PHYS 1444 – Section 003
Lecture #16

Wednesday, Oct. 26, 2005

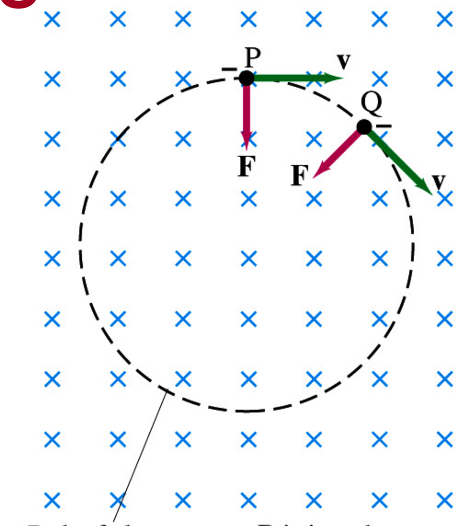
Dr. Jaehoon Yu

- Charged Particle Path in Magnetic Field
- Torque on a Current Loop
- Magnetic Dipole Moment
- Potential Energy of Magnetic Dipole
- The Hall Effect
- Magnetic field due to a straight wire
- Magnetic force between two parallel wires

Announcements

- Reading assignment
  - CH27 – 7
- The 2nd term exam
  - Date: Monday, Nov. 7
  - Time: 1 – 2:20pm
  - Location: SH 103
  - Coverage: CH 26 – whichever chapter we get to by Wednesday, Nov. 2

Charged Particle's Path in Magnetic Field



- What shape do you think is the path of a charged particle in a plane perpendicular to a uniform magnetic field?
  - Circle!! Why?
  - An electron moving to right at the point P in the figure will be pulled downward
  - At a later time, the force is still perpendicular to the velocity
  - Since the force is always perpendicular to the velocity, the magnitude of the velocity is constant
  - The direction of the force follows the right-hand-rule and is perpendicular to the direction of the magnetic field
  - Thus, the electron moves on a circular path with a centripetal force F.

Example 27 – 4

Electron's path in a uniform magnetic field. An electron travels at a speed of 2.0×10^{7} m/s in a plane perpendicular to a 0.010-T magnetic field. Describe its path.

What is formula for the centripetal force? $F = ma = m\frac{v^2}{r}$

Since the magnetic field is perpendicular to the motion of the electron, the magnitude of the magnetic force is

$$F = evB$$

Since the magnetic force provides the centripetal force, we can establish an equation with the two forces

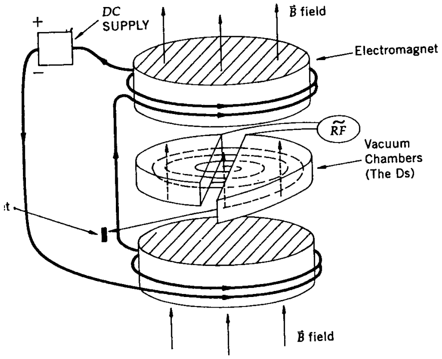
$$F = evB = m\frac{v^2}{r}$$

Solving for r

$$r = \frac{mv}{eB} = \frac{\left(9.1\times10^{-31}\,kg\right)\cdot\left(2.0\times10^{7}\,m/s\right)}{\left(1.6\times10^{-19}\,C\right)\cdot\left(0.010T\right)} = 1.1\times10^{-2}\,m$$

Cyclotron Frequency

- The time required for a particle of charge q moving w/ constant speed v to make one circular revolution in a uniform magnetic field, $\vec{B} \perp \vec{v}$, is



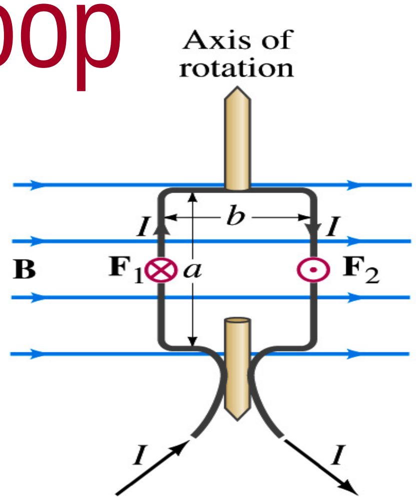
$$T = \frac{2\pi r}{v} = \frac{2\pi}{v}\frac{mv}{qB} = \frac{2\pi m}{qB}$$

