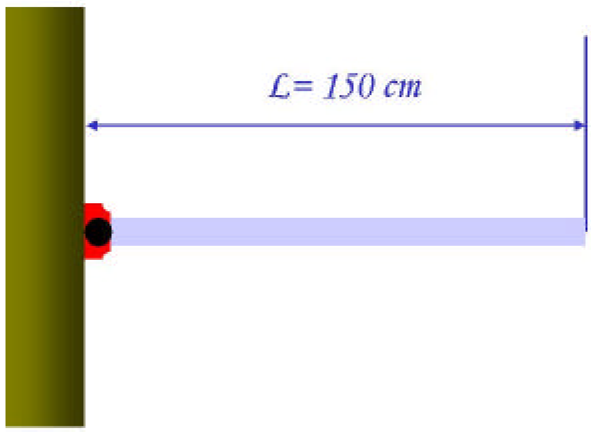
Name:
ID:

[1 – 20 points] A uniform rod of length 150 cm and mass 900g is attached at one end to a frictionless pivot and is free to rotate about the pivot in the vertical plane, as shown in the figure. Answer the following series of questions, assuming that the rod is released from rest in the horizontal position, and the magnitude of the gravitational acceleration g is $9.80 m/s^2$.



a) What is the line density of the rod?
   1. $6.00 kg/cm^3$
   2. **0.600kg/m**
   3. 0.600kg/cm
   4. $6.00 kg/m^3$

Solution: Since line density is the density per unit length, the line density for this rod is λ=m/L=0.900/1.50=0.600kg/m.

b) What is the moment of inertia of the rod in this motion?
   1. $0.267 kg \cdot m^2$
   2. $1.35 kg \cdot m$
   3. $1.35 kg \cdot m^2$
   4. $0.675 kg \cdot m^2$ (**4.**)

Solution: Moment of inertia of this rod when rotates about the axis at one end is $I=ML^2/3=0.900x(1.50)^2=0.675 kg.m^2$.

c) What is the potential energy of the rod before the release?
   1. $13.2 kg \cdot m/s$
   2. $13.2 J$
   3. $6.62 J$ (**3.**)
   4. mgh

Solution: Potential energy is mgh. The height of the CM when it is in its equilibrium position is L/2. Therefore the potential energy U=mgh=mgL/2=0.9x9.8x1.5/2=6.62J.

d) What is the angular speed of the rod when the rod reaches the bottom?
   1. $4.43/s$ (**1.**)
   2. $3.38/s$
   3. $6.62 m/s$
   4. $16.3/s$

Turn over

Solution: Since potential energy is transferred to rotational energy, at the bottom the angular speed ω becomes:

$$\frac{1}{2}Iw^2 = mgh = mgL/2$$
$$w = \sqrt{\frac{mgL}{I}} = \sqrt{\frac{13.2}{0.675}} = 4.43/s$$

e) What is the linear velocity of the center of mass when the rod is at its vertical position?

1. $7.66 m/s$
2. **$3.32 m/s$**
3. $6.64/s$
4. $3.38/s$

Solution: Since linear velocity is v=r ω=Lω/2=1.5x4.43/2=3.32m/s.

f) What is the linear speed of the lowest point of the rod when the rod is in its vertical position?

1. **$6.65 m/s$**
2. $2.33/s$
3. $6.78 m/s$
4. $3.32 m/s$

Solution: Since linear velocity is v=r ω=Lω=1.5x4.43=6.64m/s.

g) What is the magnitude of torque?

1. $9.8N$
2. $3.96N \cdot m$
3. **$6.62N \cdot m$**
4. $12.3N \cdot m$

Solution: Since torque is t=r F=L/2xmg=1.5/2x0.9x9.8=6.62N.m.

h) What is the direction of torque in this motion?

