Name:
ID:

[1 – 28 points] •An object whose mass is 1.50kg on the surface of the Earth is undergoing a projectile motion on the surface of a planet P whose mass (M_p) is half that of the Earth (M_E). The object is thrown upward from the top of a building at an angle of 30º to horizontal with initial speed of 30.0m/s. The height of the building is 20.0m, and there is no air on the planet. The magnitude of the Earth's gravitational acceleration is g=9.8m/s^2. Answer the following questions, assuming the top of the building is at y=0, and upward is +y direction.

a) What are the forces involved in this motion? (2 points)
   1. Planet force
   2. **Gravitational force**
   3. Radial, Frictional, and gravitational forces
   4. Gravitational, Tangential, and Radial forces

Solution: Since there are no other sources of forces, not even air, the gravitational force by the planet's mass is the only force in this motion.

b) What is the mass of the object on the surface of the planet (2 points)?
   1. **1.5 kg**
   2. 0.75kg
   3. 3.0kg
   4. 14.7kg

Solution: Since mass is the fundamental property of matter, it does not differ no matter where the location is.

c) What is weight of the object on the surface of the planet? (2 points)
   1. 1.5 N
   2. 14.7kg m/s^2
   3. **7.35N**
   4. 12.7N

Solution: Since the mass of the planet is half that of the Earth, the gravitational acceleration on this planet is also half. Therefore, the weight of the object is 1.5x9.8/2.

d) Which direction is the gravitational acceleration on this planet? (2 points)
   1. 30º upward to horizontal in the direction of the movement
   2. **Downward to the center of the planet**
   3. 30º upward when going up and 30º downward when coming down
   4. Upward toward the Earth

Solution: Since the gravitational force is caused by the mass of matters, it always points to the center of mass which is the center of the planet. Therefore the gravitational acceleration on this planet also points to downward, to the center of the planet.

e) What is each acceleration component on this planet? (2 points)
   1. $a_x = 4.90 m/s^2$ and $a_y = -8.49 m/s^2$
   2. $a_x = 4.90 m/s^2$ and $a_y = 8.49 m/s^2$
   3. $a_x = 0 m/s^2$ and $a_y = -9.80 m/s^2$

Turn over

4. $a_x = 0m/s^2$ **and** $a_y = -4.90m/s^2$

Solution: Since the mass of the planet is half that of the Earth, the gravitational acceleration on this planet is also half, and the gravitational acceleration points downward, the correct acceleration components are a_x=0 and a_y=-4.9m/s^2.

f) What is the potential energy of the object at the top of the building? (3 points)

1. $0J$
2. $294J$
3. $212W$
4. **4.** $147J$

Solution: Since the mass of the planet is half that of the Earth, the gravitational acceleration on this planet is also half, and the object is at the top of the building whose height is 20m. Therefore the potential energy of the object at the top of the building is U=mg'h=mgh/2=1.5x9.8x20/2=147J.

g) How long does it take for the object to reach its maximum height? (3 points)

1. $2.39s$
2. **2.** $3.06s$
3. $1.53s$
4. $1.00s$

Solution: Since the gravitational acceleration g'=1/2g, this must be taken into account in computing the time. Since at the maximum height v_y=0, we obtain

h) How long would it take for the object to hit the surface of the planet? (3 points)

1. $9.8s$
2. **2.** $7.26s$
3. $14.3s$
4. $6.12s$

Solution: Since the gravitational acceleration g'=1/2g, this must be taken into account in computing the time. Since at the maximum height v_y=0, we obtain

i) How far is the object away from the initial position when it hits the surface of the planet? (3 points)

1. $372m$
2. $94m$
3. **3.** $188\,m$
4. $159m$

j) What is the coordinate of the maximum height the object would reach? (3 points)

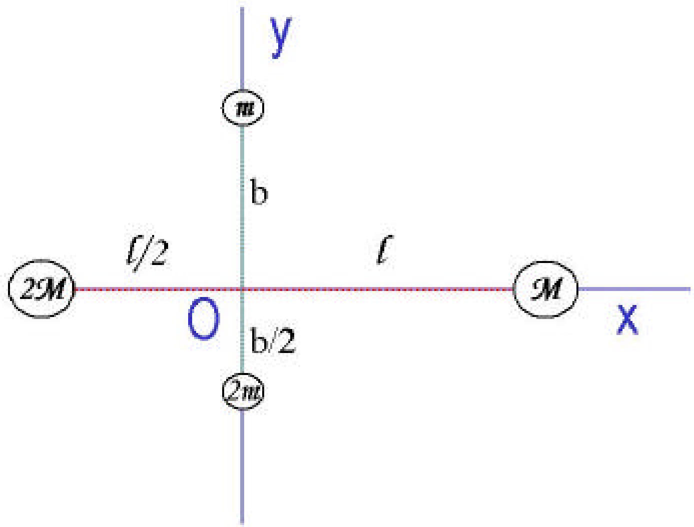
1. $11.5m$
2. $31.5m$
3. $43.0m$
4. **4.** $23.0m$

k) What is the kinetic energy of the object just before it hits the surface of the planet? (3 points)