- Since T is the period of rotation, the frequency of the rotation is

$$f = \frac{1}{T} = \frac{qB}{2\pi m}$$

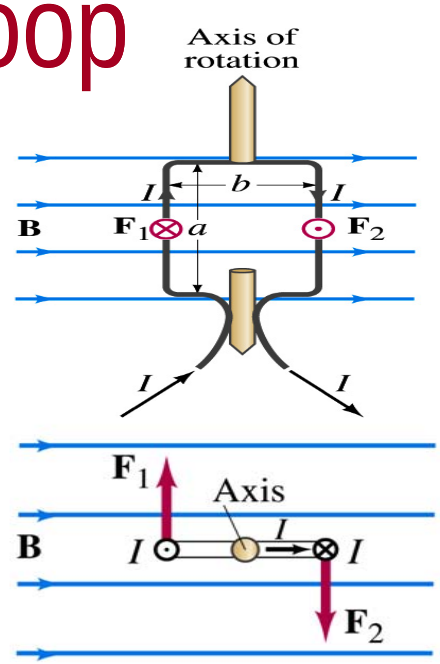
- This is the cyclotron frequency, the frequency of a particle with charge q in a cyclotron accelerator
  - While r depends on v, the frequency is independent of v and r.

Torque on a Current Loop



- What do you think will happen to a closed rectangular loop of wire with electric current as shown in the figure?
  - It will rotate! Why?
  - The magnetic field exerts a force on both vertical sections of wire.
  - Where is this principle used in?
    - Ammeters, motors, volt-meters, speedometers, etc
- The two forces on the different sections of the wire exerts net torque to the same direction about the rotational axis along the symmetry axis of the wire.
- What happens when the wire turns 90 degrees?
  - It will not turn unless the direction of the current changes

Torque on a Current Loop



- So what would be the magnitude of this torque?
  - What is the magnitude of the force on the section of the wire with length a?
    - $F_a = IaB$
    - The moment arm of the coil is b/2
  - So the total torque is the sum of the torques by each of the forces

$$\tau = IaB\frac{b}{2} + IaB\frac{b}{2} = IabB = IAB$$

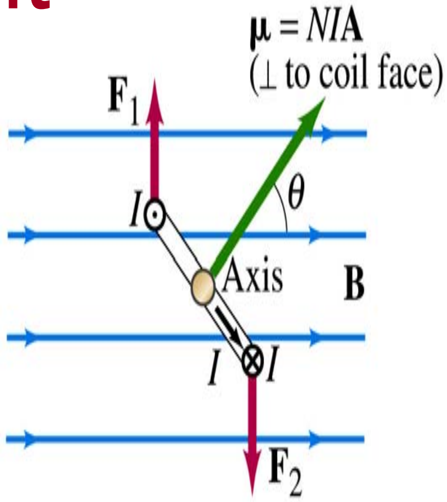
    - Where A=ab is the area of the coil
  - What is the total net torque if the coil consists of N loops of wire?

$$\tau = NIAB$$

  - If the coil makes an angle θ w/ the field $\tau = NIAB\sin\theta$

Magnetic Dipole Moment

- The formula derived in the previous page for a rectangular coil is valid for any shape of the coil
- The quantity NIA is called the **magnetic dipole moment of the coil**
  - It is considered a vector $\vec{\mu} = NI\vec{A}$
    - Its direction is the same as that of the area vector A and is perpendicular to the plane of the foil consistent with the right-hand rule
      - Your thumb points to the direction of the magnetic moment when your finer cups around the loop in the direction of the wire
  - Using the definition of magnetic moment, the torque can be written in vector form $\vec{\tau} = NI\vec{A} \times \vec{B} = \vec{\mu} \times \vec{B}$



Magnetic Dipole Potential Energy

- Where else did you see the same form of torque?
  - Remember the torque due to electric field on an electric dipole? $\vec{\tau} = \vec{p} \times \vec{E}$
  - The potential energy of the electric dipole is
  - $U = -\vec{p} \cdot \vec{E}$
- How about the potential energy of a magnetic dipole?
  - The work done by the torque is
  - $U = \int \tau d\theta = \int NIAB \sin\theta d\theta = -\mu B \cos\theta + C$
  - If we chose U=0 at $\theta = \pi/2$, then C=0
  - Thus the potential energy is $U = -\mu B \cos\theta = -\vec{\mu} \cdot \vec{B}$
    - Very similar to the electric dipole

Example 27 – 8

Magnetic moment of a hydrogen atom. Determine the magnetic dipole moment of the electron orbiting the proton of a hydrogen atom, assuming (in th eBohr model) it is in its ground state with a circular orbit of radius 0.529×10^{-10}m.

