1443-501 Spring 2002
Lecture #5

Dr. Jaehoon Yu

1. Applications of Newton's Laws
2. Forces of Friction
3. Newton's Second Law & Circular Motions
4. Motion in Accelerated Frames
5. Motion with Resistive Force

1st term exam on Monday Feb. 11, 2002, at 5:30pm, in the classroom!!
Will cover chapters 1-6!!

Newton's Laws

1st Law:
Law of Inertia

In the absence of external forces, an object at rest remains at rest and an object in motion continues in motion with a constant velocity.

2nd Law:
Law of Forces

$$\boxed{\sum_i \vec{F}_i = m\vec{a}}$$

The acceleration of an object is directly proportional to the net force exerted on it and inversely proportional to the object's mass.

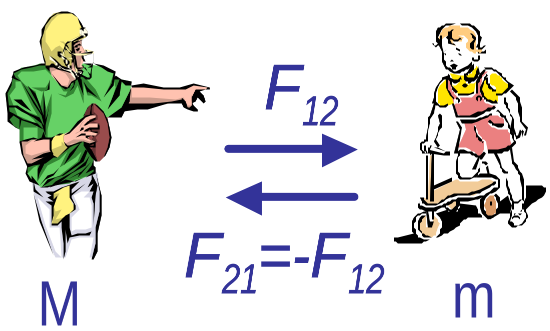
3rd Law:
Law of Action and Reaction

$$\boxed{\vec{F}_{21} = -\vec{F}_{12}}$$

If two objects interact, the force, F_{12}, exerted on object 1 by object 2 is equal magnitude to and opposite direction to the force, F_{21}, exerted on object 1 by object 2.

Example 5.3

A large man and a small boy stand facing each other on **frictionless ice**. They put their hands together and push against each other so that they move apart. a) Who moves away with the higher speed and by how much?



$$\overrightarrow{F_{12}} = -\overrightarrow{F_{21}}; \quad \left|\overrightarrow{F_{12}}\right| = \left|\overrightarrow{F_{21}}\right| = F$$

$$\overrightarrow{F_{12}} = m\overrightarrow{a_b}; \quad F_{12x} = ma_{bx}; \quad F_{12y} = ma_{by} = 0$$

$$\overrightarrow{F_{21}} = M\overrightarrow{a_M}; \quad F_{21x} = Ma_{Mx}; \quad F_{21y} = Ma_{My} = 0$$

$$\overrightarrow{F_{12}} = -\overrightarrow{F_{21}}; \quad \left|\overrightarrow{F_{12}}\right| = \left|-\overrightarrow{F_{21}}\right| = F; \quad a_{bx} = \frac{F}{m} = \frac{M}{m}a_{Mx}$$

$$v_{Mxf} = v_{Mxi} + a_{Mx}t = a_{Mx}t$$

$$v_{bxf} = v_{bxi} + a_{bx}t = a_{bx}t = \frac{M}{m}a_{Mx}t = \frac{M}{m}v_{Mxf}$$

$$\therefore v_{bxf} \rangle v_{Mxf} \text{ if } M \rangle m \text{ by the ratio of the masses}$$

b) Who moves farther while their hands are in contact?

Given in the same time interval, since the boy has higher acceleration and thereby higher speed, he moves farther than the man.

$$x_b = v_{bxf}t + \frac{1}{2}a_{bx}t^2 = \frac{M}{m}v_{Mxf}t + \frac{M}{2m}a_{Mx}t^2$$

$$= \frac{M}{m}\left(v_{Mxf}t + \frac{1}{2}a_{Mx}t^2\right) = \frac{M}{m}x_M$$

Some Basic Information

When Newton's laws are applied, *external forces* are only of interest

Why?

Because, as described in Newton's first law, an object will keep its current motion unless non-zero net external forces are applied.

Normal Force, *n:*

Reaction force that balances gravitational force, keeping objects stationary.

