1443-501 Spring 2002
Lecture #6

Dr. Jaehoon Yu

1. Motion with Resistive Forces
2. Resistive Force in Air at High Speed
3. Analytical and Numerical Methods
4. Review Examples

Physics Clinic Hours Extened!!
Mon. – Thu. till 6pm
Thursday: 6-9pm by appointments
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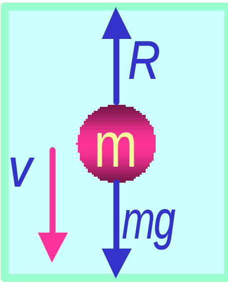
1st term exam on Monday Feb. 11, 2002, at 5:30pm, in the classroom!!
Will cover chapters 1-6!!

Resistive Force Proportional to Speed

Since the resistive force is proportional to speed, we can write R=bv



Let's consider that a ball of mass m is falling through a liquid.

$$\sum \vec{F} = \vec{F}_g + \vec{R}$$

$$\sum F_x = 0; \quad \sum F_y = mg - bv = ma = m\frac{dv}{dt}$$

$$\frac{dv}{dt} = g - \frac{b}{m}v$$

This equation also tells you that

$$\frac{dv}{dt} = g - \frac{b}{m}v = g, \text{ when } v = 0$$

What does this mean?

The above equation also tells us that as time goes on the speed increases and the acceleration decreases, eventually reaching 0.

An object moving in a viscous medium will obtain speed to a certain speed (terminal speed) and then maintain the same speed without any more acceleration.

What is the terminal speed in above case?

$$\frac{dv}{dt} = g - \frac{b}{m}v = 0; \quad v_t = \frac{mg}{b}$$

How do the speed and acceleration depend on time?

$$v = \frac{mg}{b}\left(1 - e^{-bt/m}\right); \quad v = 0 \text{ when } t = 0;$$

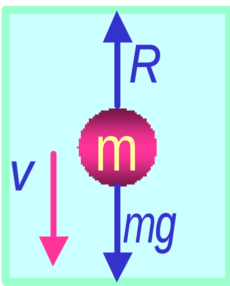
$$a = \frac{dv}{dt} = \frac{mg}{b}\frac{b}{m}e^{-bt/m} = ge^{-t/\tau}; \quad a = g \text{ when } t = 0;$$

$$\frac{dv}{dt} = \frac{mg}{b}\frac{b}{m}e^{-t/\tau} = \frac{mg}{b}\frac{b}{m}\left(1 - 1 + e^{-t/\tau}\right) = g - \frac{b}{m}v$$

The time needed to reach 63.2% of the terminal speed is defined as the time constant, τ=m/b.

Example 6.11

A small ball of mass 2.00g is released from rest in a large vessel filled with oil, where it experiences a resistive force proportional to its speed. The ball reaches a terminal speed of 5.00 cm/s. Determine the time constant τ and the time it takes the ball to reach 90% of its terminal speed.



Determine the time constant τ.

$$v_t = \frac{mg}{b}$$

$$\therefore b = \frac{mg}{v_t} = \frac{2.00\times10^{-3}kg \cdot 9.80m/s^2}{5.00\times10^{-2}m/s} = 0.392kg/s$$

$$\tau = \frac{m}{b} = \frac{2.00\times10^{-3}kg}{0.392kg/s} = 5.10\times10^{-3}s$$

Determine the time it takes the ball to reach 90% of its terminal speed.

$$v = \frac{mg}{b}\left(1-e^{-t/\tau}\right) = v_t\left(1-e^{-t/\tau}\right)$$

$$0.9v_t = v_t\left(1-e^{-t/\tau}\right)$$

$$\left(1-e^{-t/\tau}\right) = 0.9;\ e^{-t/\tau} = 0.1$$

$$t = -\tau \cdot \ln 0.1 = 2.30\tau = 2.30 \cdot 5.10\times10^{-3} = 11.7(ms)$$

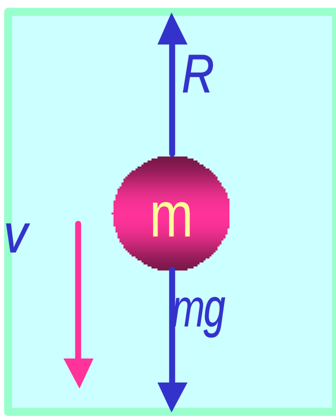
Air Drag

For objects moving through air at high speed, the resistive force is roughly proportional to the square of the speed.

