PHYS 1443 – Section 501
Lecture #2

Monday, Jan. 16, 2002
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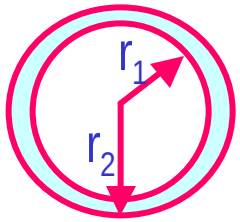
1. Some Chapter 1 problems
2. Some fundamentals
3. Displacement, Velocity, and Speed
4. Acceleration
5. Kinetic Equation of Motion

Use the CD-Rom in your book for demonstration.!!

Reading of the day
http://www.economist.com/surveys/displaystory.cfm?story_id=922278

Problems 1.4 and 1.13

- The mass of a material with density, ?, required to make a hollow spherical shell with inner radius, r_1, and outer radius, r_2?



$$V_{sphere} = \frac{4\pi}{3} r^3$$

$$M_{sphere} = \rho V_{sphere} = \frac{4\pi}{3}\rho r^3$$

$$M_{inner} = \rho V_{inner} = \frac{4\pi}{3}\rho r_1^3$$

$$M_{outer} = \rho V_{outer} = \frac{4\pi}{3}\rho r_2^3$$

$$M_{shell} = M_{outer} - M_{inner}$$

$$= \frac{4\pi}{3}\rho (r_2^3 - r_1^3)$$

- Prove that displacement of a particle moving under uniform acceleration is, $s=ka^m t^n$, is dimensionally correct if k is a dimensionless constant, $m=1$, and $n=2$.

Displacement: Dimension of Length
Acceleration a:Dimension of L/T^2

$$[s] = \left[\frac{l}{t^2}\right]^m [t]^n = [lt^{-2}]^m [t]^n = [l]^m [t]^{-2m+n}$$

$$\Rightarrow m = 1, n - 2m = 0;$$

$$\Rightarrow n - 2m = 2$$

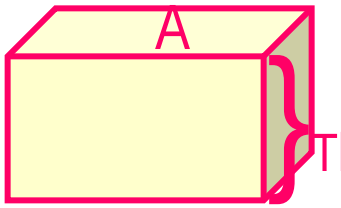
Problems 1.25 & 1.31

- Find the density, ?, of lead, in SI unit, whose mass is 23.94*g* and volume, *V*, is 2.10cm^3.

Density; $$? \; ? \; \frac{?}{V} \; ? \; \frac{23.94\ g}{2.10\ cm^3} \; ? \; 11.4 \; ? \; \frac{\frac{1}{1000} kg}{\left(\frac{1}{100} m\right)^3} \; ? \; 11.4 \; ? \; 10^3 \ kg/m^3$$

- Find the thickness of the layer covered by a gallon (*V*=3.78x10^{-3} m^3) of paint spread on an area of on the wall 25.0m^2.

- Thickness is in the dimension of Length.
- A gallon (*V*=3.78x10^{-3} m^3) of paint is covering 25.0m^2.



$$Thickness \; ? \; \frac{V}{A} \; ? \; \frac{3.78 \; ? \; 10^{?3} \ m^3}{25.0\ m^2} \; ? \; 1.51 \; ? \; 10^{?4} \ m$$

Thickness (OK, it is very skewed view!!)

Some Fundamentals

- Kinematics: Description of Motion without understanding the cause of the motion
- Dynamics: Description of motion accompanied with understanding the cause of the motion
- Vector and Scalar quantities:
  - Scalar: Physical quantities that require magnitude but no direction
    - Speed, length, mass, etc
  - Vector: Physical quantities that require both magnitude and direction
    - Velocity, Acceleration
    - It does not make sense to say "I ran at a velocity of 10miles/hour."
- Objects can be treated as point-like if their sizes are smaller than the scale in the problem
  - Earth can be treated as a point like object (or a particle)in celestial problems
  - Any other examples?

