PHYS 1444 – Section 003
Lecture #19

Monday, Nov. 14, 2005
Dr. Jaehoon Yu

- Electric Generators
- DC Generator
- Eddy Currents
- Transformer
- Mutual Inductance

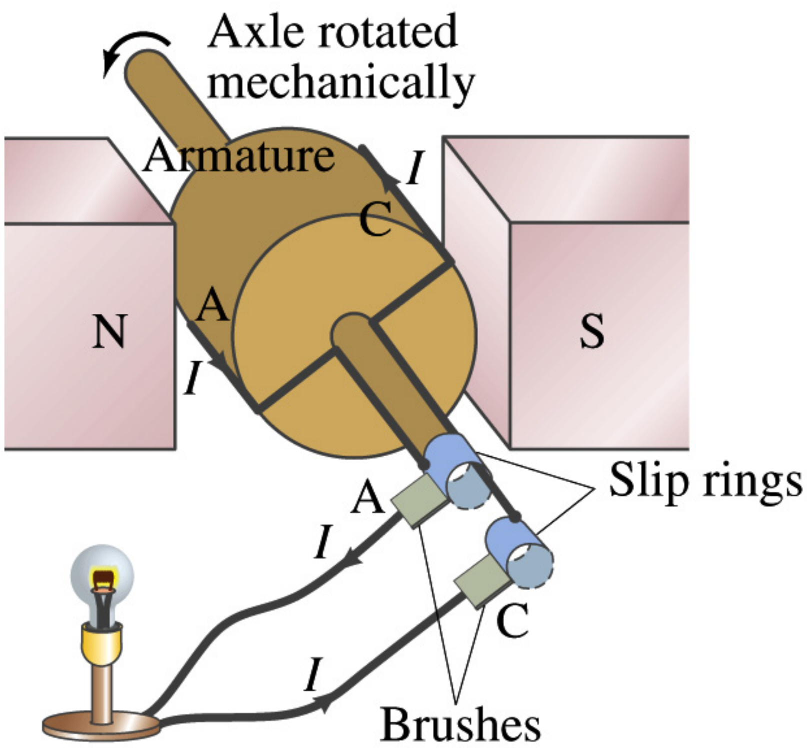
Today's homework is homework #10, due noon, next Tuesday!!

Announcements

- Instructor evaluation today
- A colloquium at 4pm this Wednesday
  - Dr. P. Nordlander from Rice University
  - About nano material and magnetic field they generate
  - Extra credit opportunity

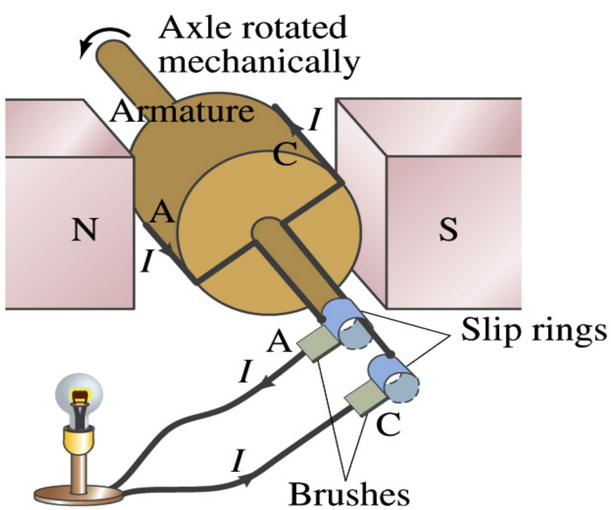
Electric Generators

- What does a generator do?
  - Transforms mechanical energy into the electrical energy
  - What does this look like?
    - An inverse of an electric motor which transforms electrical energy to mechanical energy
  - An electric generator is also called a dynamo



- Whose law does the generator based on?
  - Faraday's law of induction

How does an Electric Generator work?

