1443-501 Spring 2002
Lecture #4

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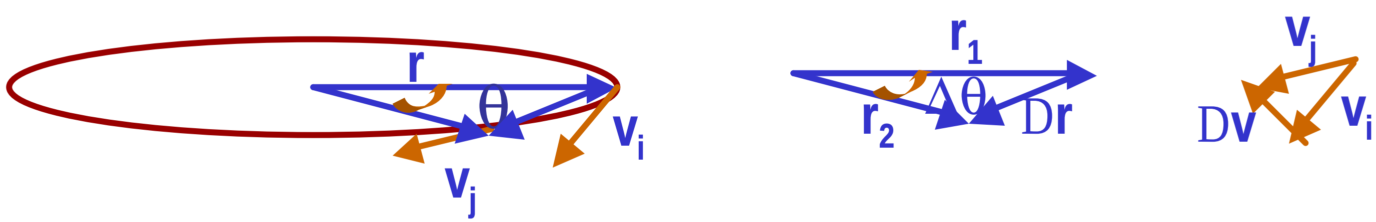
1. Uniform and Non-uniform Circular Motion
2. Newton's First Law of Motion
3. Reference Frames
4. Relative Velocity and Acceleration
5. Force
6. Mass and Newton's Second Law of Motion
7. Newton's Third Law of Motion

Remember the 1st term exam on Monday Feb. 11, 2002!!

Uniform Circular Motion

- A motion with a constant speed on a circular path.
  - The velocity of the object changes, because the direction changes
  - Therefore, there is an acceleration



Angle is Δθ

The acceleration pulls the object inward: *Centripetal Acceleration*

Average Acceleration

$$\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i} = \frac{\Delta \vec{v}}{\Delta t}$$

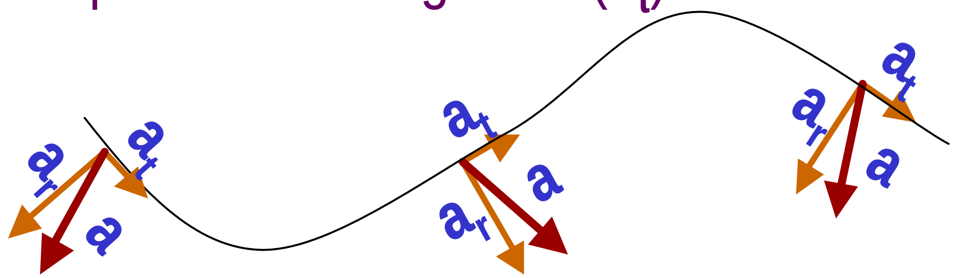
$$q = \frac{|\Delta \vec{v}|}{v} = \frac{|\Delta \vec{r}|}{r}, \quad |\Delta \vec{v}| = v\frac{|\Delta \vec{r}|}{r}, \quad \bar{a} = \frac{v}{\Delta t}\frac{|\Delta \vec{r}|}{r}$$

Instantaneous Acceleration

$$a_r = \lim_{\Delta t \to 0} \bar{a} = \lim_{\Delta t \to 0} \frac{|\Delta \vec{r}|}{\Delta t}\frac{v}{r} = v \times \frac{v}{r} = \frac{v^2}{r}$$

Non-uniform Circular Motion

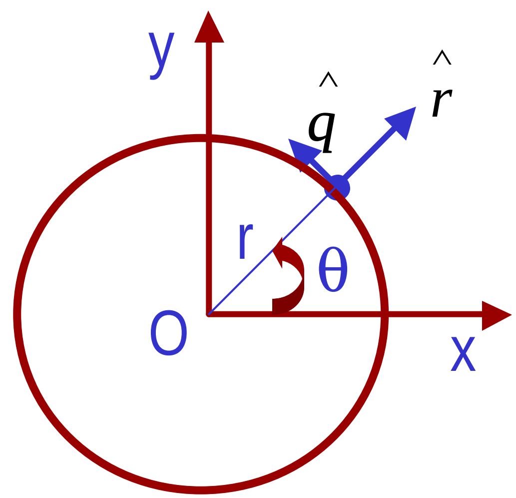
- Motion through a curved path
  - Requires both tangential (**a_t**) and radial acceleration (**a_r**)