1. **1.** $822J$
2. $969J$
3. $1240\,J$
4. $294J$

Turn over

[2– 25 points]Answer the following questions in a system consists of four small spheres as shown in the figure, assuming the radii are negligible and the rods connecting the spheres are massless.



a) Determine the coordinate of the Center of Mass of the system. (5points)

Solution

$$x_{CM} = \frac{\sum m_i x_i}{\sum m_i} = \frac{2M(-l/2) + Ml + 0\cdot 2m + 0\cdot m}{2M + M + 2m + m} = \frac{0}{3(M+m)} = 0$$

$$y_{CM} = \frac{\sum m_i y_i}{\sum m_i} = \frac{2M\cdot 0 + 0\cdot M + m\cdot b + 2m\cdot(-b/2)}{2M + M + 2m + m} = \frac{0}{3(M+m)} = 0$$

Therefore the coordinate of the center of mass of this system is

$$CM = (x_{Cm}, y_{CM}) = (0,0)$$

b) Compute the moment of inertia and the rotational kinetic energy when the system rotates clockwise about the x-axis at the angular speed ω. (5 points each)

Solution

$$I = \sum m_i r_i^2 = mb^2 + 2m\left(\frac{1}{2}b\right)^2 = \frac{3}{2}mb^2$$

Therefore the rotational kinetic energy of the system for this rotation is

$$K = \frac{1}{2}Iw^2 = \frac{1}{2}\left(\frac{3}{2}mb^2\right)w^2 = \frac{3}{4}mb^2w^2$$

Turn over

Turn over

c) Compute the moment of inertia, using parallel axis theorem, and the rotational kinetic energy when the system is rotating clockwise about the z axis at the angular speed ω. (5 points each)

Solution

$$I = Md^2 + I_{Cm} = M \cdot 0^2 + \sum m_i r_i^2 = Ml^2 + 2M\left(\frac{1}{2}l\right)^2 + mb^2 + 2m\left(\frac{1}{2}b\right)^2 = \frac{3}{2}\left(mb^2 + Ml^2\right)$$

Therefore the rotational kinetic energy of this motion is

$$K = \frac{1}{2}Iw^2 = \frac{1}{2}\left(\frac{3}{2}\left(mb^2 + Ml^2\right)\right)w^2 = \frac{3}{4}\left(mb^2 + Ml^2\right)w^2$$

Turn over

[3– 25 points] The motion of a random point on a sinusoidal wave is expressed as:

$$y = (1.50)\cos\left(\frac{2p}{0.500}x + 16.0pt\right)m$$

The mass of the string is 500g, and its length is 2.5m.

a) What kind of wave does this equation of motion represent (2 points)?

1. Standing wave   2. Free Fall
3. Harmonic Oscillation   **4. Traveling wave**

b) What is the wavelength of this wave (2 points)?

1. 1.5m   2. 2π/0.50m   **3.0.50m**   4. 16π m/s

c) What is the frequency of this motion (2 points)?

1. 2π/0.50s   **2. 8.00/s**   3. 16/s   4. 16π s

d) What is the maximum amplitude of this motion (2 points)?

1. 1.50cm   2. 3.00m   **3. 1.50m**   4. 16πm

e) What is the transverse speed of the point (2 points)?

1. $v_y = (-24.0p)\sin\left(\frac{2p}{0.500}x + 16.0pt\right)m/s$

2. $v_y = (-6p)\sin\left(\frac{2p}{0.500}x + 16.0pt\right)m/s$

3. $v_y = (-1.5)\sin\left(\frac{2p}{0.500}x + 16.0pt\right)/s$

4. $v_y = (24.0p)\cos\left(\frac{2p}{0.500}x + 16.0pt\right)m/s$

f) What is the transverse acceleration of the point (2 points)?

1. $a_y = (-24.0p)\cos\left(\frac{2p}{0.500}x + 16.0pt\right)m/s$

2. $a_y = (-384p^2)\cos\left(\frac{2p}{0.500}x + 16.0pt\right)m/s^2$

3. $a_y = (-36.0p)\cos\left(\frac{2p}{0.500}x + 16.0pt\right)m/s^2$

4. $a_y = (24.0p)\sin\left(\frac{2p}{0.500}x + 16.0pt\right)m/s$

g) What is the angular wave number (2 points)?