The magnitude of such resistive force

$$R = \frac{1}{2} D \rho A v^2$$

D: Drag Coefficient (dim.less)
ρ: Density of Air
A: Cross section of the object



Let's analyze a falling object through the air

Force

$$\sum F_y = m a_y = \frac{1}{2} D \rho A v^2 - mg$$

Acceleration

$$a_y = \left(\frac{D \rho A}{2m} \right) v^2 - g$$

Terminal Speed

$$a_y = \left(\frac{D \rho A}{2m} \right) v^2 - g = 0; \quad v_t = \sqrt{\frac{2mg}{D \rho A}}$$

Example 6.4

A pitcher hurls a 0.145 kg baseball past a batter at 40.2m/s (=90 mi/h). Determine the drag coefficients of the air, density of air is 1.29kg/m^3, the radius of the ball is 3.70cm, and the terminal velocity of the ball in the air is 43.0 m/s.

Using the formula for terminal speed

$$A = pr^2 = p\left(0.0370\right)^2 = 4.30\times 10^{-3}\left(m^2\right)$$

$$D = \frac{2mg}{rAv_t^2} = \frac{2\cdot 0.145\cdot 9.80}{1.29\cdot 4.30\times 10^{-3}\cdot\left(43.0\right)^2} = 0.277$$

Find the resistive force acting on the ball at this speed.

$$R = \frac{1}{2}DrAv^2 = \frac{1}{2}\cdot 0.277\cdot 1.29\cdot 4.30\times 10^{-3}\cdot\left(40.2\right)^2 = 1.24\,N$$

Analytical Method vs Numerical Method

Analytical Method:

The method of solving problems using cause and effect relationship and mathematical expressions

When solving for a motion of an object we follow the procedure:

1. Find all the forces involved in the motion
2. Compute the net force
3. Compute the acceleration
4. Integrate the acceleration in time to obtain velocity
5. Integrate the velocity in time to obtain position

But not all problems are analytically solvable due to complex conditions applied to the given motion, e.g. position dependent acceleration, etc.

Numerical Method:

The method of solving problems using approximation and computational tools, such as computer programs, stepping through infinitesimal intervals ← *Integration by numbers…*

The Euler Method

Simplest Numerical Method for solving differential equations, by approximating the derivatives as ratios of finite differences.

Speed and the magnitude of Acceleration can be approximated in a small increment of time, Δt

$$a(t) \approx \frac{\Delta v}{\Delta t} = \frac{v(t+\Delta t) - v(t)}{\Delta t}$$

Thus the Speed at time $t+\Delta t$ is

$$v(t+\Delta t) \approx v(t) + a(t)\Delta t$$

In the same manner, the position of an object and speed are approximated in a small time increment, Δt

$$v(t) \approx \frac{\Delta x}{\Delta t} = \frac{x(t+\Delta t) - x(t)}{\Delta t}$$

And the position at time $t+\Delta t$ is

$$x(t+\Delta t) \approx x(t) + v(t)\Delta t + \frac{1}{2}a(t)(\Delta t)^2$$

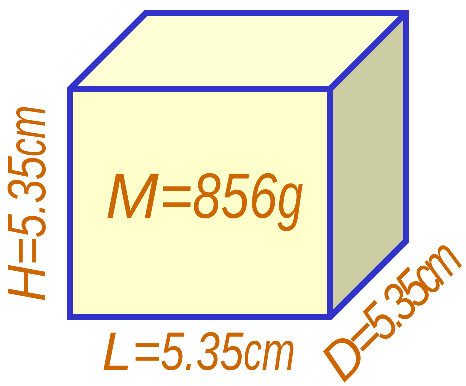
$$\xrightarrow[(\Delta t)^2 \approx 0]{} x(t+\Delta t) \approx x(t) + v(t)\Delta t + 0$$

The $(\Delta t)^2$ is ignored in Euler method because it is closed to 0.