Tangential Acceleration: $$a_t = \frac{d\left|\vec{v}\right|}{dt}$$

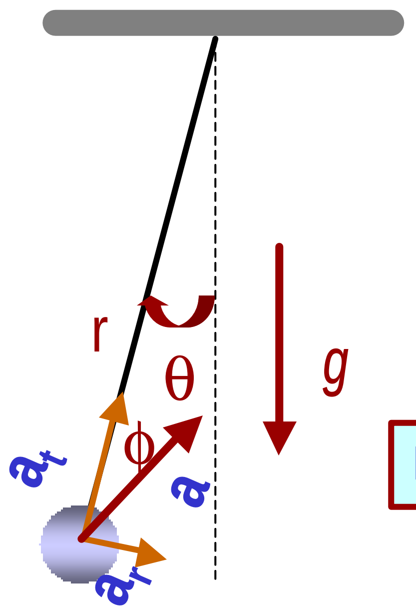
Radial Acceleration: $$a_r = \frac{v^2}{r}$$

Total Acceleration: $$\vec{a} = \vec{a_r} + \vec{a_t} = \frac{d\left|\vec{v}\right|}{dt}\hat{q} - \frac{v^2}{r}\hat{r}$$



Example 4.8

A ball tied to the end of a string of length 0.5m swings in a vertical circle under the influence of gravity, -*g*. When the string makes an angle θ=20° wrt vertical axis the ball has a speed of 1.5m/s. Find the **magnitude of the radial component of acceleration** at this time.



$$a_r = \frac{v^2}{r} = \frac{(1.5)^2}{0.5} = 4.5(m/s^2)$$

What is the magnitude of tangential acceleration when θ=20°?

$$a_t = g\sin q = g\sin(20^\circ) = 3.4 m/s^2$$

Find the magnitude and direction of the total acceleration *a* at θ=20°.

$$\left|\vec{a}\right| = \sqrt{a_r^2 + a_t^2} = \sqrt{(4.5)^2 + (3.4)^2} = 5.6 m/s^2$$

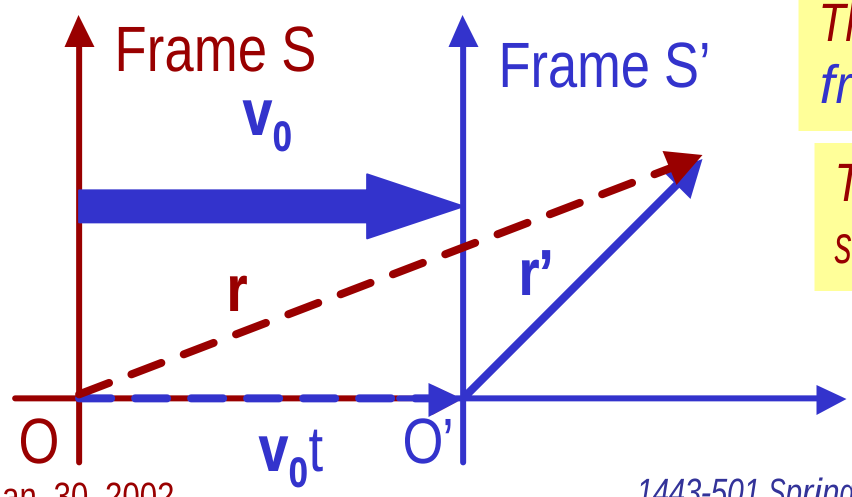
$$f = \tan^{-1}\left(\frac{a_t}{a_r}\right) = \tan^{-1}\left(\frac{3.4}{4.5}\right) = 37^\circ$$

Observations in Different Reference Frames

Results of Physical measurements in different reference frames could be different

Observations of the same motion in a stationary frame would be different than the ones made in the frame moving together with the moving object.

Consider that you are driving a car. To you the objects in the car do not move while to the person outside the car they are moving in the same speed and direction as your car is.