Tension, *T:*

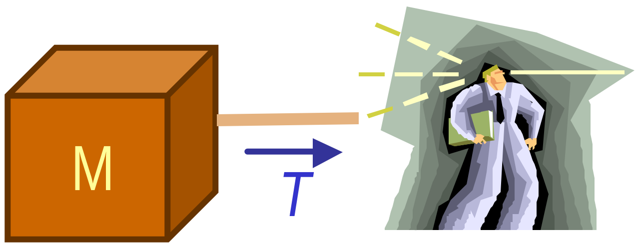
Magnitude of the force exerted on an object by a string or a rope.

Free-body diagram

*A graphical tool which is a **diagram of external forces on an object** and is extremely useful analyzing forces and motion!! Drawn only on the object.*

Applications of Newton's Laws

Suppose you are pulling a box on frictionless ice, using a rope.



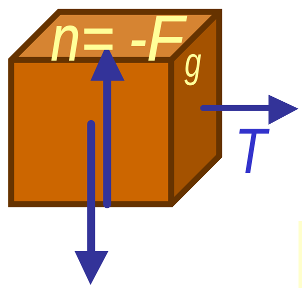
What are the forces being exerted on the box?

Gravitational force: F_g

Normal force: n

Tension force: T

Free-body diagram



$F_g=Mg$

Total force:
$F=F_g+n+T=T$

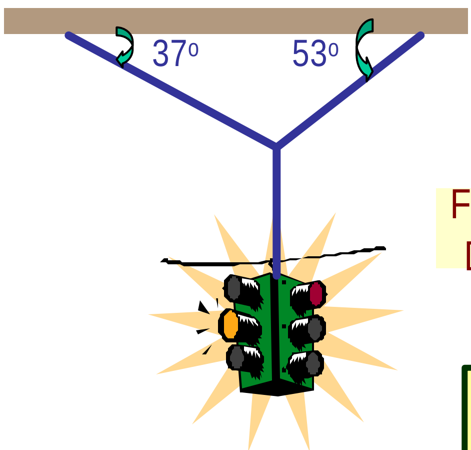
$$\sum F_x = T = Ma_x; \quad a_x = \frac{T}{M}$$
$$\sum F_y = F_g + n = 0; \quad a_y = 0$$

If T is a constant force, a_x, is constant

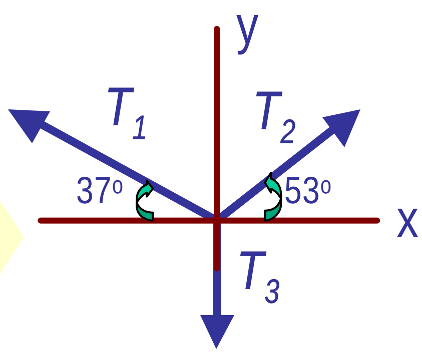
$$v_{xf} = v_{xi} + a_x t = v_{xi} + \left(\frac{T}{M}\right)t$$
$$\Delta x = x_f - x_i = v_{xi}t + \frac{1}{2}\left(\frac{T}{M}\right)t^2$$

Example 5.4

A traffic light weighing 125 N hangs from a cable tied to two other cables fastened to a support. The upper cables make angles of 37.0º and 53.0º with the horizontal. Find the tension in the three cables.



Free-body Diagram



$$\vec{F} = \vec{T_1} + \vec{T_2} + \vec{T_3}; \quad F_x = \sum_{i=1}^{i=3} T_{ix} = 0; \quad F_y = \sum_{i=1}^{i=3} T_{iy} = 0$$