1. Upward
2. Out of the page
3. **Into the page**
4. Downward

Solution: Using the right-hand rule, the direction of the torque vector is into the page.

i)What is the angular momentum of the rod when it is at its vertical position?

1. **$2.99 kg \cdot m^2/s$**
2. $3.23 kg \cdot m/s$
3. $9.80 kg \cdot m/s^2$
4. $6.00 kg \cdot m^2/s$

Solution: Since angular momentum is L=Iω=0.675x4.43=2.99kg.m2/s.

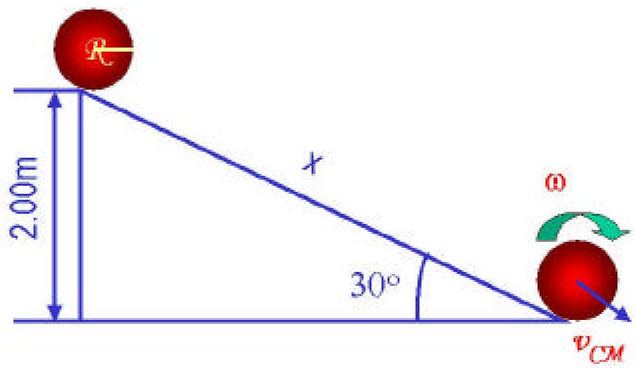
j) What is the direction of angular momentum in this motion?

1. Upward
2. Out of the page
3. **Into the page**
4. Downward

Solution: Using the right-hand rule, the direction of the angular momentum vector is into the page.

Turn over

[2 – 25 points] •A solid sphere of mass 1.00kg and radius R=50.0cm was released at the top of the incline from rest, as shown in the picture, and is rolling down the incline without slipping. . The magnitude of the gravitational acceleration is g=9.8m/s^2. The surface of the incline has friction. Answer the following series of questions.



a) What are the forces involved in this motion? (3 points)
   1. Gravitational, Tangential, and Radial forces
   2. Radial, Frictional, and Tangential forces
   3. Gravitational force only
   4. **Gravitational, Frictional, and normal forces**

b) What is the condition for the sphere to undergo a "pure rolling" motion? (4points)

   1. $\vec{F} = m\vec{a}$ 2. $t = I a$ **3.** $v_{CM} = R v$ 4. $\frac{1}{2} m v_{CM}^2 = mgh$

   Solution: Since pure rolling motion requires the arc length be the same is the distance traveled by the CM at v_{CM}, s=rθ, ds/dt=v_{CM}=rdθ/dt=rω.

c) What is the magnitude of the gravitational force acting on the direction of sphere's movement. (2 points)
   1. 49.0 N **2. 4.90N**
   3. 9.80N 4. 5.66N

   Solution: The force component along the incline is F=mgcos60=4.90N

d) Which direction is the gravitational acceleration in this motion? (2 points)
   1. 30° upward to horizontal
   2. **Downward**
   3. Horizontal
   4. Up before the middle of the incline and down past the middle

Solution: Gravitational acceleration is always downward on the surface of the Earth.

e) What is the potential energy of the sphere before it is released, ignoring the radius of the sphere? (2 points)
   1. 9.80J **2.** 19.6J
   3. 39.2J 4. 19.6$kg \cdot m/s^2$

Turn over

Solution: U=mgh=1.00x9.8x2.00=19.6J

f) What is the linear velocity of the center of mass of the sphere at the bottom of the incline? Use the moment of inertia of a solid sphere given in the formula (4 points)

1. $28.0 m/s$
2. **2.** $5.29 m/s$
3. $39.2 m/s$
4. $6.26 m/s$

Solution: Since the potential energy is completely converted to kinetic and rotational energy. Therefore

$$\frac{1}{2}mv_{CM}^2 + \frac{1}{2}Iw^2 = \frac{1}{2}\left(mv_{CM}^2 + \frac{2}{5}mr^2w^2\right) = \frac{7}{10}mv_{CM}^2 = mgh$$

$$v_{CM} = \sqrt{\frac{10}{7}gh} = \sqrt{\frac{10}{7}9.80 \times 2.00} = 5.29 m/s$$

g) What is the angular speed of the sphere at the bottom of the incline? (4 points)

1. $5.30 m/s$
2. $0.106 /sm$
3. $2.79 /s$
4. **4.** $10.6 /s$

Solution: Since angular speed $\omega = v_{CM}/r = 5.29/0.50 = 10.6/s$.