The position vector **r'** *is still* **r'** *in the moving frame S'.no matter how much time passed!!*

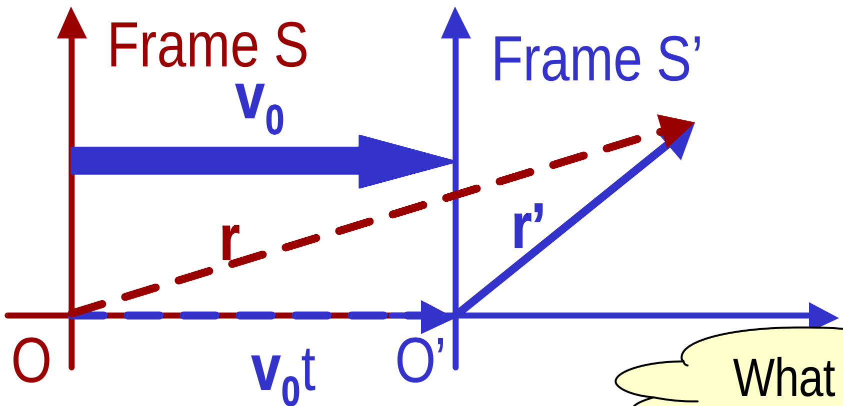
The position vector **r** *is no longer* **r** *in the stationary frame S when time* **t** *has passed.*

How are these position vectors related to each other?

$$\vec{r}(t) = \vec{r}' + \vec{v}_0 t$$

Relative Velocity and Acceleration

The velocity and acceleration in two different frames of references can be denoted, using the formula in the previous slide:



Galilean transformation equation

$$\vec{r}\,' = \vec{r} - \vec{v}_0 t$$
$$\frac{d\vec{r}\,'}{dt} = \frac{d\vec{r}}{dt} - \vec{v}_0$$
$$\vec{v}\,' = \vec{v} - \vec{v}_0$$

What does this tell you?

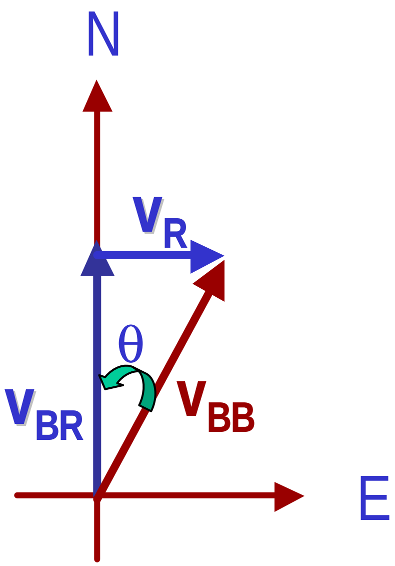
$$\vec{r}\,' = \vec{r} - \vec{v}_0 t$$
$$\frac{d\vec{v}\,'}{dt} = \frac{d\vec{v}}{dt} - \frac{d\vec{v}_0}{dt}$$
$$\vec{a}\,' = \vec{a}, \text{ when } \vec{v}_0 \text{ is constant}$$

The accelerations measured in two frames are the same when the frames move at a constant velocity with respect to each other!!!

The earth's gravitational acceleration is the same in a frame moving at a constant velocity wrt the earth.

Example 4.9

A boat heading due north with a speed 10.0km/h is crossing the river whose stream has a uniform speed of 5.00km/h due east. Determine the velocity of the boat seen by the observer on the bank.



$$\vec{v}_{BB} = \vec{v}_{BR} + \vec{v}_{R}$$

$$\left|\vec{v}_{BB}\right| = \sqrt{\left|\vec{v}_{BR}\right|^2 + \left|\vec{v}_{R}\right|^2} = \sqrt{(10.0)^2 + (5.00)^2} = 11.2\,km/h$$

$$\because\ \vec{v}_{BR} = 10.0\,\hat{j} \text{ and } \vec{v}_{R} = 5.00\,\hat{i}$$

$$\vec{v}_{BB} = 5.00\,\hat{i} + 10.0\,\hat{j}$$

$$q = \tan^{-1}\left(\frac{v_{BBy}}{v_{BBx}}\right) = \tan^{-1}\left(\frac{5.00}{10.0}\right) = 26.6^\circ$$

How long would it take for the boat to cross the river if the width is 3.0km?

$$v_{BB}\cos q \bullet t = 3.0km$$

$$t = \frac{3.0}{v_{BB}\cos q} = \frac{3.0}{11.2\times\cos(26.6^\circ)} = 0.30hrs = 18\min$$

Force

We've been learning kinematics; describing motion without understanding what the cause of the motion was. Now we are going to learn dynamics!!