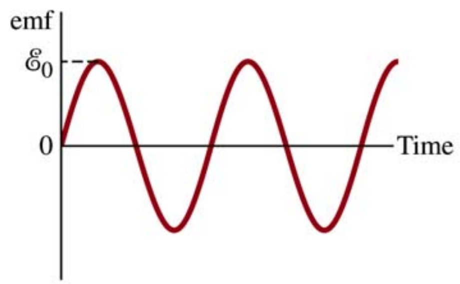


- An electric generator consists of
  - Many coils of wires wound on an armature that can rotate by mechanical means in a magnetic field
- An emf is induced in the rotating coil
- Electric current is the output of a generator

- Which direction does the output current flow when the armature rotates counterclockwise?
  - The conventional current flows outward on wire A toward the brush
  - After half the revolution the wire A will be where the wire C is and the current flow on A is reversed
- Thus the current produced is alternating its direction

How does an Electric Generator work?

- Let's assume the loop is rotating in a uniform B field w/ constant angular velocity ω. The induced emf is
- $$\varepsilon = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\int \vec{B}\cdot d\vec{A} = -\frac{d}{dt}\left[BA\cos\theta\right]$$
- What is the variable that changes above?
  - The angle θ. So what is $d\theta/dt$?
    - The angular speed ω.
  - So $\theta=\theta_0+\omega t$
  - If we choose $\theta_0=0$, we obtain
  - $\varepsilon = -BA\frac{d}{dt}\left[\cos\varpi t\right] = BA\varpi \sin\varpi t$
  - If the coil contains N loops: $\varepsilon = -N\frac{d\Phi_B}{dt} = NBA\varpi\sin\varpi t = \varepsilon_0 \sin\varpi t$
  - What is the shape of the output?
    - Sinusoidal w/ amplitude $\varepsilon_0=NBA\omega$



- US ac frequency is 60Hz. Europe is at 50Hz
  - Most the U.S. power is generated at steam plants

Example 29 – 5

An AC generator. The armature of a 60-Hz ac generator rotates in a 0.15-T magnetic field. If the area of the coil is $2.0\times10^{-2}m^2$, how many loops must the coil contain if the peak output is to be ε_0=170V?

The maximum emf of a generator is $\varepsilon_0 = NBA\varpi$

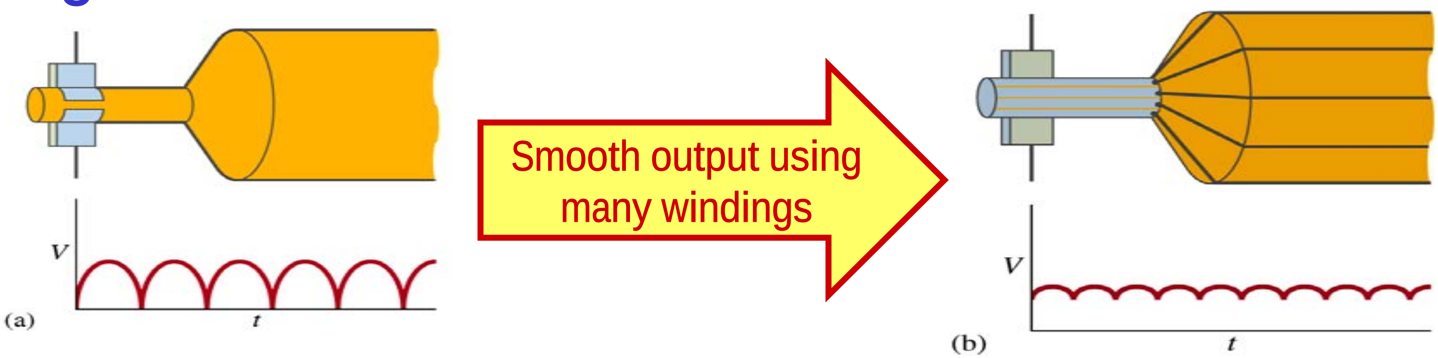
Solving for N $N = \dfrac{\varepsilon_0}{BA\varpi}$