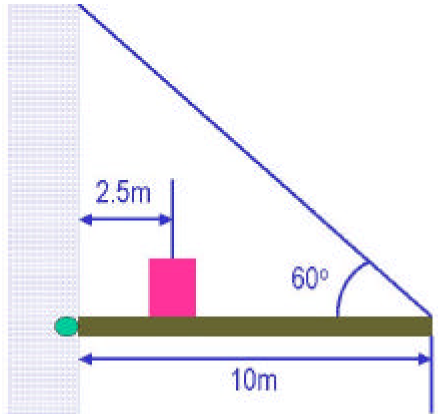
h) What is the total energy of the sphere at the bottom of the hill? (4 points)

1. $14.0 J$
2. $5.62 J$
3. $39.2 J$
4. **4.** $19.6 J$

Solution: Since total energy is the same as the initial potential energy, it is 19.6J.

Turn over

[3 – 20 points] A box of mass 75kg, is put on a steel beam of mass 150kg which is connected to the wall by a frictionless pivot and is supported by a steel cable as shown in the figure. Assuming that entire system is at its equilibrium, and the gravitational acceleration is g=9.8m/s^2, answer the following series of questions.

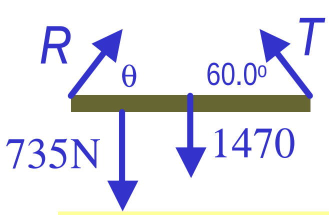


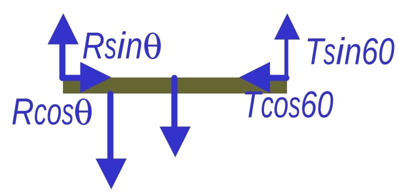
a) What are the weights of the box and the steel beam? (2 points each)

Weight is the gravitational force exerted on the given masses. Therefore,

$$W_{box} = M_{box}g = 75 \times 9.8 = 735N$$
$$W_{beam} = M_{beam}g = 150 \times 9.8 = 1470N$$

b) Draw a free body diagram and write down the sum of net force components and net torque involved in this system. (8 points)





$$\sum F_x = R\cos\theta - T\cos 60.0^\circ = 0$$
$$\sum F_y = R\sin\theta + T\sin 60.0^\circ - 735N - 1470N = 0$$

$$\sum \tau = T\sin 60.0^\circ \times 10.0 - 375N \times 2.50 - 1470N \cdot 5.00 = 0$$

Turn over

c) What is the magnitude of the tension on the cable required to keep the system balanced? (3 points)

From the net torque equation above one can get:

$$\sum t = T\sin 60.0^{\circ} \times 10.00 - 375N \times 2.50 - 1470N \cdot 5.00m = 0$$
$$T = 1061N$$

d) What are the magnitude and the direction of the force exerted by the wall on the beam? (5 points)

Using the net force equations that provides translational equilibrium conditions

$$R\cos q = T\cos 60.0^{\circ} = 1061 \times \cos 60.0^{\circ} = 531.0N$$
$$R\sin q = -T\sin 60.0^{\circ} + 375N + 1470N = 1286N$$
$$q = \tan^{-1}\left(\frac{1845 - 1061 \times \sin 60.0^{\circ}}{1061 \times \cos 60.0^{\circ}}\right) = 67.6^{\circ}$$
$$|R| = \sqrt{(R\cos q)^2 + (R\sin q)^2} = \sqrt{(531.0)^2 + (1286)^2}$$

