PHYS 1444 – Section 003
Lecture #17

Monday, Oct. 31, 2005
Dr. Jaehoon Yu

- Example for Magnetic force between two parallel wires
- Ampére's Law
- Solenoid and Toroid Magnetic Field
- Biot-Savart Law

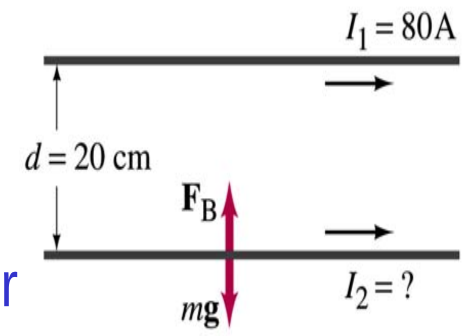
Today's homework is homework #9, due noon, next Thursday!!

Announcements

- Reading assignments
  - CH28 – 7, 28 – 8, and 28 – 10
- The 2nd term exam
  - Date: Monday, Nov. 7
  - Time: 1 – 2:20pm
  - Location: SH 103
  - Coverage: CH 26 – whichever chapter we get to by Wednesday, Nov. 2
- Your textbooks
  - UTA bookstore agreed to exchange your books with the ones that has complete chapters
    - You need to provide a proof of purchase
      - Receipts, copy of cancelled checks, credit card statement, etc.

Example 28 – 2

Suspending a current with a current. A horizontal wire carries a current I_1=80A dc. A second parallel wire 20cm below it must carry how much current I_2 so that it doesn't fall due to the gravity? The lower has a mass of 0.12g per meter of length.



Which direction is the gravitational force? Downward

This force must be balanced by the magnetic force exerted on the wire by the first wire.

$$\frac{F_g}{l} = \frac{mg}{l} = \frac{F_M}{l} = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d}$$

Solving for I_2

$$I_2 = \frac{mg}{l}\frac{2\pi d}{\mu_0 I_1} =$$

$$\frac{2\pi\left(9.8\,m/s^2\right)\cdot\left(0.12\times 10^{-3}\,kg\right)\cdot(0.20m)}{\left(4\pi\times 10^{-7}\,T\cdot m/A\right)\cdot(80A)} = 15A$$

Which direction should the current flow? The same direction as I_1.

Operational Definition of Ampere and Coulomb

- The permeability of free space is defined to be exactly

$$\mu_0 = 4\pi \times 10^{-7}\ T \cdot m/A$$

- The unit of current, ampere, is defined using the definition of the force between two wires each carrying 1A of current and separated by 1m

$$\frac{F}{l} = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d} = \frac{4\pi \times 10^{-7}\ T \cdot m/A}{2\pi}\frac{1A \cdot 1A}{1m} = 2 \times 10^{-7}\ N/m$$

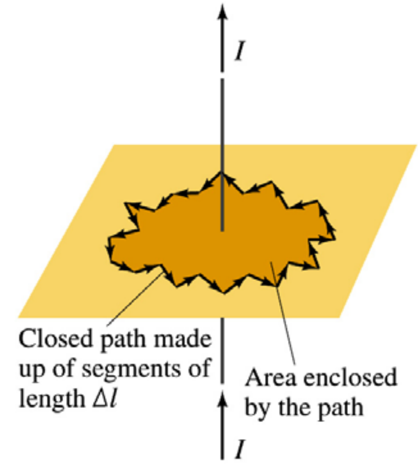
  – So 1A is defined as: the current flowing each of two long parallel conductors 1m apart, which results in a force of exactly 2×10^{-7}N/m.

- Coulomb is then defined as exactly 1C=1A.s.
- We do it this way since current is measured more accurately and controlled more easily than charge.