$$T_1 \sin(37^\circ) + T_2 \sin(53^\circ) - mg = 0$$

$$-T_1 \cos(37^\circ) + T_2 \cos(53^\circ) = 0$$

$$\therefore T_1 = \frac{\cos(53^\circ)}{\cos(37^\circ)} T_2 = 0.754\ T_2$$

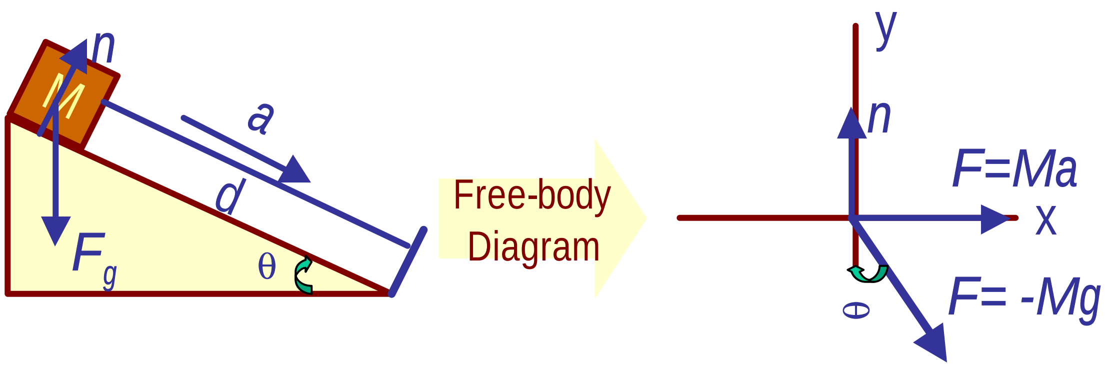
$$T_2 \left[\sin(53^\circ) + 0.754 \times \sin(37^\circ)\right] = 1.25\ T_2 = 125\ N$$

$$T_2 = 100\ N; \quad T_1 = 0.754\ T_2 = 75.4\ N$$

Example 5.6

A crate of mass M is placed on a frictionless inclined plane of angle θ.

a) Determine the acceleration of the crate after it is released.



$$\vec{F} = \vec{F_g} + \vec{n}$$

$$F_y = Ma_y = n + F_{gy} = 0$$

$$F_x = Ma_x = F_{gx} = Mg\sin\theta$$

$$a_x = g\sin\theta$$

Supposed the crate was released at the top of the incline, and the length of the incline is d. How long does it take for the crate to reach the bottom and what is its speed at the bottom?

$$d = v_{ix}t + \frac{1}{2}a_x t^2 = \frac{1}{2}g\sin\theta t^2$$

$$\therefore t = \sqrt{\frac{2d}{g\sin\theta}}$$

$$v_{xf} = v_{ix} + a_x t = g\sin\theta\sqrt{\frac{2d}{g\sin\theta}} = \sqrt{2dg\sin\theta}$$

Forces of Friction

Resistive force exerted on a moving object due to viscosity or other types frictional property of the medium in or surface on which the object moves.

These forces are either proportional to velocity or normal force

Force of static friction, f_s: *The resistive force exerted on the object until just before the beginning of its movement*

Empirical Formula

$$\left|\vec{f}_s\right| \le m_s \left|\vec{n}\right|$$

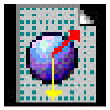
What does this formula tell you?

Frictional force increases till it reaches to the limit!!

Beyond the limit, there is no more static frictional force but kinetic frictional force takes it over.