Turn over

[4 – 18 points] An object of mass 500g is undergoing a motion along the x-axis without friction. Its displacement from the origin varies with time according to the equation;

$$x = -(3.50m)\sin\left(0.5pt + \frac{5p}{4}\right)$$

where t is in seconds, and the angles in the parentheses are in radians. Answer the following questions

a) What kind of motion is this?
1. Simple motion 2. Free Fall
3. Harmonic Oscillation 4. Perfect motion

b) What is the amplitude of this motion?
1. **3.50*m*** 2. 2.00π *m* 3.3.50π *m* 4. 7.00π *m/s*

c) What is the period of this motion?
1. 4.00s 2. 2.00/s 3. 4.00/s 4. 2.00s

d) What is the frequency of this motion?
1. 0.25s 2. 0.50/s **3. 0.25/s** 4. 2.00s

e) What is the linear velocity of the motion?
1. $v = -(3.50)\cos\left(0.5pt + \frac{5p}{4}\right)m/s$ **2.** $v = -(1.75p)\cos\left(0.5pt + \frac{5p}{4}\right)m/s$
3. $v = -(1.75p)\sin\left(0.5pt + \frac{5p}{4}\right)m/s$ 4. $v = (1.75p)\cos\left(0.5pt + \frac{5p}{4}\right)m/s$

f) What is the linear acceleration of the motion?
1. $a = (3.50p^2)\sin\left(0.5pt + \frac{5p}{4}\right)m/s^2$ 2. $a = -(0.88p^2)\cos\left(0.5pt + \frac{5p}{4}\right)m/s^2$
3. $a = (0.88p^2)\sin\left(0.5pt + \frac{5p}{4}\right)m/s^2$ 4. $a = -(3.50p^2)\cos\left(0.5pt + \frac{5p}{4}\right)m/s^2$

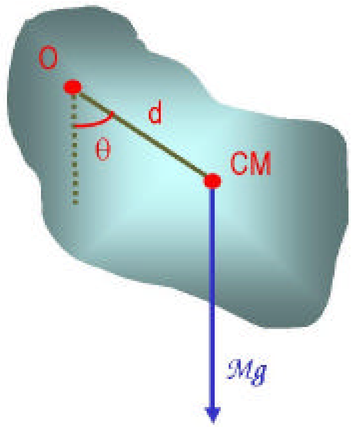
g) What is maximum kinetic energy of the system?
1. $K_{Max} = 7.56J$ 2. $K_{Max} = 74.8J$
3. $K_{Max} = 3.06J$ 4. $K_{Max} = 8.64J$
Solution: Since $K_{max} = kA^2/2$ and k=sqrt(ω^2.m), K_{max}= sqrt(ω^2.m)$(3.5)^2$/2=7.56J

h) What is maximum potential energy of the system?
1. $U_{Max} = 7.56J$ 2. $U_{Max} = 74.8J$
3. $U_{Max} = 3.06J$ 4. $U_{Max} = 8.64J$
Solution: Since the maximum potential energy is the same as the total energy which is again the same as maximum kinetic energy, U_{max}=7.56J

i) What is total mechanical energy of the system?
1. $E = 7.56J$ 2. $E = 15.2J$
3. $E = 3.06J$ 4. $E = 17.3J$
Solution: Since the maximum potential energy is the same as the total energy which is again the same as maximum kinetic energy, E=7.56J

Turn over

[5 – 20 points]A rigid object of mass M is pivoted at the point O. The pivot O is a distance d away from the center of mass of the object, as shown in the figure. Answer the following series of questions, assuming the gravitational acceleration is g and is uniform throughout the body of the object, and the moment of inertial for this object is I.



a) Prove that this motion is a simple harmonic oscillation. (5points)