We then use the approximated expressions to compute position at infinitesimal time interval, Δt, with computer to find the position of an object at any given time

Unit Conversion: Example 1.4

- US and UK still use British Engineering units: foot, lbs, and seconds
  - 1.0 in= 2.54 cm, 1ft=0.3048m=30.48cm
  - 1m=39.37in=3.281ft~1yd, 1mi=1609m=1.609km
  - 1lb=0.4535kg=453.5g, 1oz=28.35g=0.02835kg
  - Online unit converter: http://www.digitaldutch.com/unitconverter/
- Example 1.4: Determine density in basic SI units (*m,kg*)



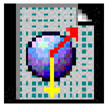
$$r = \frac{M}{V}$$

$$V = L \times D \times H = (5.35\ cm) \times (5.35\ cm) \times (5.35\ cm) = (5.35)^3\ cm^3$$

$$= 153.13\ cm^3 = \frac{153.13\ cm^3}{(100\ cm/m)^3} = 153.13 \times 10^{-6}\ m^3$$

$$M = 856\ g = \frac{856\ g}{1000\ g/kg} = 0.856\ kg$$

$$r = \frac{M}{V} = \frac{0.856\ kg}{153.13 \times 10^{-6}\ m^3} = 5.59 \times 10^3\ kg/m^3$$



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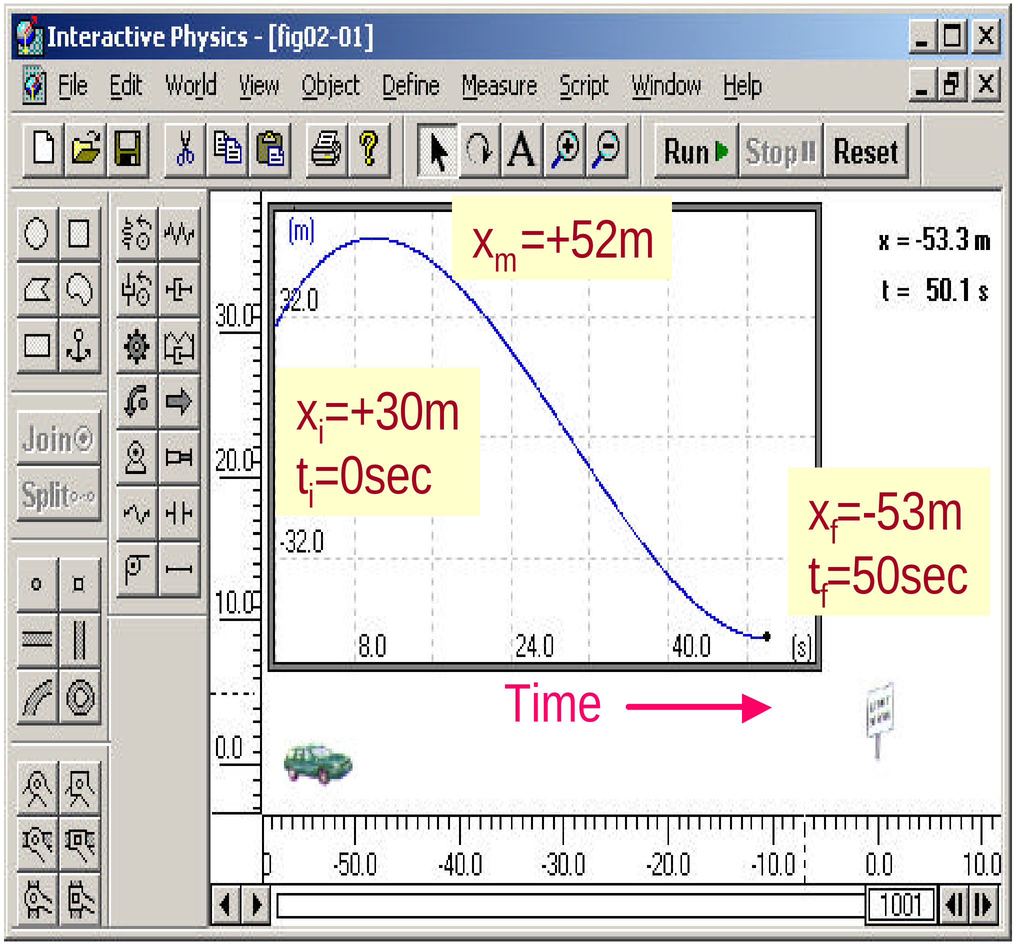
Example 2.1