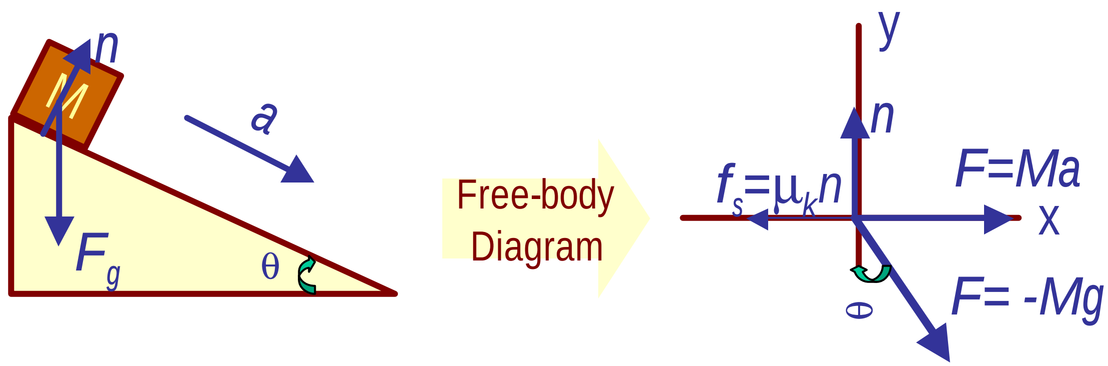
Force of kinetic friction, f_k *The resistive force exerted on the object during its movement*

$$\left|\vec{f}_k\right| = m_k \left|\vec{n}\right|$$



Example 5.12

Suppose a block is placed on a rough surface inclined relative to the horizontal. The inclination angle is increased till the block starts to move. Show that by measuring this critical angle, θ_c, one can determine coefficient of static friction, μ_s.



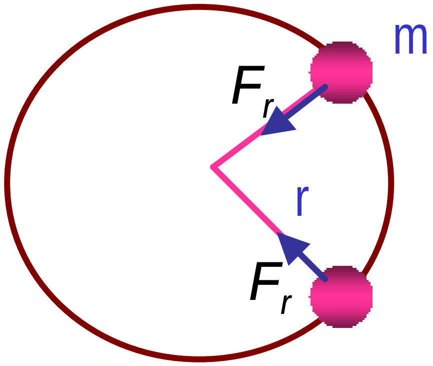
$$\vec{F} = \vec{F_g} + \vec{n} + f_s$$

$$F_y = Ma_y = n + F_{gy} = 0; \quad n = -F_{gy} = Mg\cos\theta_c$$

$$F_x = F_{gx} - f_s = 0; \quad f_s = \mu_s n = Mg\sin\theta_c$$

$$\mu_s = \frac{Mg\sin\theta_c}{n} = \frac{Mg\sin\theta_c}{Mg\cos\theta_c} = \tan\theta_c$$

Newton's Second Law & Uniform Circular Motion



The centripetal acceleration is always perpendicular to velocity vector, v, for uniform circular motion.

$$a_r = \frac{v^2}{r}$$

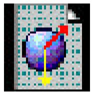
Is there force in this motion? If there is, what does it do?

*The force that causes the centripetal acceleration acts toward the center of the circular path and causes a change in the direction of the velocity vector. This force is called **centripetal force**.*

$$\sum F_r = ma_r = m\frac{v^2}{r}$$

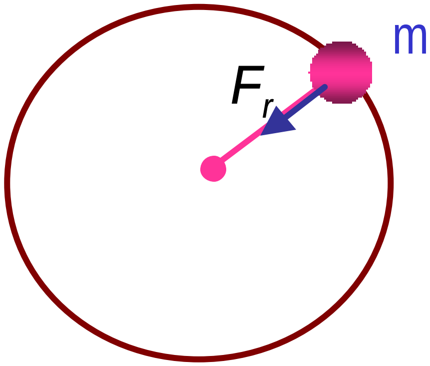
What do you think will happen to the ball if the string that holds the ball breaks? Why?

Based on Newton's 1st law, since the external force no longer exist, the ball will continue its motion without change and will fly away following the tangential direction to the circle.



Example 6.2

A ball of mass 0.500kg is attached to the end of a cord 1.50m long. The ball is moving in a horizontal circle. If the string can withstand maximum tension of 50.0 N, what is the maximum speed the ball can attain before the cord breaks?



Centripetal acceleration:

$$a_r = \frac{v^2}{r}$$

When does the string break?

$$\sum F_r = ma_r = m\frac{v^2}{r} = T$$

When the centripetal force is greater than the sustainable tension.