What provides the centripetal force? Coulomb force

So we can obtain the speed of the electron from

$$F = \frac{e^2}{4\pi\varepsilon_0 r^2} = \frac{mv^2}{r}$$

Solving for v

$$v = \sqrt{\frac{e^2}{4\pi\varepsilon_0 mr}} = \sqrt{\frac{\left(8.99\times10^9\,N\cdot m^2/C^2\right)\cdot\left(1.6\times10^{-19}\,C\right)^2}{\left(9.1\times10^{-31}\,kg\right)\cdot\left(0.529\times10^{-10}\,m\right)}} = 2.19\times10^6\ m/s$$

Since current is the charge that passes given point per unit time, we can obtain the current

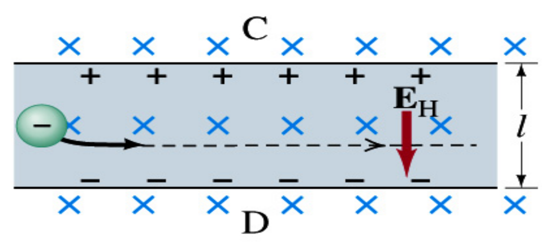
$$I = \frac{e}{T} = \frac{ev}{2\pi r}$$

Since the area of the orbit is $A=\pi r^2$, we obtain the hydrogen magnetic moment

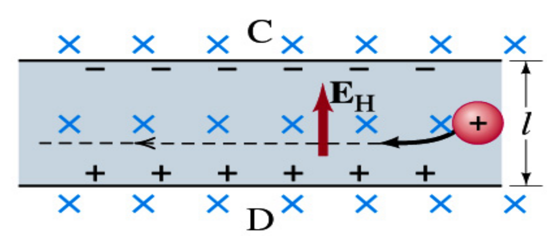
$$\mu = IA = \frac{ev}{2\pi r}\pi r^2 = \frac{evr}{2} = \frac{er}{2}\sqrt{\frac{e^2}{4\pi\varepsilon_0 mr}} = \frac{e^2}{4}\sqrt{\frac{r}{\pi\varepsilon_0 m}}$$

The Hall Effect

- What do you think will happen to the electrons flowing through a conductor immersed in a magnetic field?
  - Magnetic force will push the electrons toward one side of the conductor. Then what happens?
    - $\vec{F}_B = -e\vec{v}_d \times \vec{B}$
  - A potential difference will be created due to continued accumulation of electrons on one side. Till when? Forever?
  - Nope. Till the electric force inside the conductor is equal and opposite to the magnetic force



(a)

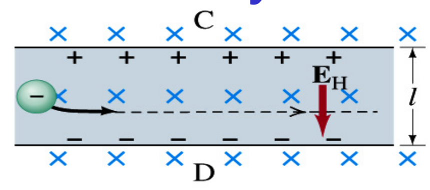


(b)

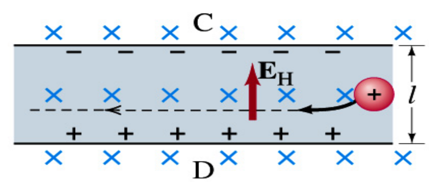
- This is called the **Hall Effect**
  - The potential difference produced is called
    - The Hall emf
  - The electric field due to the separation of charge is called the Hall field, E_H and points to the direction opposite to the magnetic force

The Hall Effect

- In equilibrium, the force due to Hall field is balanced by the magnetic force ev_dB, so we obtain
- $eE_H = ev_d B$ and $E_H = v_d B$
- The Hall emf is then $\varepsilon_H = E_H l = v_d Bl$
  - Where l is the width of the conductor



(a)



(b)

- What do we use the Hall effect for?
  - The current of negative charge moving to the right is equivalent to the positive charge moving to the left
  - The Hall effect can distinguish these since the direction of the Hall field or direction of the Hall emf is opposite
  - Since the magnitude of the Hall emf is proportional to the magnetic field strength ➔ can measure the b-field strength
    - Hall probe

Sources of Magnetic Field

- We have learned so far about the effects of magnetic field on electric currents and moving charge
- We will now learn about the dynamics of magnetism
- How to determine magnetic field strengths in certain situations?
- How two wires with electric current interacts?
- A general approach to finding the connection between current and magnetic field?