Ampére's Law

- What is the relationship between magnetic field strength and the current? $B = \frac{\mu_0 I}{2\pi r}$
  - Does this work in all cases?
    - Nope!
    - OK, then when?
    - Only valid for a long straight wire
- Then what would be the more generalized relationship between the current and the magnetic field for any shape of the wire?
  - French scientist André Marie Ampére proposed such a relationship soon after Oersted's discovery

Ampére's Law



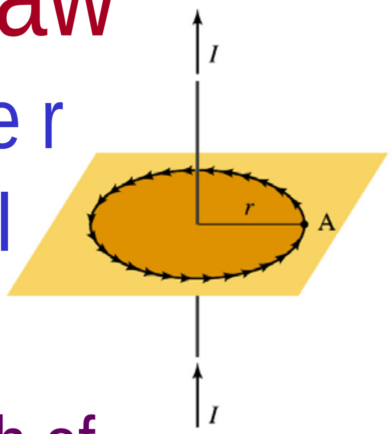
- Let's consider an arbitrary closed path around the current as shown in the figure.
  - Let's split this path with small segments each of Δl long.
  - The sum of all the products of the length of each segment and the component of B parallel to that segment is equal to μ_0 times the net current I_{encl} that passes through the surface enclosed by the path
  - $$\sum B_{||} \Delta l = \mu_0 I_{encl}$$
  - In the limit $\Delta l \rightarrow 0$, this relation becomes
  - $$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$$

Ampére's Law

Looks very similar to a law in the electricity. Which law is it?

Gauss' Law

Verification of Ampére's Law



- Let's find the magnitude of B at a distance r away from a long straight wire w/ current I
  - This is a verification of Ampere's Law
  - We can apply Ampere's law to a circular path of radius r.

$$\mu_0 I_{encl} = \oint \vec{B} \cdot d\vec{l} = \oint B dl = B \oint dl = 2\pi r B$$

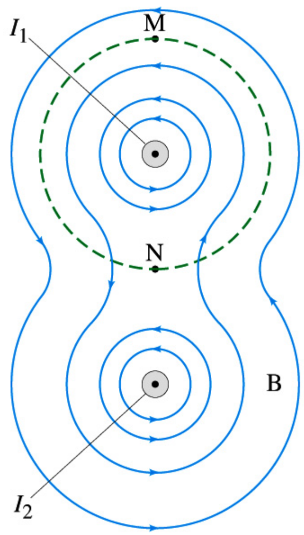
Solving for $\mathcal{B}$

$$B = \frac{\mu_0 I_{encl}}{2\pi r} = \frac{\mu_0}{2\pi} \frac{I}{r}$$

  - We just verified that Ampere's law works in a simple case
  - Experiments verified that it works for other cases too
  - The importance, however, is that it provides means to relate magnetic field to current

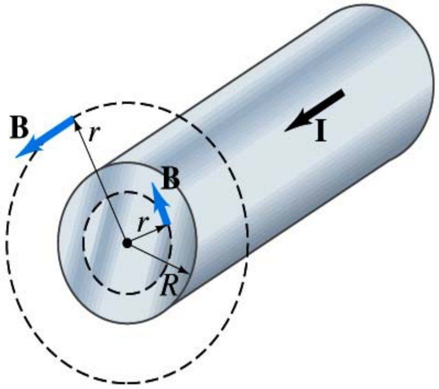
Verification of Ampére's Law

- Since Ampere's law is valid in general, B in Ampere's law is not just due to the current I_{encl}.
- B is the field at each point in space along the chosen path due to all sources
  - Including the current I enclosed by the path but also due to any other sources
  - How do you obtain B in the figure at any point?
    - Vector sum of the field by the two currents
  - The result of the closed path integral in Ampere's law for green dashed path is still $\mu_0 I_1$. Why?
  - While B in each point along the path varies, the integral over the closed path still comes out the same whether there is the second wire or not.



Example 28 – 4

Field inside and outside a wire. A long straight cylindrical wire conductor of radius R carries a current I of uniform current density in the conductor. Determine the magnetic field at (a) points outside the conductor (r>R) and (b) points inside the conductor (r<R). Assume that r, the radial distance from the axis, is much less than the length of the wire. (c) If R=2.0mm and I=60A, what is B at r=1.0mm, r=2.0mm and r=3.0mm?



Since the wire is long, straight and symmetric, the field should be the same at any point the same distance from the center of the wire.

Since B must be tangent to circles around the wire, let's choose a circular path of closed-path integral outside the wire (r>R). What is I_{encl}? $I_{encl} = I$

So using Ampere's law

$$\mu_0 I = \oint \vec{B}\cdot d\vec{l} = 2\pi r B$$

Solving for B

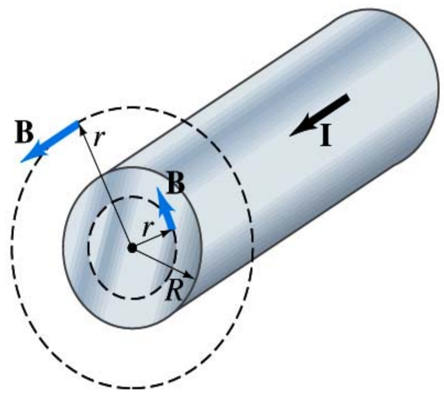
$$B = \frac{\mu_0}{2\pi}\frac{I}{r}$$

Example 28 – 4

For r<R, the current inside the closed path is less than I.