- Find the displacement, average velocity, and average speed.
- Displacement:

$$\Delta x \equiv x_f - x_i = -53 - 30 = -83(m)$$

- Average Velocity:

$$v_x \equiv \frac{x_f - x_i}{t_f - t_i} = \frac{\Delta x}{\Delta t} = \frac{-83}{50} = -1.7(m/s)$$

- Average Speed:

$$v \equiv \frac{\text{Total Distance Traveled}}{\text{Total Time Interval}} = \frac{22+52+53}{50} = \frac{127}{50} = 2.5(m/s)$$

Example 2.2

- Particle is moving along x-axis following the expression: $\boxed{x = -4t + 2t^2}$
- Determine the displacement in the time intervals t=0 to t=1s and t=1 to t=3s:

For interval t=0 to t=1s

$$x_{t=0} = 0, x_{t=1} = -4 \times (1) + 2 \times (1)^2 = -2$$
$$\Delta x_{t=0,1} = x_{t=1} - x_{t=0} = -2 - 0 = -2(m)$$

For interval t=1 to t=3s

$$x_{t=1} = -2, x_{t=3} = -4 \times (3) + 2 \times (3)^2 = 6$$
$$\Delta x_{t=1,3} = x_{t=3} - x_{t=1} = 6 + 2 = 8(m)$$

- Compute the average velocity in the time intervals t=0 to t=1s and t=1 to t=3s:

$$v_x = \frac{\Delta x_{t=0,1}}{\Delta t} = \frac{-2}{1}(m/s)$$

$$v_x = \frac{\Delta x_{t=1,3}}{\Delta t} = \frac{8}{2} = +4(m/s)$$

- Compute the instantaneous **velocity at t=2.5s:**

Instantaneous velocity at any time *t*

$$v_x(t) = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} = \frac{d}{dt}\left(-4t + 2t^2\right) = -4 + 4t$$

Instantaneous velocity at *t*=2.5s

$$v_x(t = 2.5) = -4 + 4 \times (2.5) = +6(m/s)$$

Kinetic Equation of Motion in a Straight Line Under Constant Acceleration

$$\boxed{v_{xf}(t) = v_{xi} + a_x t}$$

Velocity as a function of time

$$\boxed{x_f - x_i = \frac{1}{2}\overline{v_x}t = \frac{1}{2}\left(v_{xf} + v_{xi}\right)t}$$

Displacement as a function of velocity and time

$$\boxed{x_f - x_i = v_{xi}t + \frac{1}{2}a_x t^2}$$

Displacement as a function of time, velocity, and acceleration

$$\boxed{v_{xf}^2 = v_{xi}^2 + 2a_x\left(x_f - x_i\right)}$$

Velocity as a function of Displacement and acceleration

You may use different forms of Kinetic equations, depending on the information given to you for specific physical problems!!