Some More Fundamentals

- Motions:Can be described as long as the position is known at any time (or position is expressed as a function of time)
  - Translation: Linear motion
  - Rotation: Circular or elliptical motion
  - Vibration: Oscillation
- Dimensions
  - 0 dimension: A point
  - 1 dimension: Linear drag of a point, resulting in a line ✍ Motion in one-dimension is a motion on a line
  - 2 dimension: Linear drag of a line resulting in a surface
  - 3 dimension: Perpendicular Linear drag of a surface, resulting in a stereo object

Velocity and Speed

One dimensional displacement is defined as:

$$\Delta x = x_f - x_i$$

Displacement is the difference between initial and final potions of motion and is a vector quantity

Average velocity is defined as:

$$v_x = \frac{x_f - x_i}{t_f - t_i} = \frac{\Delta x}{\Delta t}$$

Displacement per unit time in the period throughout the motion

Average speed is defined as:

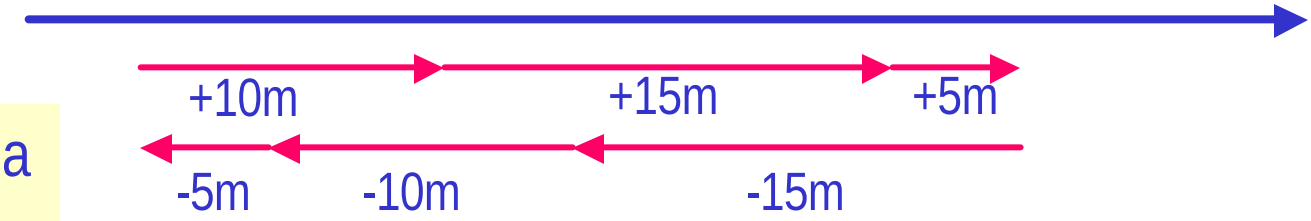
$$v = \frac{\text{Total Distance Traveled}}{\text{Total Time Interval}}$$

Can someone tell me what the difference between speed and velocity is?

The difference between Speed and Velocity

- Let's take a simple one dimensional translation that has many steps:

Let's call this line as X-axis



Let's have a couple of motions in total time interval of 20seconds

Total Displacement: $?x ? x_f ? x_i ? x_i ? x_i ? 0$

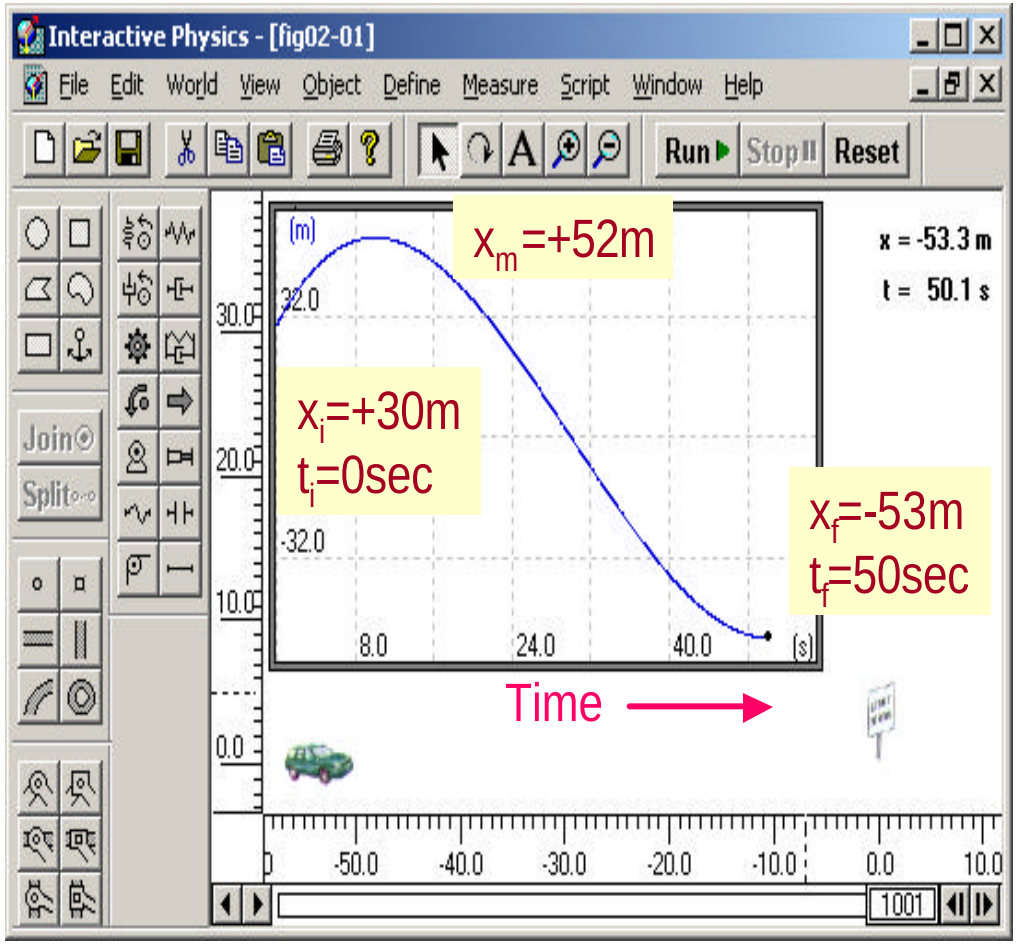
Average Velocity: $v_x ? \frac{x_f ? x_i}{t_f ? t_i} ? \frac{?x}{?t} ? \frac{0}{20} ? 0(m/s)$