FORCEs are what cause an object to move

Can someone tell me what FORCE is?

The above statement is not entirely correct. Why?

Because when an object is moving with a constant velocity no force is exerted on the object!!!

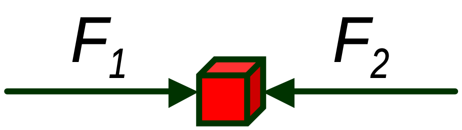
FORCEs are what cause any change in the velocity of an object!!

What does this statement mean?

When there is force, there is change of velocity. Forces cause acceleration.

What happens there are several forces being exerted on an object?

Forces are vector quantities, so vector sum of all forces, the NET FORCE, determines the motion of the object.



NET FORCE, $F = F_1 + F_2$

When net force on an objectis 0, the has constant velocity and is at its equilibrium!!

More Force

There are various classes of forces

Contact Forces: Forces exerted by physical contact of objects

Examples of Contact Forces: Baseball hit by a bat, Car collisions

Field Forces: Forces exerted without physical contact of objects

Examples of Field Forces: Gravitational Force, Electro-magnetic force

What are possible ways to measure strength of Force?

A calibrated spring whose length changes linearly with the force exerted .

Forces are vector quantities, so addition of multiple forces must be done following the rules of vector additions.

Newton's First Law and Inertial Frames

Galileo's statement on natural states of matter:

Any velocity once imparted to a moving body will be rigidly maintained as long as the external causes of retardation are removed!!

This statement is formulated by Newton into the **1st law of motion (Law of Inertia)**:

In the absence of external forces, an object at rest remains at rest and an object in motion continues in motion with a constant velocity.

What does this statement tell us?

1. *When no force is exerted on an object, the acceleration of the object is 0.*
2. *Any isolated object, the object that do not interact with its surrounding, is either at rest or moving at a constant velocity.*
3. *Objects would like to keep its current state of motion, as long as there is no force that interferes with the motion. This tendency is called the Inertia.*

A frame of reference that is moving at constant velocity is called an *Inertial Frame*

Mass

Mass: An inherent property of an object

1. Independent of the object's surroundings: The same no matter where you go.
2. Independent of method of measurement: The same no matter how you measure it

The heavier an object gets the bigger the inertia!!

It is harder to make changes of motion of a heavier object than the lighter ones.

The same forces applied to two different masses result in different acceleration depending on the mass.

$$\frac{m_1}{m_2} \equiv \frac{a_2}{a_1}$$

Note that mass and weight of an object are two different quantities!!

Weight of an object is the magnitude of gravitational force exerted on the object.

Not an inherent property of an object!!!

Weight will change if you measure on the Earth or on the moon.

Newton's Second Law of Motion

The acceleration of an object is directly proportional to the net force exerted on it and inversely proportional to the object's mass.

How do we write the above statement in a mathematical expression?

$$\sum_i \vec{F}_i = m\vec{a}$$

Since it's a vector expression the components should also satisfy:

$$\sum_i F_{ix} = ma_x; \quad \sum_i F_{iy} = ma_y; \quad \sum_i F_{iz} = ma_z$$

From the above vector expression, what do you conclude the dimension and unit of force are?