Example 2.12

Stone was thrown straight upward at t=0 with +20.0m/s initial velocity on the roof of a 50.0m high building,

1. Find the time the stone reaches at maximum height (*v=0*)
2. Find the maximum height
3. Find the time the stone reaches its original height
4. Find the velocity of the stone when it reaches its original height
5. Find the velocity and position of the stone at t=5.00s

$g=-9.80m/s^2$

1

$$v_f = v_{yi} + a_y t = +20.0 - 9.80t = 0.00$$

$$t = \frac{20.0}{9.80} = 2.04s$$

2

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$

$$= 50.0 + 20 \times 2.04 + \frac{1}{2} \times (-9.80) \times (2.04)^2$$

$$= 50.0 + 20.4 = 70.4(m)$$

3

$$t = 2.04 \times 2 = 4.08s$$

Other ways?

4

$$v_{yf} = v_{yi} + a_y t = 20.0 + (-9.80) \times 4.08 = -20.0(m/s)$$

5-Velocity

$$v_{yf} = v_{yi} + a_y t$$

$$= 20.0 + (-9.80) \times 5.00$$

$$= -29.0(m/s)$$

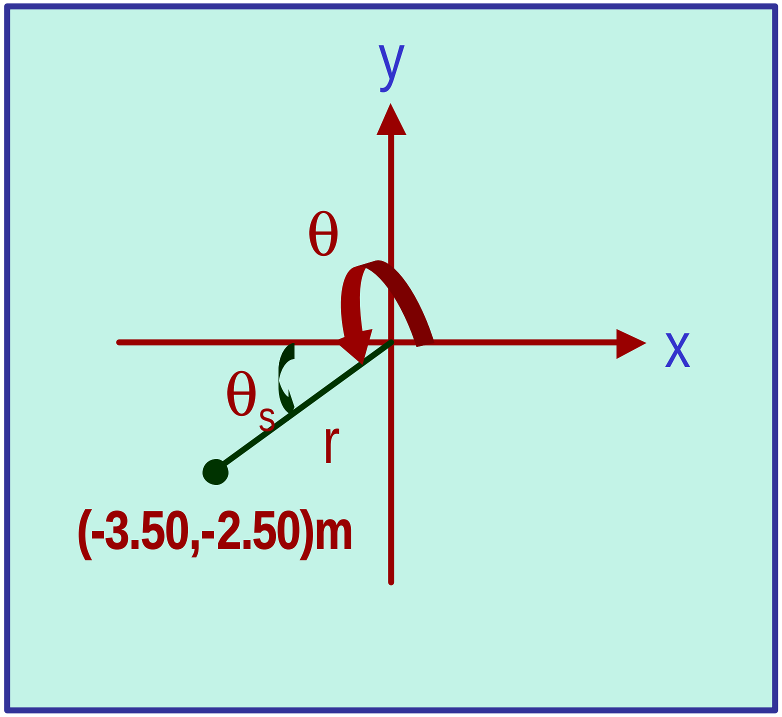
5-Position

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$

$$= 50.0 + 20.0 \times 5.00 + \frac{1}{2} \times (-9.80) \times (5.00)^2 = +27.5(m)$$

Example 3.1

Cartesian Coordinate of a point in the xy plane are (x,y)= (-3.50,-2.50)m. Find the polar coordinates of this point.



$$r = \sqrt{\left(x_1^2 + y_1^2\right)} =$$

$$\sqrt{\left((-3.50)^2 + (-2.50)^2\right)}$$

$$= \sqrt{18.5} = 4.30(m)$$

$$q = 180 + q_s$$

$$\tan q_s = \frac{-2.50}{-3.50} = \frac{5}{7}$$

$$q_s = \tan^{-1}\left(\frac{5}{7}\right) = 35.5^\circ$$

$$\therefore q = 180 + q_s = 180^\circ + 35.5^\circ = 216^\circ$$

2-dim Motion Under Constant Acceleration

- Position vectors in xy plane: $\vec{r_i} = x_i \vec{i} + y_i \vec{j}$ $\quad \vec{r_f} = x_f \vec{i} + y_f \vec{j}$
- Velocity vectors in xy plane: $\vec{v_i} = v_{xi} \vec{i} + v_{yi} \vec{j}$ $\quad \vec{v_f} = v_{xf} \vec{i} + v_{yf} \vec{j}$

$$v_{xf} = v_{xi} + a_x t, v_{yf} = v_{yi} + a_y t$$
$$\vec{v_f} = (v_{xi} + a_x t)\vec{i} + (v_{yi} + a_y t)\vec{j} = \vec{v_i} + \vec{a}t$$