Total Distance: $D ? 10 ? 15 ? 5 ? 15 ? 10 ? 5 ? 60(m)$

Average Speed: $v ? \frac{\text{Total Distance Traveled}}{\text{Total Time Interval}} ? \frac{60}{20} ? 3(m/s)$

Fig02-01.ip

Example 2.1



- Find the displacement, average velocity, and average speed.
- Displacement:

$$?x ? x_f ? x_i ? ?53? 30? ?83(m)$$

- Average Velocity:

$$v_x ? \frac{x_f ? x_i}{t_f ? t_i} ? \frac{?x}{?t} ? \frac{?83}{50} ? ?1.7(m/s)$$

- Average Speed:

$$v ? \frac{\text{Total Distance Traveled}}{\text{Total Time Interval}} ?$$

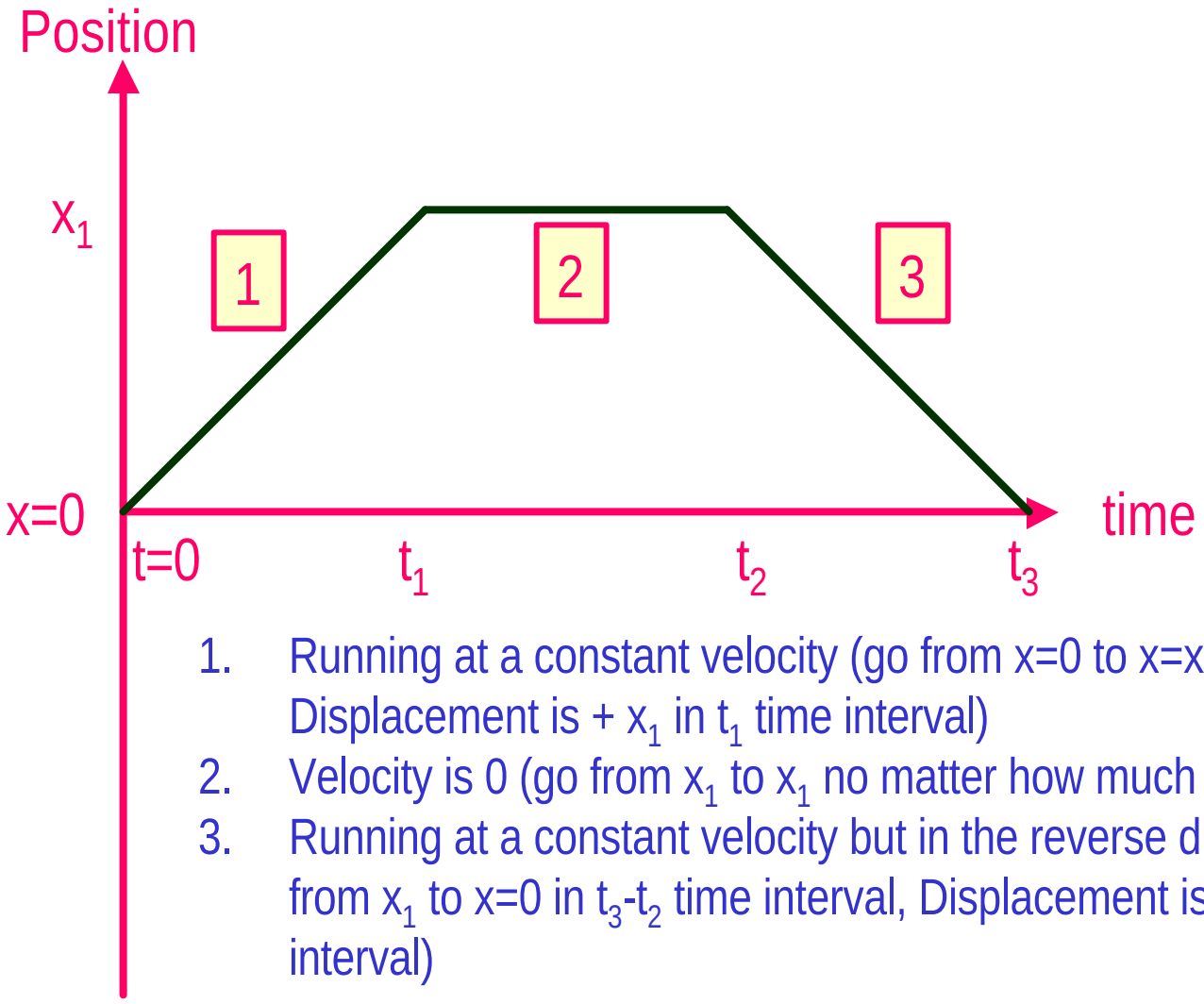
$$\frac{22 ? 52 ? 53}{50} ? \frac{127}{50} ? 2.5(m/s)$$

Instantaneous Velocity and Speed

- Here is where calculus comes in to help understanding the concept of "instantaneous quantities"
- Instantaneous velocity is defined as: $v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$
  - What does this mean?
    - Displacement in an infinitesimal time interval
    - Mathematically: Slope of the position variation as a function of time
- Instantaneous speed is the size (magnitude) of the velocity vector: $|v_x| = \lim_{\Delta t \to 0} \left|\frac{\Delta x}{\Delta t}\right| = \left|\frac{dx}{dt}\right|$