The net torque exerted on the object by the gravitational force is

$$\sum \tau = -mgd\sin\theta$$

Using the relationship between torque and angular acceleration we can rewrite

$$\sum \tau = I\alpha = I\frac{d^2\theta}{dt^2} = -mgd\sin\theta$$

Since angle θ is small

$$\frac{d^2\theta}{dt^2} = -\frac{mgd}{I}\sin\theta \approx -\left(\frac{mgd}{I}\right)\theta = -\omega^2\theta$$

The above equation shows that the angular acceleration is proportional to angular displacement and is an opposite direction to the change. These two properties satisfy the conditions for harmonic oscillation. Therefore this motion is a simple harmonic oscillation.

b) What is the period, T, of this motion in terms of known quantities?. (5 points)

As shown in the previous solution, the angular frequency ω is

$$\omega = \sqrt{\frac{mgd}{I}}$$

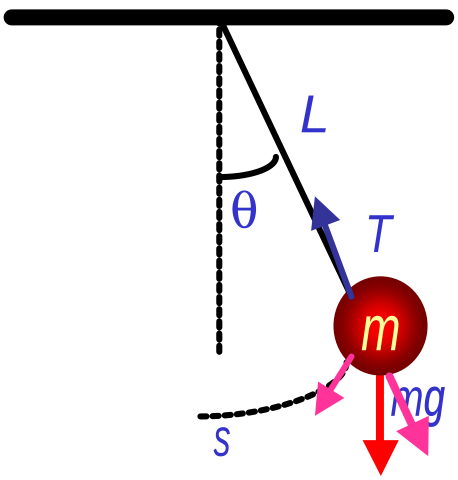
Therefore the period for this harmonic oscillation is

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{mgd}}$$

Turn over

c) Show that this formula for period is also valid for simple pendulum where the bob of mass m is hanging at the end of massless string of length L. (5 points)

The period of a simple pendulum as shown in the figure can be obtained through the conditions for equilibrium:



$$\sum F_r = T - mg\cos\theta_A = 0$$
$$\sum F_t = -mg\sin\theta_A = ma = m\frac{d^2s}{dt^2}$$

Since the arc length s=Lθ, we can write

$$\frac{d^2s}{dt^2} = L\frac{d^2\theta}{dt^2} = -g\sin\theta \approx -g\theta = \omega^2\theta$$

Since angular frequency ω is

$$\omega = \sqrt{\frac{g}{L}}$$

The period for this pendulum is

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}}$$

Now, sine the moment of inertia for this pendulum is $I=mL^2$, the period from the previous problem becomes

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{mgd}} = 2\pi\sqrt{\frac{mL^2}{mgL}} = 2\pi\sqrt{\frac{L}{g}}$$

which is identical to the period of a simple pendulum. Therefore, this generalized formula of period of a physical pendulum is also valid for a simple pendulum.

Turn over

Useful Formulae

Velocity: $\vec{v}_f = \vec{v}_i + \vec{a}t$

Position: $\vec{r}_f = \vec{r}_i + \vec{v}_i t + \frac{1}{2}\vec{a}t^2$

Center of Mass: $\vec{r}_{CM} = \frac{\sum m_i \vec{r}_i}{\sum m_i}$

Linear Momentum: $\vec{p} = m\vec{v}$

Kinetic Energy: $K = \frac{1}{2}mv^2$

Gravitational Potential Energy: $U = mgh$

Moment of Inertia: $I = \int r^2 dm$

Moment of Inertia of a Solid Sphere with mass M and radius R: $I = \frac{2}{5}MR^2$

Torque: $t = Fd = Ia$ where d is moment arm and α is the angular acceleration

Rotational Kinetic Energy: $K_R = \frac{1}{2}Iw^2$

Angular Momentum: $\vec{L} = \vec{r} \times \vec{p}$
$|L| = Iv$

The solutions for a 2-dimensional equation:

$$ax^2 + bx + c = 0$$

are:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Turn over