The dimension of force is

$$[m][a] = [M][LT^{-2}]$$

The unit of force in SI is

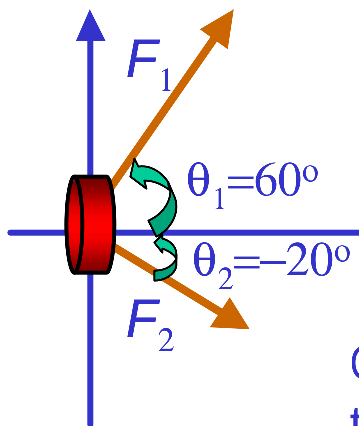
$$F = ma = [M][LT^{-2}] = kg \cdot m/s^2$$

$$1\,N \equiv 1\,kg \cdot m/s^2 \approx \frac{1}{4}\,lbs$$

See Table 5.1 for lbs to kgm/s^2 conversion.

Example 5.1

Determine the magnitude and direction of acceleration of the puck whose mass is 0.30kg and is being pulled by two forces, **F1** and **F2,** as shown in the picture, whose magnitudes of the forces are 8.0 N and 5.0 N, respectively.



Components of F_1

$$F_{1x} = \left|\vec{F_1}\right| \cos\theta_1 = 8.0 \times \cos\left(60^\circ\right) = 4.0\ N$$
$$F_{1y} = \left|\vec{F_1}\right| \sin\theta_1 = 8.0 \times \sin\left(60^\circ\right) = 6.9\ N$$

Components of F_2

$$F_{2x} = \left|\vec{F_2}\right| \cos\theta_2 = 5.0 \times \cos\left(-20^\circ\right) = 4.7\ N$$
$$F_{2y} = \left|\vec{F_2}\right| \sin\theta_2 = 5.0 \times \sin\left(-20^\circ\right) = -1.7\ N$$

Components of total force F

$$F_x = F_{1x} + F_{2x} = 4.0 + 4.7 = 8.7\ N = ma_x$$
$$F_y = F_{1y} + F_{2y} = 6.9 - 1.7 = 5.2\ N = ma_y$$

Magnitude and direction of acceleration a

$$a_x = \frac{F_x}{m} = \frac{8.7}{0.3} = 29\ m/s^2, \quad a_y = \frac{F_y}{m} = \frac{5.2}{0.3} = 17\ m/s^2$$
$$\left|\vec{a}\right| = \sqrt{(29)^2 + (17)^2} = 34\ m/s^2, \quad \theta = \tan^{-1}\left(\frac{a_y}{a_x}\right) = \tan^{-1}\left(\frac{17}{29}\right) = 30^\circ$$
$$\vec{a} = a_x\hat{i} + a_y\hat{j} = \left(29\hat{i} + 17\hat{j}\right) m/s^2$$

Gravitational Force and Weight

Gravitational Force, F_g

The attractive force exerted on an object by the Earth

$$\vec{F_g} = m\vec{a} = m\vec{g}$$

Weight of an object with mass M is

$$W = \left|\vec{F_g}\right| = M\left|\vec{g}\right| = Mg$$

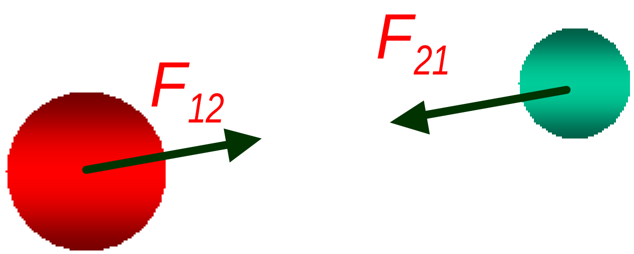
Since weight depends on the magnitude of gravitational acceleration, g, it varies depending on geographical location.

By measuring the forces one can determine masses. This is why you can measure mass using spring scale.

Actual unit of weight is in the unit of force but the unit of mass is commonly used in place of force.

Newton's Third Law (Law of Action and Reaction)

If two objects interact, the force, F_{12}, exerted on object 1 by object 2 is equal magnitude to and opposite direction to the force, F_{21}, exerted on object 1 by object 2.



$$\overrightarrow{F_{21}} = -\overrightarrow{F_{12}}$$

The action force is equal in magnitude to the reaction force but in opposite direction. These two forces always act on different objects.

What is the reaction force to the force of a free fall object?

The force exerted by the ground when it completed the motion.

Stationary objects on top of a table has a reaction force (normal force) from table to balance the action force, the gravitational force.