1. $k = 16pm$ 2. $k = 16p/m$

3. $k = \frac{2p}{0.500}/m$ 4. $k = \frac{2p}{0.500}m$

h) What is speed of the wave (3 points)?

1. $v = 4.00m/s$ 2. $v = 16pm/s$

3. $v = \frac{2p}{0.500}s/m$ 4. $v = 8.00m/s$

Turn over

i) What is the line density of the string (2 points)?

1. 0.500kg/m 2. 1.00kg/m **3. 0.200kg/m** 4.3.00kg

j) What is the power supplied by the tension on the string to keep this motion (3 points)?

1. $P = 512W$
2. $P = 2490W$
3. **3.** $P = 2274W$
4. $P = 2747J$

k) What is the total energy put into the system to sustain this wave for 10 seconds (3 points)?

1. $E = 2.49\times10^{4} J$
2. **2.** $E = 2.27\times10^{4} J$
3. $E = 5.12\times10^{3} J$
4. $E = 2.75\times10^{4} J$

Turn over

[4– 22 points]After a storm the water level at the Hoover dam on planet K, whose mass is twice that of the Earth, is reaching to a height of 50m. The gravitational acceleration on the surface of the Earth is $9.8m/s^2$. The density of water is $r = 1.00\times10^3 kg/m^3$. Width of the dam is 250m. Answer the following questions.

a) Determine the pressure exerted on the dam at 20m from the surface of the water, ignoring the air pressure. (5points)

Solution

Since the dam is on the surface of a planet whose mass is twice of that of the Earth, the gravitational acceleration on the surface of this planet is different than the one on the Earth and is twice the gravitational acceleration on the Earth's surface. Thus, the planet's gravitational acceleration g' =2g.

$$P = rg'h = 2rgh = 2\times1.00\times10^3\times9.80\times20.0 = 3.92\times10^5 N/m^2$$

b) Determine the total force exerted on the dam by the water. (7 points)

Solution

Since the force varies as a function of the depth, one needs to integrate the forces on the infinitesimal area of the dam's wall. The small force on the area defined by the width w and the height dy is *dF=PdA=Pwdy*. Thus by integrating dF through the entire height one can obtain the total force

$$F = \int_0^H dF = \int_0^H PdA = \int_0^H rg'hwdy = \int_0^H 2rg(H-y)wdy$$

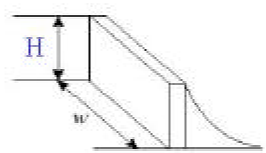
$$= 2rg\left(Hy - \frac{1}{2}y^2\right)w\Big|_0^H = rgH^2w = 2\times1.00\times10^3\times9.80\times(50)^2\times250 = 6.10\times10^9 N$$

Turn over

c) Assuming the width of the dam is constant throughout the entire height of the dam, design a dam that can hold the water and explain why you designed the dam the way you did. (10 points)

Solution

Since the pressure exerted on the dam by the water increases as a function of depth, the total force applied on the dam also increases. The total force exerted on the wall of the dam is proportional to square of the height H of the dam. Therefore the dam must be designed to balance this variation of the force as a function of the depth. Therefore the shape of the wall of the dam will follow a parabola as answer given in the previous problem $F = rgH^2w$**.**
So the design of the dam would look like



Turn over

Turn over

Useful Formulae

Velocity: $\vec{v}_f = \vec{v}_i + \vec{a}t$

Position: $\vec{r}_f = \vec{r}_i + \vec{v}_i t + \frac{1}{2}\vec{a}t^2$

Center of Mass: $\vec{r}_{CM} = \frac{\sum m_i \vec{r}_i}{\sum m_i}$

Linear Momentum: $\vec{p} = m\vec{v}$

Kinetic Energy: $K = \frac{1}{2}mv^2$

Moment of Inertia: $I = \int r^2 dm$

Torque: $t = Fd = Ia$ where d is moment arm and α is the angular acceleration

Rotational Kinetic Energy: $K_R = \frac{1}{2}Iw^2$

Angular Momentum: $\vec{L} = \vec{r} \times \vec{p}$
$|L| = Iv$

Gravitational force between two objects with masses M_1 and M_2 at the distance r is

$$F = \frac{GM_1M_2}{r^2}$$

Pressure exerted by a liquid of density ρ in the depth h is $P = rgh$

Energy transfer by a sinusoidal wave on a string is $P = \frac{1}{2}mw^2A^2v$ where μ is the density of the string, ω is the angular frequency, A is the amplitude, and v is the speed of wave.

The solutions for a 2-dimensional equation:

$$ax^2 + bx + c = 0$$

are:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Turn over