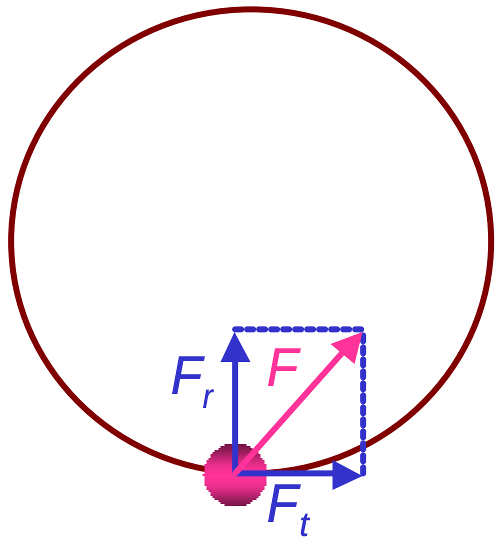
$$m\frac{v^2}{r} = T$$

$$v = \sqrt{\frac{Tr}{m}} = \sqrt{\frac{50.0 \times 1.5}{0.500}} = 12.2\,(m/s)$$

Calculate the tension of the cord when speed of the ball is 5.00m/s.

$$T = m\frac{v^2}{r} = 0.500 \times \frac{(5.00)^2}{1.5} = 8.33\,(N)$$

Forces in Non-uniform Circular Motion



The object has both tangential and radial accelerations.

What does this statement mean?

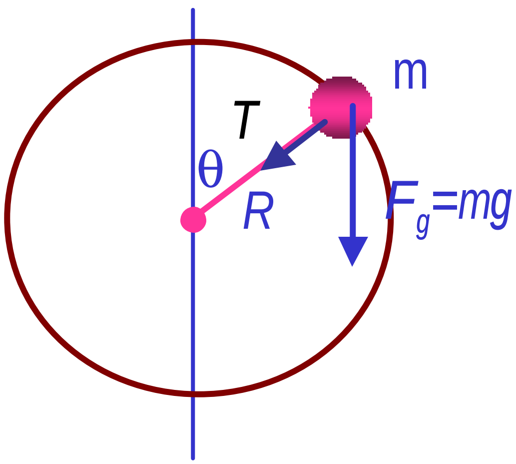
The object is moving under both tangential and radial forces.

$$\boxed{\vec{F} = \vec{F}_r + \vec{F}_t}$$

These forces cause not only the velocity but also the speed of the ball to change. The object undergoes a curved motion under the absence of constraints, such as a string.

Example 6.8

A ball of mass m is attached to the end of a cord of length R. The ball is moving in a vertical circle. Determine the tension of the cord at any instant when the speed of the ball is *v* and the cord makes an angle θ with vertical.



What are the forces involved in this motion?

The gravitational force F_g and the radial force, T, providing tension.

$$\sum F_t = ma_t = mg \sin q$$

$$a_t = g \sin q$$

$$\sum F_r = T - mg \cos q = ma_r = m\frac{v^2}{R}$$

$$T = m\left(\frac{v^2}{R} + g \cos q\right)$$

At what angles the tension becomes maximum and minimum. What are the tension?

Motion in Accelerated Frames

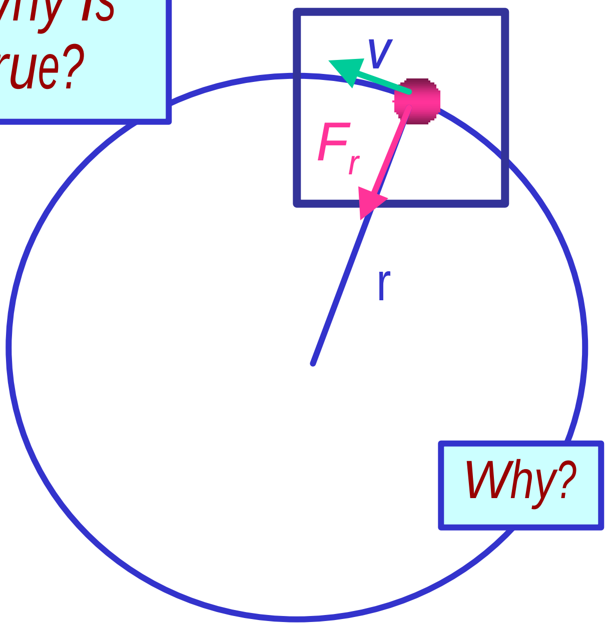
Newton's laws are valid only when observations are made in an inertial frame of reference. What happens in a non-inertial frame?