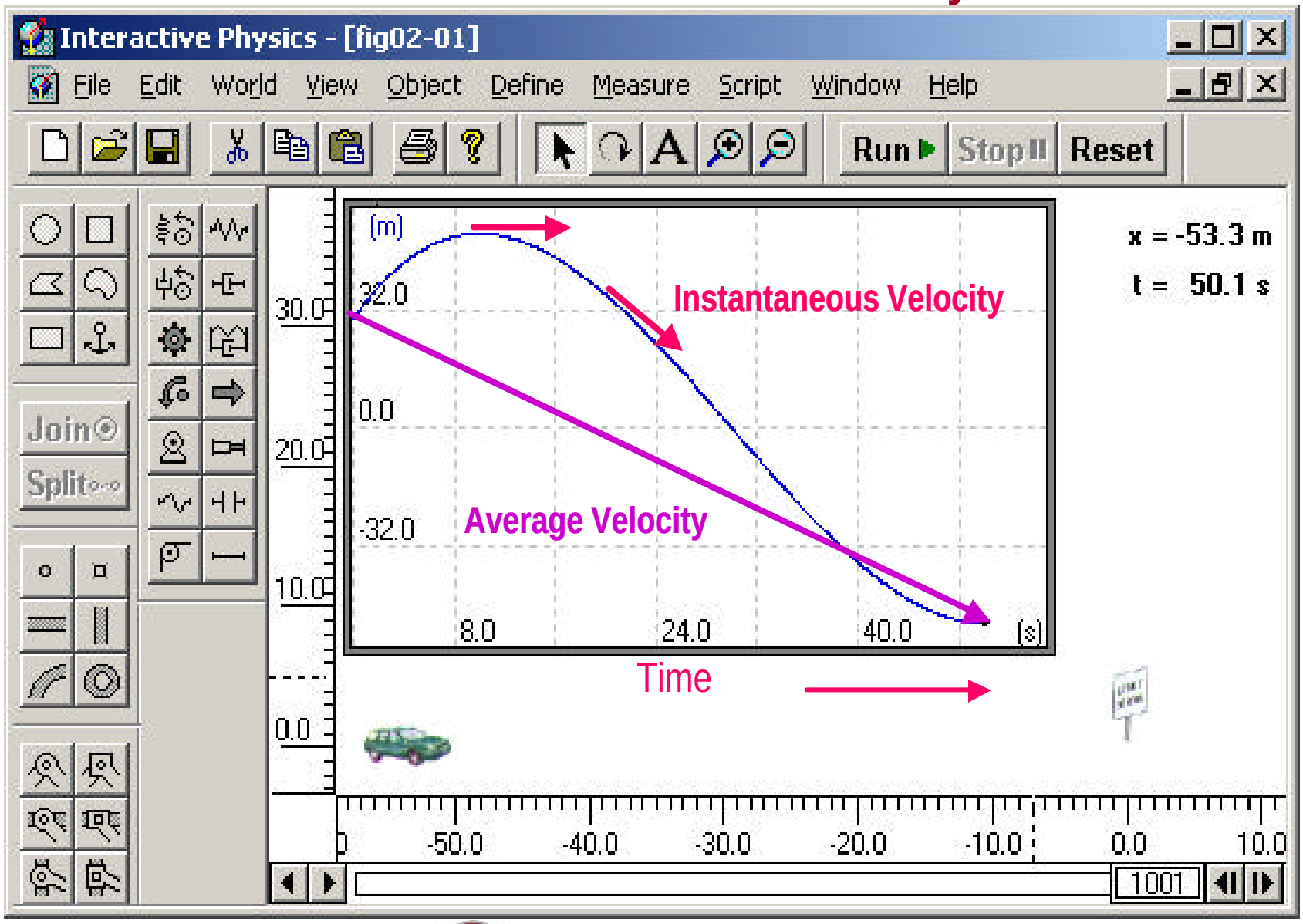
*Magnitude of Vectors are Expressed in absolute values

Position vs Time Plot



1. Running at a constant velocity (go from x=0 to $x=x_1$ in t_1, Displacement is + x_1 in t_1 time interval)
2. Velocity is 0 (go from x_1 to x_1 no matter how much time changes)
3. Running at a constant velocity but in the reverse direction as 1. (go from x_1 to x=0 in t_3-t_2 time interval, Displacement is - x_1 in t_3-t_2 time interval)

Instantaneous Velocity



Example 2.2

- Particle is moving along x-axis following the expression: $x = 4t - 2t^2$
- Determine the displacement in the time intervals t=0 to t=1s and t=1 to t=3s:

For interval t=0 to t=1s

$$x_{t=0} = 0, x_{t=1} = -4 \cdot (1) - 2 \cdot (1)^2 = -2$$
$$\Delta x_{t=0,1} = x_{t=1} - x_{t=0} = -2 - 0 = -2(m)$$