How much is it?

$$I_{encl} = I\frac{\pi r^2}{\pi R^2} = I\left(\frac{r}{R}\right)^2$$



So using Ampere's law

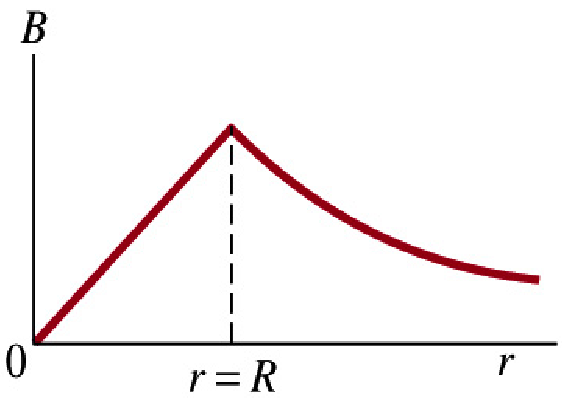
$$\mu_0 I\left(\frac{r}{R}\right)^2 = \oint \vec{B}\cdot d\vec{l} = 2\pi r B$$

Solving for B

$$B = \frac{\mu_0}{2\pi}\frac{I}{r}\left(\frac{r}{R}\right)^2 = \frac{\mu_0}{2\pi}\frac{Ir}{R^2}$$

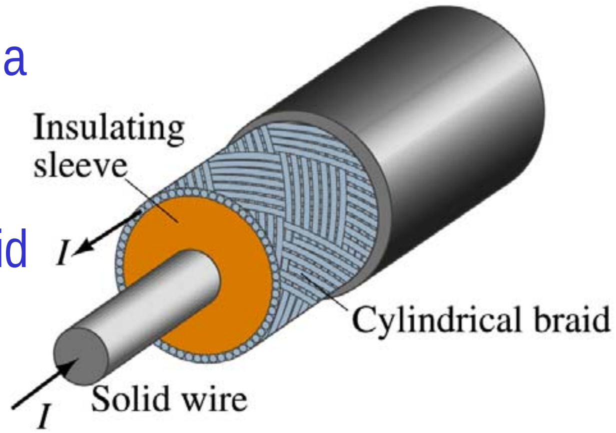
What does this mean?

The field is 0 at r=0 and increases linearly as a function of the distance from the center of the wire up to r=R then decreases as 1/r beyond the radius of the conductor.



Example 28 – 5

Coaxial cable. A coaxial cable is a single wire surrounded by a cylindrical metallic braid, as shown in the figure. The two conductors are separated by an insulator. The central wire carries current to the other end of the cable, and the outer braid carries the return current and is usually considered ground. Describe the magnetic field (a) in the space between the conductors and (b) outside the cable.



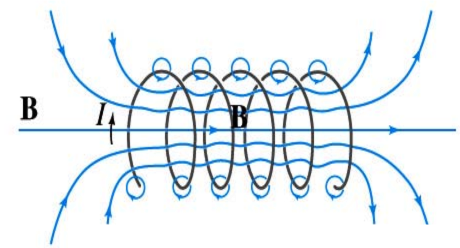
(a) The magnetic field between the conductors is the same as the long, straight wire case since the current in the outer conductor does not impact the enclosed current.

$$B = \frac{\mu_0}{2\pi}\frac{I}{r}$$

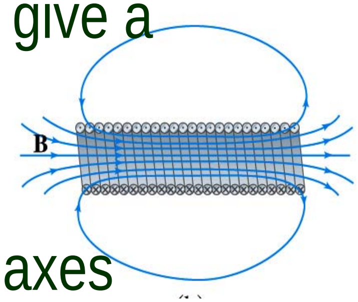
(b) Outside the cable, we can draw a similar circular path, since we expect the field to have a circular symmetry. What is the sum of the total current inside the closed path? $I_{encl} = I - I = 0.$

So there is no magnetic field outside a coaxial cable. In other words, the coaxial cable self-shields. The outer conductor also shields against an external electric field. Cleaner signal and less noise.