Since $\varpi = 2\pi f$ We obtain

$$N = \frac{\varepsilon_0}{2\pi BAf} = \frac{170V}{2\pi \cdot (0.15T) \cdot \left(2.0\times10^{-2}m^2\right) \cdot \left(60s^{-1}\right)} = 150 turns$$

A DC Generator

- A DC generator is almost the same as an ac generator except the slip rings are replaced by split-ring commutators



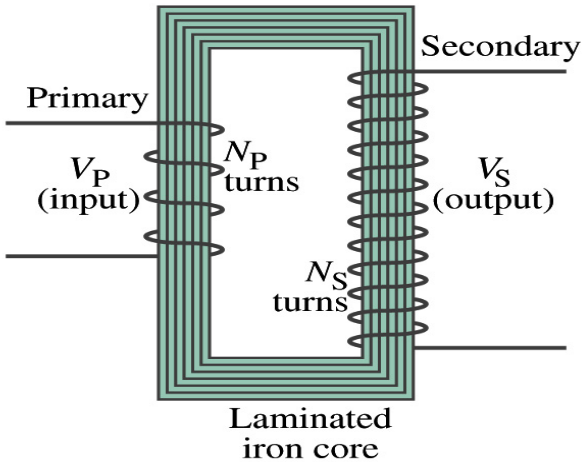
- Output can be smoothed out by placing a capacitor on the output
  - More commonly done using many armature windings

Eddy Currents (read more in 29-5)

- Induced currents are not always confined to well-defined path
- In some cases where a conductor is moving in and out of the magnetic field, the Lenz's law causes flow of electrons that opposes the change in magnetic flux
  - This change is in the direction that impedes the production of emf
  - And thus causes energy losses
- These currents are called eddy currents
  - Just like the eddy currents in the water that pulls the boat in the opposite direction of the movement

Transformer

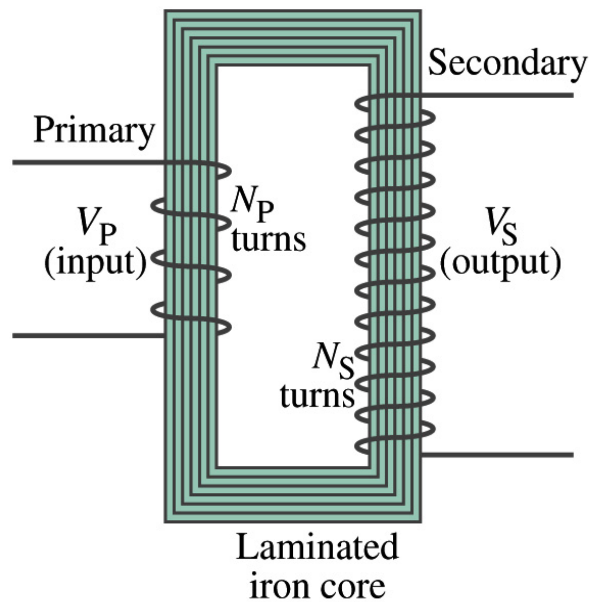
- What is a transformer?
  - A device for increasing or decreasing an ac voltage
  - A few examples?
    - TV sets to provide HV to picture tubes, portable electronic device converters, transformers on the pole, etc
- A transformer consists of two coils of wires known as primary and secondary
  - The two coils can be interwoven or linked by a laminated soft iron core to reduce eddy current losses
- Transformers are designed so that all magnetic flux produced by the primary coil pass through the secondary



How does a transformer work?