- How are the position vectors written in acceleration vectors?

$$x_f = x_i + v_{xi} t + \frac{1}{2} a_x t^2, y_f = y_i + v_{yi} t + \frac{1}{2} a_y t^2$$
$$\vec{r_f} = \left(x_i + v_{xi} t + \frac{1}{2} a_x t^2 \right)\vec{i} + \left(y_i + v_{yi} t + \frac{1}{2} a_y t^2 \right)\vec{j}$$
$$= \vec{r_i} + \vec{v}t + \frac{1}{2}\vec{a}t^2$$

Ex04-01.ip

Example 4.1

A particle starts at origin when t=0 with an initial velocity **v**=(20**i**-15**j**)m/s. The particle moves in the xy plane with a_x=4.0m/s². Determine the components of velocity vector at any time, t.

$$v_{xf} = v_{xi} + a_x t = 20 + 4.0t\ (m/s)$$

$$v_{yf} = v_{yi} + a_y t = -15\ (m/s)$$

$$\vec{v}(t) = \{(20 + 4.0t)\vec{i} - 15\vec{j}\}m/s$$

Compute the velocity and speed of the particle at t=5.0 s.

$$\vec{v} = \{(20 + 4.0\times 5.0)\vec{i} - 15\vec{j}\}m/s = (40\vec{i} - 15\vec{j})m/s$$

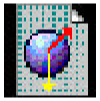
$$speed = |\vec{v}| = \sqrt{(v_x)^2 + (v_y)^2} = \sqrt{(40)^2 + (-15)^2} = 43\ m/s$$

$$q = \tan^{-1}\left(\frac{-15}{40}\right) = \tan^{-1}\left(\frac{-3}{8}\right) = -21^\circ$$

Determine the *x* and *y* components of the particle at t=5.0 s.

$$x_f = v_{xi}t + \frac{1}{2}a_x t^2 = 20\times 5 + \frac{1}{2}\times 4\times 5^2 = 150\ m,\quad y_f = v_{yi}t = -15\times 5 = -75\ m$$

$$\vec{r}_f = x_f\vec{i} + y_f\vec{j} = (150\vec{i} - 75\vec{j})m$$



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Example 4.5

- A stone was thrown upward from the top of a building at an angle of 30° to horizontal with initial speed of 20.0m/s. If the height of the building is 45.0m, how long is it before the stone hits the ground?

$$v_{xi} = v_i \cos q_i = 20.0 \times \cos 30^\circ = 17.3\, m/s$$

$$v_{yi} = v_i \sin q_i = 20.0 \times \sin 30^\circ = 10.0\, m/s$$

$$y_f = -45.0 = v_{yi} t - \frac{1}{2} g t^2$$

$$g t^2 - 20.0 t - 90.0 = 9.80 t^2 - 20.0 t - 90.0 = 0$$

$$t = \frac{20.0 \pm \sqrt{(-20)^2 - 4 \times 9.80 \times (-90)}}{2 \times 9.80}$$

$$t = -2.18\, s \text{ or } t = 4.22\, s$$

$$\therefore t = 4.22\, s$$

- What is the speed of the stone just before it hits the ground?

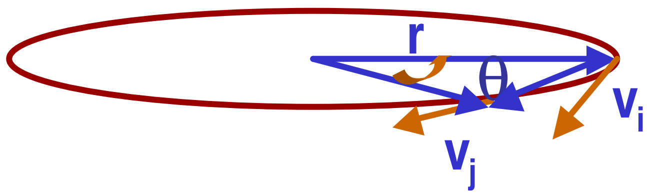
$$v_{xf} = v_{xi} = v_i \cos q_i = 20.0 \times \cos 30^\circ = 17.3\, m/s$$