Fictitious forces are needed to apply Newton's second law in an accelerated frame.

This force does not exist when the observations are made in an inertial reference frame.

What does this mean and why is this true?

Let's consider a free ball inside a box under uniform circular motion.



How does this motion look like in an inertial frame (or frame outside a box)?

We see that the ball has a radial force exerted on it.

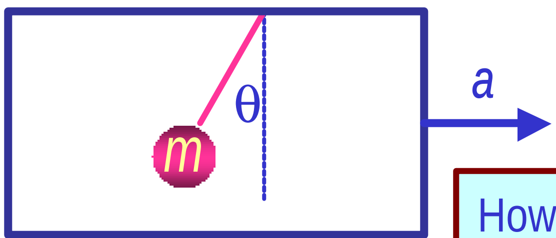
How does this motion look like in the box?

The ball is tumbled over to the wall of the box and feels that it is getting force that pushes it toward the wall.

According to Newton's first law, the ball wants to continue on its original movement but since the box is turning, the ball feels like it is being pushed toward the wall relative to everything else in the box.

Example 6.9

A ball of mass m is is hung by a cord to the ceiling of a boxcar that is moving with an acceleration *a*. What do the inertial observer at rest and the non-inertial observer traveling inside the car conclude? How do they differ?

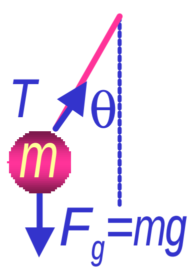


This is how the ball looks like no matter which frame you are in.

How do the free-body diagrams look for two frames?

How do the motions interpreted in these two frames? Any differences?

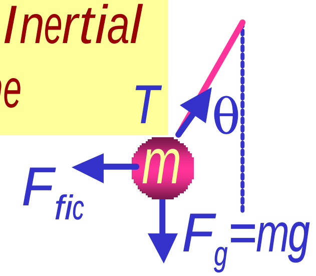
Inertial Frame



$$\sum \vec{F} = \vec{F}_g + \vec{T}$$
$$\sum F_x = ma_x = ma_c = T\sin\theta$$
$$\sum F_y = T\cos\theta - mg = 0$$
$$T = \frac{mg}{\cos\theta}; \quad a_c = g\tan\theta$$

For an inertial frame observer, the forces being exerted on the ball are only **T** and $\mathbf{F_g}$. The acceleration of the ball is the same as that of the box car and is provided by the x component of the tension force.

Non-Inertial Frame



$$\sum \vec{F} = \vec{F}_g + \vec{T} + \vec{F}_{fic}$$
$$\sum F_x = T\sin\theta - F_{fic} = 0; \; F_{fic} = ma_{fic} = T\sin\theta$$
$$\sum F_y = T\cos\theta - mg = 0$$
$$T = \frac{mg}{\cos\theta}; \quad a_{fic} = g\tan\theta$$

In the non-inertial frame observer, the forces being exerted on the ball are **T**, $\mathbf{F_g}$, and $\mathbf{F_{fic}}$.

For some reason the ball is under a force, $\mathbf{F_{fic}}$, that provides acceleration to the ball.

While the mathematical expression of the acceleration of the ball is identical to that of inertial frame observer's, the cause of the force is dramatically different.