- When an ac voltage is applied to the primary, the changing B it produces will induce voltage of the same frequency in the secondary
- So how would we make the voltage different?
  - By varying the number of loops in each coil
  - From Faraday's law, the induced emf in the secondary is
  - $V_S = N_S \frac{d\Phi_B}{dt}$
  - The input primary voltage is
  - $V_P = N_P \frac{d\Phi_B}{dt}$
  - Since $d\Phi_B/dt$ is the same, we obtain
  - $\boxed{\frac{V_S}{V_P} = \frac{N_S}{N_P}}$

Transformer Equation



Transformer Equation

- The transformer equation does not work for dc current since there is no change of magnetic flux
- If $N_S > N_P$, the output voltage is greater than the input so it is called a step-up transformer while $N_S < N_P$ is called step-down transformer
- Now, it looks like energy conservation is violated since we can get more emf from smaller ones, right?
  - Wrong! Wrong! Wrong! Energy is always conserved!
  - A well designed transformer can be more than 99% efficient
  - The power output is the same as the input:
  - $V_P I_P = V_S I_S$
  - $\boxed{\dfrac{I_S}{I_P} = \dfrac{V_P}{V_S} = \dfrac{N_P}{N_S}}$

Example 29 – 8

Portable radio transformer. A transformer for home use of a portable radio reduces 120-V ac to 9.0V ac. The secondary contains 30 turns, and the radio draws 400mA. Calculate (a) the number of turns in the primary; (b) the current in the primary; and (c) the power transformed.

(a) What kind of a transformer is this? A step-down x-former

Since $\frac{V_P}{V_S}=\frac{N_P}{N_S}$ We obtain $N_P=N_S\frac{V_P}{V_S}=30\frac{120V}{9V}=400turns$

(b) Also from the transformer equation $\frac{I_S}{I_P}=\frac{V_P}{V_S}$ We obtain

$$I_P=I_S\frac{V_S}{V_P}=0.4A\frac{9V}{120V}=0.03A$$

(c) Thus the power transformed is

$$P=I_SV_S=(0.4A)\cdot(9V)=3.6W$$

How about the input power? The same assuming 100% efficiency.

Example 29 – 9: Power Transmission

Transmission lines. An average of 120kW of electric power is sent to a small town from a power plant 10km away. The transmission lines have a total resistance of 0.4Ω. Calculate the power loss if the power is transmitted at (a) 240V and (b) 24,000V.

We cannot use $P=V^2/R$ since we do not know the voltage along the transmission line. We, however, can use $P=I^2R$.

(a) If 120kW is sent at 240V, the total current is

$$I=\frac{P}{V}=\frac{120\times10^3}{240}=500A.$$

Thus the power loss due to transmission line is

$$P=I^2R=(500A)^2\cdot(0.4\Omega)=100kW$$

(b) If 120kW is sent at 24,000V, the total current is

$$I=\frac{P}{V}=\frac{120\times10^3}{24\times10^3}=5.0A.$$

Thus the power loss due to transmission line is

$$P=I^2R=(5A)^2\cdot(0.4\Omega)=10W$$

The higher the transmission voltage, the smaller the current, causing less loss of energy. This is why power is transmitted w/ HV, as high as 170kV.

Electric Field due to Magnetic Flux Change

- When electric current flows through a wire, there is an electric field in the wire that moves electrons
- We saw, however, that changing magnetic flux induces a current in the wire. What does this mean?
  - There must be an electric field induced by the changing magnetic flux.
- In other words, a changing magnetic flux produces an electric field
- This results apply not just to wires but to any conductor or any region in space

Generalized Form of Faraday's Law

- Recall the relation between electric field and the potential difference $V_{ab} = \int_a^b \vec{E} \cdot d\vec{l}$
- Induced emf in a circuit is equal to the work done per unit charge by the electric field
- $\varepsilon = \oint \vec{E} \cdot d\vec{l}$
- So we obtain

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

- The integral is taken around a path enclosing the area through which the magnetic flux Φ_B is changing.