For interval t=1 to t=3s

$$x_{t=1} = -2, x_{t=3} = -4 \cdot (3) - 2 \cdot (3)^2 = -6$$
$$\Delta x_{t=1,3} = x_{t=3} - x_{t=1} = -6 - 2 = -8(m)$$

- Compute the average velocity in the time intervals t=0 to t=1s and t=1 to t=3s:

$$v_x = \frac{\Delta x_{t=0,1}}{\Delta t} = \frac{-2}{1}(m/s)$$

$$v_x = \frac{\Delta x_{t=1,3}}{\Delta t} = \frac{-8}{2} = -4(m/s)$$

- Compute the instantaneous **velocity at t=2.5s:**

Instantaneous velocity at any time *t*

$$v_x(t) = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} = \frac{d}{dt}(-4t - 2t^2) = -4 - 4t$$

Instantaneous velocity at *t=2.5s*

$$v_x(t = 2.5) = -4 - 4(2.5) = -6(m/s)$$

Acceleration

- Change of velocity in time
- Average acceleration:

$$a_x \equiv \frac{v_{xf} - v_{xi}}{t_f - t_i} \equiv \frac{\Delta v_x}{\Delta t}$$ analogs to $$v_x \equiv \frac{x_f - x_i}{t_f - t_i} \equiv \frac{\Delta x}{\Delta t}$$

- Instantaneous Acceleration

$$a_x \equiv \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} \equiv \frac{dv_x}{dt} \equiv \frac{d}{dt}\left(\frac{dx}{dt}\right) \equiv \frac{d^2x}{dt^2}$$ analogs to $$v_x \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} \equiv \frac{dx}{dt}$$

- In calculus terms: A slope (derivative) of velocity with respect to time or change of slopes of position as a function of time

Example 2.4

- Velocity, v_x, is express in: $v_x(t) = (40 - 5t^2)\ m/s$
- Find average acceleration in time interval, t=0 to t=2.0s

$$v_{xi}(t_i = 0) = 40\ (m/s)$$
$$v_{xf}(t_f = 2.0) = (40 - 5 \cdot 2^2) = 20\ (m/s)$$
$$a_x = \frac{v_{xf} - v_{xi}}{t_f - t_i} = \frac{\Delta v_x}{\Delta t} = \frac{20 - 40}{2 - 0} = 10\ (m/s^2)$$

- Find instantaneous acceleration at any time t and t=2.0s

Instantaneous Acceleration at any time

$$a_x(t) = \frac{dv_x}{dt} = \frac{d}{dt}(40 - 5t^2) = -10t$$

Instantaneous Acceleration at any time t=2.0s

$$a_x(t = 2.0) = -10 \cdot (2.0) = -20(m/s^2)$$


Fig02-09.ip

Meanings of Acceleration

- When an object is moving in a constant velocity ($v=v_0$), there is no acceleration ($a=0$)
  - Is there any acceleration when an object is not moving?
- When an object is moving faster as time goes on, ($v=v(t)$), acceleration is positive ($a>0$)
- When an object is moving slower as time goes on, ($v=v(t)$), acceleration is negative ($a<0$)
- In all cases, velocity is positive, unless the direction of the movement changes.
- Is there acceleration if an object moves in a constant speed but changes direction?

The answer is YES!!