Solenoid and Its Magnetic Field



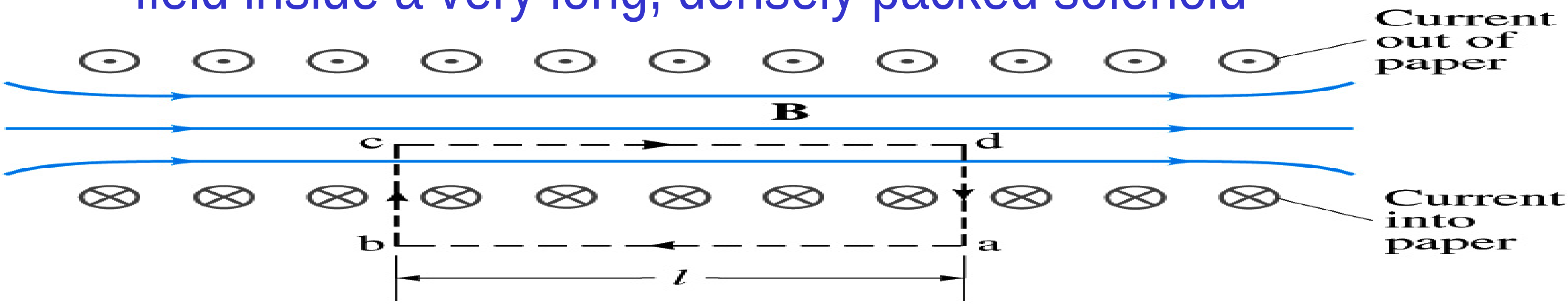
- What is a solenoid?
  - A long coil of wire consisting of many loops
  - If the space between loops are wide
    - The field near the wires are nearly circular
    - Between any two wires the fields due to each loop cancel
    - Toward the center of the solenoid, the fields add up to give a field that can be fairly large and uniform



  - For a long, densely packed loops
    - The field is nearly uniform and parallel to the solenoid axes within the entire cross section
    - The field outside the solenoid is very small compared to the field inside, except the ends
      - The same number of field lines spread out to an open space

Solenoid Magnetic Field

- Now let's use Ampere's law to determine the magnetic field inside a very long, densely packed solenoid



- Let's choose the path abcd, far away from the ends
  - We can consider four segments of the loop for integral
  - $\oint \vec{B}\cdot d\vec{l} = \int_a^b \vec{B}\cdot d\vec{l} + \int_b^c \vec{B}\cdot d\vec{l} + \int_c^d \vec{B}\cdot d\vec{l} \quad \int_d^a \vec{B}\cdot d\vec{l}$
  - The field outside the solenoid is negligible. So the integral on a→b is 0.
  - Now the field B is perpendicular to the bc and da segments. So these integrals become 0, also.

Solenoid Magnetic Field

– So the sum becomes: $\oint \vec{B} \cdot d\vec{l} = \int_c^d \vec{B} \cdot d\vec{l} = Bl$

– If a current I flows in the wire of the solenoid, the total current enclosed by the closed path is NI

  • Where N is the number of loops (or turns of the coil) enclosed

– Thus Ampere's law gives us $Bl = \mu_0 NI$

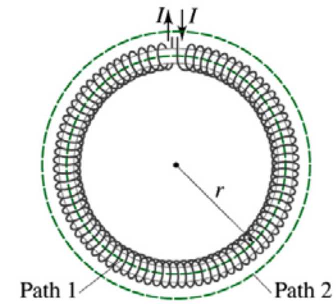
– If we let n=N/l be the number of loops per unit length, the magnitude of the magnetic field within the solenoid becomes

– $$B = \mu_0 nI$$

  • B depends on the number of loops per unit length, n, and the current

  • But does not depend on the position within the solenoid but uniform inside it, very similar to a bar of magnet

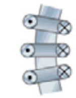
Example 28 – 8



(a)

Toroid. Use Ampere's law to determine the magnetic field (a) inside and (b) outside a toroid, which is like a solenoid bent into the shape of a circle.

(a) How do you think the magnetic field lines inside the toroid look?


(b)

Since it is a bent solenoid, it should be a circle concentric with the toroid.