Motion in Resistive Forces

Medium can exert resistive forces on an object moving through it due to viscosity or other types frictional property of the medium.

Some examples? Air resistance, viscous force of liquid, etc

These forces are exerted on moving objects in opposite direction of the movement.

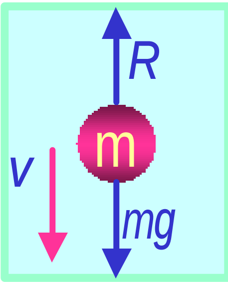
These forces are proportional to such factors as speed. They almost always increase with increasing speed.

Two different cases of proportionality:

1. Forces linearly proportional to speed: Slowly moving or very small objects
2. Forces proportional to square of speed: Large objects w/ reasonable speed

Resistive Force Proportional to Speed

Since the resistive force is proportional to speed, we can write R=bv



Let's consider that a ball of mass m is falling through a liquid.

$$\sum \vec{F} = \vec{F}_g + \vec{R}$$

$$\sum F_x = 0; \quad \sum F_y = mg - bv = ma = m\frac{dv}{dt}$$

$$\frac{dv}{dt} = g - \frac{b}{m}v$$

This equation also tells you that

$$\frac{dv}{dt} = g - \frac{b}{m}v = g, \text{ when } v = 0$$

What does this mean?

The above equation also tells us that as time goes on the speed increases and the acceleration decreases, eventually reaching 0.

An object moving in a viscous medium will obtain speed to a certain speed (terminal speed) and then maintain the same speed without any more acceleration.

What is the terminal speed in above case?

$$\frac{dv}{dt} = g - \frac{b}{m}v = 0; \quad v_t = \frac{mg}{b}$$

How do the speed and acceleration depend on time?

$$v = \frac{mg}{b}\left(1 - e^{-bt/m}\right); \quad v = 0 \text{ when } t = 0;$$

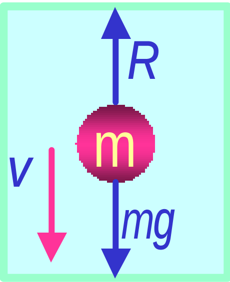
$$a = \frac{dv}{dt} = \frac{mg}{b}\frac{b}{m}e^{-bt/m} = ge^{-t/\tau}; \quad a = g \text{ when } t = 0;$$

$$\frac{dv}{dt} = \frac{mg}{b}\frac{b}{m}e^{-t/\tau} = \frac{mg}{b}\frac{b}{m}\left(1 - 1 + e^{-t/\tau}\right) = g - \frac{b}{m}v$$

The time needed to reach 63.2% of the terminal speed is defined as the time constant, τ=m/b.

Example 6.11

A small ball of mass 2.00g is released from rest in a large vessel filled with oil, where it experiences a resistive force proportional to its speed. The ball reaches a terminal speed of 5.00 cm/s. Determine the time constant τ and the time it takes the ball to reach 90% of its terminal speed.



Determine the time constant τ.

$$v_t = \frac{mg}{b}$$

$$\therefore b = \frac{mg}{v_t} = \frac{2.00\times10^{-3}kg \cdot 9.80m/s^2}{5.00\times10^{-2}m/s} = 0.392kg/s$$

$$t = \frac{m}{b} = \frac{2.00\times10^{-3}kg}{0.392kg/s} = 5.10\times10^{-3}s$$

Determine the time it takes the ball to reach 90% of its terminal speed.

$$v = \frac{mg}{b}\left(1 - e^{-t/t}\right) = v_t\left(1 - e^{-t/t}\right)$$

$$0.9v_t = v_t\left(1 - e^{-t/t}\right)$$

$$\left(1 - e^{-t/t}\right) = 0.9;\ e^{-t/t} = 0.1$$

$$t = -t \cdot \ln 0.1 = 2.30t = 2.30 \cdot 5.10\times10^{-3} = 11.7(ms)$$