One Dimensional Motion

- Let's start with simplest case: acceleration is constant ($a=a_0$)
- Using definitions of average acceleration and velocity, we can draw equation of motion (description of motion, position *wrt* time)

$$a_x = \frac{v_{xf} - v_{xi}}{t_f - t_i} = \frac{v_{xf} - v_{xi}}{t}$$

If $t_f=t$ and $t_i=0$ $\quad v_{xf} = v_{xi} + a_x t$

For constant acceleration, simple numeric average

$$\bar{v}_x = \frac{v_{xi} + v_{xf}}{2} = \frac{2v_{xi} + a_x t}{2} = v_{xi} + \frac{1}{2} a_x t$$

$$\bar{v}_x = \frac{x_f - x_i}{t_f - t_i} = \frac{x_f - x_i}{t}$$

If $t_f=t$ and $t_i=0$ $\quad x_f = x_i + \bar{v}_x t$

Resulting Equation of Motion becomes

$$x_f = x_i + \bar{v}_x t = x_i + v_{xi} t + \frac{1}{2} a_x t^2$$

Kinetic Equation of Motion in a Straight Line Under Constant Acceleration

$$v_{xf} = v_{xi} + a_x t$$

Velocity as a function of time

$$x_f = x_i + \frac{1}{2}(v_{xi} + v_{xf})t$$

Displacement as a function of velocity and time

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$$

Displacement as a function of time, velocity, and acceleration

$$v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$$

Velocity as a function of Displacement and acceleration

You may use different forms of Kinetic equations, depending on the information given to you for specific physical problems!!

Ex02-08.ip

Example 2.8

- Problem: Car traveling at constant speed of 45.0m/s (~162km/hr or ~100miles/hr), police starts chasing the car at the constant acceleration of 3.00m/s^2, one second after the car passes him. How long does it take for police to catch the violator?
- Let's call the time interval for police to catch; T
- Set up an equation:Police catches the violator when his final position is the same as the violator's.

$$x_f^{Police} = \frac{1}{2}aT^2 = \frac{1}{2}(3.00)T^2$$

$$x_f^{Car} = v(T-1) = 45.0(T-1)$$

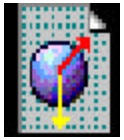
$$x_f^{Police} = x_f^{Police}; \quad \frac{1}{2}(3.00)T^2 = 45.0(T-1);$$

$$1.00T^2 = 30.0T - 30.0 = 0;$$

$$T = 31.0 \text{ (possible)} \quad \text{or} \quad T = -1.00 \text{ (not possible)}$$

Solutions for $ax^2 + bx + c = 0$ are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$



Falling body with variable initial velocity.IP

Free Fall

- Free fall is a motion under the influence of gravitational pull (gravity) only; Which direction is a freely falling object moving?
- Gravitational acceleration is inversely proportional to the distance between the object and the center of the earth
- The gravitational acceleration is $g=9.80m/s^2$ on the surface of the earth, most the time
- The direction of gravity is toward the center of the earth, which we normally call (-y); where up and down direction are indicated as the variable "y"
- Thus the correct denotation of gravitational acceleration on the surface of the earth is $g=-9.80m/s^2$

Example 2.12

Stone was thrown straight upward at t=0 with +20.0m/s initial velocity on the roof of a 50.0m high building,

1. Find the time the stone reaches at maximum height (*v=0*)
2. Find the maximum height
3. Find the time the stone reaches its original height
4. Find the velocity of the stone when it reaches its original height
5. Find the velocity and position of the stone at t=5.00s

$g=-9.80 m/s^2$

1 $v_f = v_{yi} + a_y t = +20.0 - 9.80t = 0.00$

$t = \frac{20.0}{9.80} = 2.04s$

2 $y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$

$= 50.0 + 20 \cdot 2.04 + \frac{1}{2} \cdot (-9.80) \cdot (2.04)^2$

$= 50.0 + 20.4 = 70.4(m)$

3 $t = 2.04 \cdot 2 = 4.08s$ Other ways?

4 $v_{yf} = v_{yi} + a_y t = 20.0 + (-9.80) \cdot 4.08 = -20.0(m/s)$

5-Velocity $v_{yf} = v_{yi} + a_y t$

$= 20.0 + (-9.80) \cdot 5.00$

$= -29.0(m/s)$

5-Position $y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$

$= 50.0 + 20.0 \cdot 5.00 + \frac{1}{2} \cdot (-9.80) \cdot (5.00)^2 = -27.5(m)$