If we choose path of integration one of these field lines of radius r inside the toroid, path 1, to use the symmetry of the situation, making B the same at all points on the path. So from Ampere's law

$$\oint \vec{B} \cdot d\vec{l} = B(2\pi r) = \mu_0 I_{encl} = \mu_0 NI$$

Solving for B

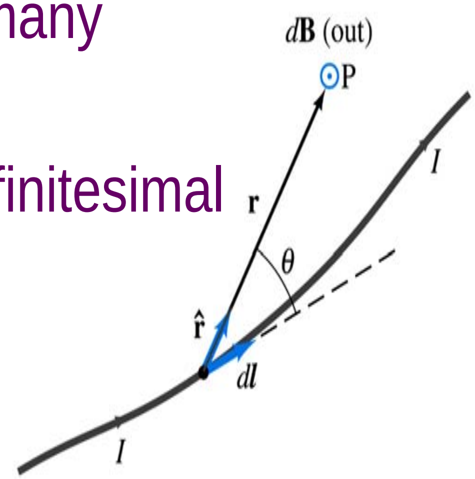
$$B = \frac{\mu_0 NI}{2\pi r}$$

So the magnetic field inside a toroid is not uniform. It is larger on the inner edge. However, the field will be uniform if the radius is large and the toroid is thin and $B = \mu_0 nI$.

(b) Outside the solenoid, the field is 0 since the net enclosed current is 0.

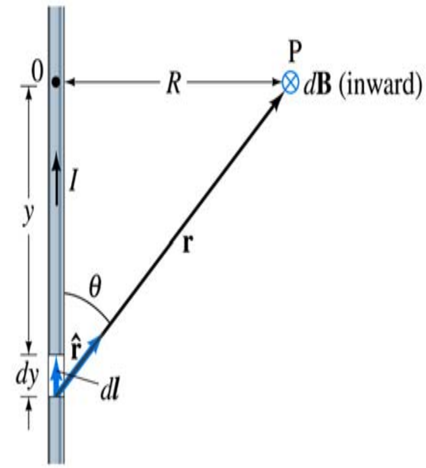
Biot-Savart Law

- Ampere's law is useful in determining magnetic field utilizing symmetry
- But sometimes it is useful to have another method of using infinitesimal current segments for B field
  - Jean Baptiste Biot and Feilx Savart developed a law that a current I flowing in any path can be considered as many infinitesimal current elements
  - The infinitesimal magnetic field d**B** caused by the infinitesimal length $d\ell$ that carries current I is
  - $$d\vec{B} = \frac{\mu_0 I}{4\pi}\frac{d\vec{l} \times \hat{r}}{r^2}$$ **Biot-Savart Law**
    - **r** is the displacement vector from the element $d\ell$ to the point P
    - Biot-Savart law is the magnetic equivalent to Coulomb's law



B field in Biot-Savart law is only that by the current nothing else.

Example 28 – 9



B due to current I in straight wire. For the field near a long straight wire carrying a current I, show that the Biot-Savarat law gives the same result as the simple long straight wire, $B=\mu_0 I/2\pi R$.

What is the direction of the field **B** at point P? Going into the page.

All dB at point P has the same direction based on right-hand rule.

The magnitude of B using Biot-Savart law is

$$B=\oint dB=\frac{\mu_0 I}{4\pi}\int_{-\infty}^{+\infty}\frac{d\vec{l}\times\hat{r}}{r^2}=\frac{\mu_0 I}{4\pi}\int_{y=-\infty}^{+\infty}\frac{dy\sin\theta}{r^2}$$

Where $dy=d\ell$ and $r^2=R^2+y^2$ and since $y=-R\cot\theta$ we obtain

$$dy=+R\csc^2\theta d\theta=\frac{Rd\theta}{\sin^2\theta}=\frac{Rd\theta}{(R/r)^2}=\frac{r^2d\theta}{R}$$

Integral becomes

$$B=\frac{\mu_0 I}{4\pi}\int_{y=-\infty}^{+\infty}\frac{dy\sin\theta}{r^2}=\frac{\mu_0 I}{4\pi}\frac{1}{R}\int_{\theta=0}^{\pi}\sin\theta d\theta=-\frac{\mu_0 I}{4\pi}\frac{1}{R}\cos\theta\Big|_0^{\pi}=\frac{\mu_0 I}{2\pi}\frac{1}{R}$$

The same as the simple, long straight wire!! It works!!