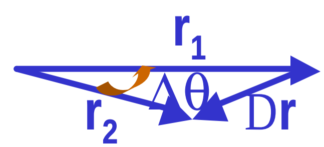
$$v_{yf} = v_{yi} - g t = v_i \sin q_i - g t = 10.0 - 9.80 \times 4.22 = -31.4\, m/s$$

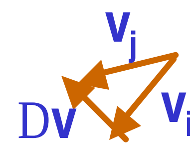
$$|v| = \sqrt{v_{xf}^2 + v_{yf}^2} = \sqrt{17.3^2 + (-31.4)^2} = 35.9\, m/s$$

Uniform Circular Motion

- A motion with a constant speed on a circular path.
  - The velocity of the object changes, because the direction changes
  - Therefore, there is an acceleration







Angle is Δθ

The acceleration pulls the object inward: *Centripetal Acceleration*

Average Acceleration

$$\vec{a} = \frac{\overrightarrow{v_f} - \overrightarrow{v_i}}{t_f - t_i} = \frac{\Delta\vec{v}}{\Delta t}$$

$$q = \frac{\left|\Delta\vec{v}\right|}{v} = \frac{\left|\Delta\vec{r}\right|}{r}, \quad \left|\Delta\vec{v}\right| = v\frac{\left|\Delta\vec{r}\right|}{r}, \quad \bar{a} = \frac{v}{\Delta t}\frac{\left|\Delta\vec{r}\right|}{r}$$

Instantaneous Acceleration

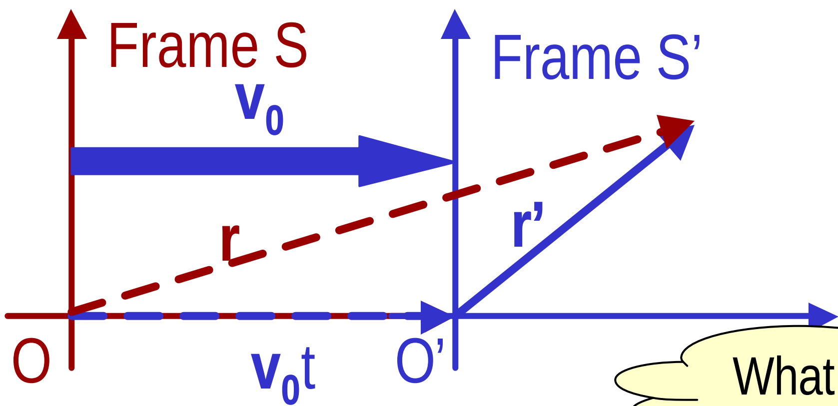
$$a_r = \lim_{\Delta t \to 0} \bar{a} = \lim_{\Delta t \to 0} \frac{\left|\Delta\vec{r}\right|}{\Delta t}\frac{v}{r} = v \times \frac{v}{r} = \frac{v^2}{r}$$

Is this correct in dimension?

What story is this expression telling you?

Relative Velocity and Acceleration

The velocity and acceleration in two different frames of references can be denoted:



Galilean transformation equation

$$\vec{r}' = \vec{r} - \vec{v}_0 t$$

$$\frac{d\vec{r}'}{dt} = \frac{d\vec{r}}{dt} - \vec{v}_0$$

$$\vec{v}' = \vec{v} - \vec{v}_0$$

What does this tell you?

$$\vec{r}' = \vec{r} - \vec{v}_0 t$$

$$\frac{d\vec{v}'}{dt} = \frac{d\vec{v}}{dt} - \frac{d\vec{v}_0}{dt}$$

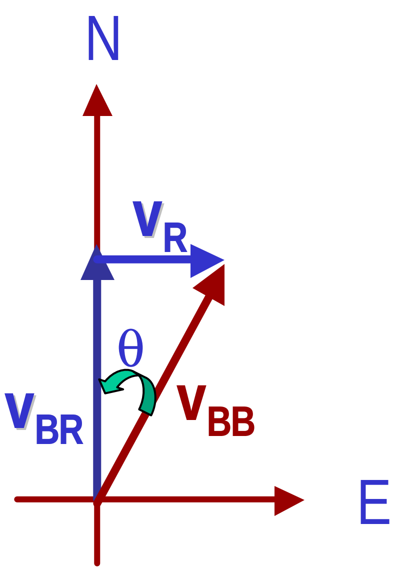
$$\vec{a}' = \vec{a}, \text{ when } \vec{v}_0 \text{ is constant}$$

The accelerations measured in two frames are the same when the frames move at a constant velocity with respect to each other!!!

