaning?
    • The time required for the capacitor to reach \( (1-e^{-1}) = 0.63 \) or 63% of the full charge

• The current is \( I = \frac{dQ}{dt} = \frac{\varepsilon}{R} e^{-t/RC} \)
**Example 26 – 12**

**RC circuit, with emf.** The capacitance in the circuit of the figure is $C=0.30\,\mu\text{F}$, the total resistance is $20\,\text{k}\Omega$, and the battery emf is $12\,\text{V}$. Determine (a) the time constant, (b) the maximum charge the capacitor could acquire, (c) the time it takes for the charge to reach 99% of this value, (d) the current $I$ when the charge $Q$ is half its maximum value, (e) the maximum current, and (f) the charge $Q$ when the current $I$ is 0.20 its maximum value.

(a) Since $\tau = RC$ We obtain 
$$\tau = 20 \times 10^3 \cdot 0.30 \times 10^{-6} = 6.0 \times 10^{-3} \text{ sec}$$

(b) Maximum charge is 
$$Q_{\text{max}} = C\varepsilon = 0.30 \times 10^{-6} \cdot 12 = 3.6 \times 10^{-6} \, \text{C}$$

(c) Since 
$$Q = C\varepsilon \left(1 - e^{-t/RC}\right)$$
For 99% we obtain 
$$0.99C\varepsilon = C\varepsilon \left(1 - e^{-(t/RC)}\right)$$
$$e^{-t/RC} = 0.01; \quad -t/RC = -2\ln 10; \quad t = RC \cdot 2\ln 10 = 4.6RC = 28 \times 10^{-3} \text{ sec}$$

(d) Since 
$$\varepsilon = IR + Q/C$$
We obtain 
$$I = \left(\varepsilon - Q/C\right)/R$$

The current when $Q$ is $0.5Q_{\text{max}}$ 
$$I = \left(12 - 1.8 \times 10^{-6}/0.30 \times 10^{-6}\right)/20 \times 10^3 = 3 \times 10^{-4} \, \text{A}$$

(e) When is $I$ maximum? when $Q=0$: 
$$I = 12/20 \times 10^3 = 6 \times 10^{-4} \, \text{A}$$

(f) What is $Q$ when $I=120\,\text{mA}$? 
$$Q = C\left(\varepsilon - IR\right) =$$
$$0.30 \times 10^{-6} \left(12 - 1.2 \times 10^{-4} \cdot 2 \times 10^4\right) = 2.9 \times 10^{-6} \, \text{C}$$

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Discharging RC Circuits

- When a capacitor is already charged, it is allowed to discharge through a resistance $R$.
  - When the switch $S$ is closed, the voltage across the resistor at any instant equals that across the capacitor. Thus $IR=Q/C$.
  - The rate at which the charge leaves the capacitor equals the negative the current flows through the resistor
    - $I=-dQ/dt$. Why negative?
    - Since the current is leaving the capacitor
  - Thus the voltage equation becomes a differential equation
    $$-rac{dQ}{dt} \cdot R = \frac{Q}{C} \quad \Rightarrow \quad \frac{dQ}{Q} = -\frac{dt}{RC}$$
    Rearrange terms
Discharging RC Circuits

– Now, let’s integrate from \( t=0 \) when the charge is \( Q_0 \) to \( t \) when the charge is \( Q \)

\[
\int_{Q_0}^{Q} \frac{dQ}{Q} = -\int_{0}^{t} \frac{dt}{RC}
\]

– The result is

\[
\ln |Q|_{Q_0}^{Q} = \ln \frac{Q}{Q_0} = -\frac{t}{RC}
\]

– Thus, we obtain

\[
Q(t) = Q_0 e^{-t/RC}
\]

– What does this tell you about the charge on the capacitor?

• It decreases exponentially with time and with time constant \( RC \)
• Just like the case of charging

– The current is:

\[
I = -\frac{dQ}{dt} = \frac{Q_0}{RC} e^{-t/RC}
\]

\[
I(t) = I_0 e^{-t/RC}
\]

• The current also decreases exponentially with time with constant \( RC \)
Example 26 – 13

Discharging RC circuit. In the RC circuit shown in the figure the battery has fully charged the capacitor, so $Q_0 = C \varepsilon$. Then at $t=0$, the switch is thrown from position a to b. The battery emf is 20.0V, and the capacitance $C=1.02 \mu F$. The current $I$ is observed to decrease to 0.50 of its initial value in 40$\mu$s. (a) what is the value of $R$? (b) What is the value of $Q$, the charge on the capacitor, at $t=0$? (c) What is $Q$ at $t=60\mu$s?

(a) Since the current reaches to 0.5 of its initial value in 40$\mu$s, we can obtain

$$I(t) = I_0 e^{-t/RC}$$

For $0.5I_0$

$$0.5I_0 = I_0 e^{-t/RC}$$

Rearrange terms

$$-t/RC = \ln 0.5 = -\ln 2$$

Solve for $R$

$$R = t/(C \ln 2) = 40 \times 10^{-6}/(1.02 \times 10^{-6} \cdot \ln 2) = 56.6 \Omega$$

(b) The value of $Q$ at $t=0$ is

$$Q_0 = Q_{\text{max}} = C \varepsilon = 1.02 \times 10^{-6} \cdot 20.0 = 20.4 \mu C$$

(c) What do we need to know first for the value of $Q$ at $t=60\mu$s?

The RC time

$$\tau = RC = 56.6 \cdot 1.02 \times 10^{-6} = 57.7 \mu s$$

Thus

$$Q(t = 60 \mu s) = Q_0 e^{-t/RC} = 20.4 \times 10^{-6} \cdot e^{-60 \mu s/57.7 \mu s} = 7.2 \mu C$$
Application of RC Circuits

What do you think the charging and discharging characteristics of RC circuits can be used for?

- To produce voltage pulses at a regular frequency
- How?

- The capacitor charges up to a particular voltage and discharges
- A simple way of doing this is to use breakdown of voltage in a gas filled tube
  - The discharge occurs when the voltage breaks down at $V_0$
  - After the completion of discharge, the tube no longer conducts
  - Then the voltage is at $V_0'$ and it starts charging up
  - How do you think the voltage as a function of time look?
    » A sawtooth shape

- Pace maker, intermittent windshield wiper, etc
Magnetism

- What are magnets?
  - Objects with two poles, north and south poles
    - The pole that points to geographical north is the north pole and the other is the south pole
      - Principle of compass
    - These are called magnets due to the name of the region, Magnesia, where rocks that attract each other were found

- What happens when two magnets are brought to each other?
  - They exert force onto each other
  - What kind?
  - Both repulsive and attractive forces depending on the configurations
    - Like poles repel each other while the unlike poles attract
Magnetism

• So the magnet poles are the same as the electric charge?
  – No. Why not?
  – While the electric charges (positive and negative) can be isolated the magnet poles cannot be isolated.
  – So what happens when a magnet is cut?
    • If a magnet is cut, two magnets are made.
    • The more they get cut, the more magnets are made
  – Single pole magnets are called the monopole but it has not been seen yet

• Ferromagnetic materials: Materials that show strong magnetic effects
  – Iron, cobalt, nickel, gadolinium and certain alloys

• Other materials show very weak magnetic effects