Magnetic Field due to a Straight Wire

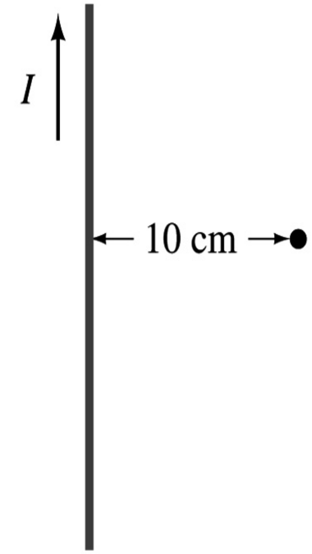
- The magnetic field due to a current flowing a straight wire forms a circular pattern around the wire
  - What do you imagine the strength of the field is as a function of the distance from the wire?
    - It must be weaker as the distance increases
  - How about as a function of current?
    - Directly proportional to the current
  - Indeed, the above are experimentally verified $B \propto \frac{I}{r}$
    - This is valid as long as r << the length of the wire
  - The proportionality constant is $\mu_0/2\pi$, thus the field strength becomes

$$B = \frac{\mu_0 I}{2\pi r}$$

  - μ_0 is the permeability of free space $\mu_0 = 4\pi \times 10^{-7}\ T \cdot m/A$

Example 28 – 1

Calculation of B near wire. A vertical electric wire in the wall of a building carries a dc current of 25A upward. What is the magnetic field at a point 10cm due north of this wire?



Using the formula for the magnetic field near a straight wire

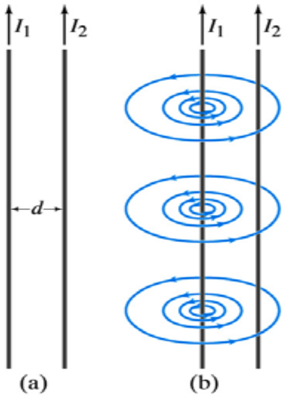
$$B = \frac{\mu_0 I}{2\pi r}$$

So we can obtain the magnetic field at 10cm away as

$$B = \frac{\mu_0 I}{2\pi r} = \frac{\left(4\pi \times 10^{-7}\ T \cdot m/A\right) \cdot \left(25A\right)}{\left(2\pi\right) \cdot \left(0.01m\right)} = 5.0 \times 10^{-5}\,T$$

Force Between Two Parallel Wires

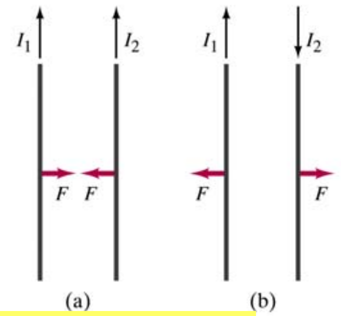
- We have learned that a wire carrying a current produces magnetic field
- Now what do you think will happen if we place two current carrying wires next to each other?
  - They will exert force onto each other. Repel or attract?
  - Depending on the direction of the currents



- This was first pointed out by Ampére.
- Let's consider two long parallel conductors separated by a distance d, carrying currents I_1 and I_2.
- At the location of the second conductor, the magnitude of the magnetic field produced by I_1 is $$B_1 = \frac{\mu_0 I_1}{2\pi d}$$

Force Between Two Parallel Wires

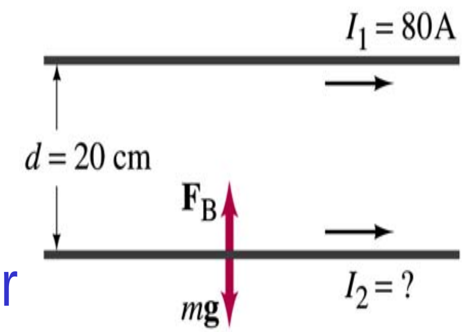
- The force F by a magnetic field B_1 on a wire of length l, carrying a current I_2 when the field and the current are perpendicular to each other is: $F = I_2 B_1 l$
  - So the force per unit length is $\dfrac{F}{l} = I_2 B_1$
  - This force is only due to the magnetic field generated by the wire carrying the current I1
    - There is the force exerted on the wire carrying the current I1 by the wire carrying current I2 of the same magnitude but in opposite direction
- So the force on unit length is $\dfrac{F}{l} = \dfrac{\mu_0}{2\pi}\dfrac{I_1 I_2}{d}$
- How about the direction of the force?



If the currents are in the same direction, the attractive force. If opposite, repulsive.

Example 28 – 2

Suspending a current with a current. A horizontal wire carries a current I_1=80A dc. A second parallel wire 20cm below it must carry how much current I_2 so that it doesn't fall due to the gravity? The lower has a mass of 0.12g per meter of length.



Which direction is the gravitational force? Downward

This force must be balanced by the magnetic force exerted on the wire by the first wire.
$$\frac{F_g}{l} = \frac{mg}{l} = \frac{F_M}{l} = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d}$$

Solving for I_2

$$I_2 = \frac{mg}{l}\frac{2\pi d}{\mu_0 I_1} =$$

$$\frac{2\pi\left(9.8\,m/s^2\right)\cdot\left(0.12\times10^{-3}\,kg\right)\cdot\left(0.20m\right)}{\left(4\pi\times10^{-7}\,T\cdot m/A\right)\cdot\left(80A\right)} = 15A$$