Inductance

- Changing magnetic flux through a circuit induce an emf in that circuit
- An electric current produces a magnetic field
- From these, we can deduce
  - A changing current in one circuit must induce an emf in a nearby circuit ➔ Mutual inductance
  - Or induce an emf in itself ➔ Self inductance

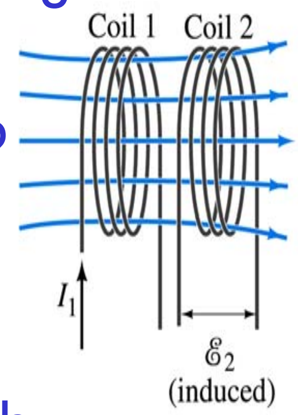
Mutual Inductance

- If two coils of wire are placed near each other, a changing current in one will induce an emf in the other.
- What does the induced emf, ε_2, in coil2 proportional to?
  - Rate of the change of the magnetic flux passing through it
- This flux is due to current I_1 in coil 1
- If Φ_{21} is the magnetic flux in each loop of coil2 created by coil1 and N_2 is the number of closely packed loops in coil2, then $N_2\Phi_{21}$ is the total flux passing through coil2.
- If the two coils are fixed in space, $N_2\Phi_{21}$ is proportional to the current I_1 in coil 1. The proportionality constant for this is called the Mutual Inductance and defined by

$$\boxed{M_{21} = \frac{N_2\Phi_{21}}{I_1}}$$

- The emf induced in coil2 due to the changing current in coil1 is

$$\boxed{\varepsilon_2 = -N_2\frac{d\Phi_{21}}{dt} = -\frac{d\left(N_2\Phi_{21}\right)}{dt} = -M_{21}\frac{dI_1}{dt}}$$

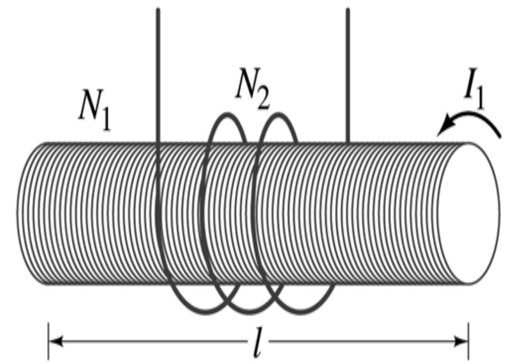


Mutual Inductance

- The mutual induction of coil2 with respect to coil1, M_{21},
  - is a constant and does not depend on I1.
  - depends on "geometric" factors such as the size, shape, number of turns and relative position of the two coils, and whether a ferromagnetic material is present
    - The farther apart the two coils are the less flux can pass through coil, 2, so M_{21} will be less.
  - Most cases mutual inductance are determined experimentally
- Conversely, the changing current in coil2 will induce an emf in coil1
- $$\varepsilon_1 = -M_{12}\frac{dI_2}{dt}$$
  - M_{12} is the mutual inductance of coil1 with respect to coil2 and $M_{12} = M_{21}$
  - We can put $M=M_{12}=M_{21}$ and obtain $\varepsilon_1 = -M\frac{dI_2}{dt}$ and $\varepsilon_2 = -M\frac{dI_1}{dt}$
  - SI unit for mutual inductance is henry (H) $1H = 1V\cdot s/A = 1\Omega\cdot s$

Example 30 – 1

Solenoid and coil. A long thin solenoid of length ℓ and cross-sectional area A contains N_1 closely packed turns of wire. Wrapped around it is an insulated coil of N_2 turns. Assume all the flux from coil1 (the solenoid) passes through coil2, and calculate the mutual inductance.



First we need to determine the flux produced by the solenoid.

What is the magnetic field inside the solenoid? $$B=\frac{\mu_0 N_1 I_1}{l}$$

Since the solenoid is closely packed, we can assume that the field lines are perpendicular to the surface area of the coils 2. Thus the flux through coil2 is

$$\Phi_{21}=BA=\frac{\mu_0 N_1 I_1}{l}A$$

Thus the mutual inductance of coil2 is

$$M_{21}=\frac{N_2\Phi_{21}}{I_1}=\frac{N_2}{I_1}\frac{\mu_0 N_1 I_1}{l}A=\frac{\mu_0 N_1 N_2}{l}A$$

Note that M_{21} only depends on geometric factors!