The earth's gravitational acceleration is the same in a frame moving at a constant velocity wrt the earth.

Example 4.9

A boat heading due north with a speed 10.0km/h is crossing the river whose stream has a uniform speed of 5.00km/h due east. Determine the velocity of the boat seen by the observer on the bank.



$$\vec{v}_{BB} = \vec{v}_{BR} + \vec{v}_{R}$$

$$\left|\vec{v}_{BB}\right| = \sqrt{\left|\vec{v}_{BR}\right|^2 + \left|\vec{v}_{R}\right|^2} = \sqrt{(10.0)^2 + (5.00)^2} = 11.2\, km/h$$

$$\because \vec{v}_{BR} = 10.0\,\hat{j} \text{ and } \vec{v}_{R} = 5.00\,\hat{i}$$

$$\vec{v}_{BB} = 5.00\,\hat{i} + 10.0\,\hat{j}$$

$$q = \tan^{-1}\left(\frac{v_{BBy}}{v_{BBx}}\right) = \tan^{-1}\left(\frac{5.00}{10.0}\right) = 26.6^{\circ}$$

How long would it take for the boat to cross the river if the width is 3.0km?

$$v_{BB}\cos q \bullet t = 3.0km$$

$$t = \frac{3.0}{v_{BB}\cos q} = \frac{3.0}{11.2 \times \cos(26.6^{\circ})} = 0.30hrs = 18\min$$

Force

We've been learning kinematics; describing motion without understanding what the cause of the motion was. Now we are going to learn dynamics!!

Can someone tell me what FORCE is?

FORCEs are what cause an object to move

The above statement is not entirely correct. Why?

Because when an object is moving with a constant velocity no force is exerted on the object!!!

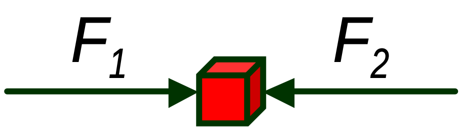
FORCEs are what cause any change in the velocity of an object!!

What does this statement mean?

When there is force, there is change of velocity. Forces cause acceleration.

What happens there are several forces being exerted on an object?

Forces are vector quantities, so vector sum of all forces, the NET FORCE, determines the motion of the object.



NET FORCE, $F = F_1 + F_2$

When net force on an objectis 0, the has constant velocity and is at its equilibrium!!

Newton's Laws

1st Law:
Law of Inertia

In the absence of external forces, an object at rest remains at rest and an object in motion continues in motion with a constant velocity.

2nd Law:
Law of Forces

$$\sum_i \vec{F}_i = m\vec{a}$$

The acceleration of an object is directly proportional to the net force exerted on it and inversely proportional to the object's mass.

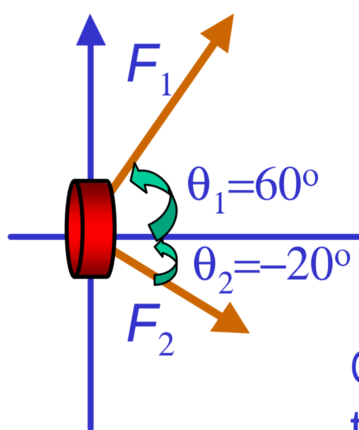
3rd Law:
Law of Action and Reaction

$$\vec{F}_{21} = -\vec{F}_{12}$$

If two objects interact, the force, F_{12}, exerted on object 1 by object 2 is equal magnitude to and opposite direction to the force, F_{21}, exerted on object 1 by object 2.

Example 5.1

Determine the magnitude and direction of acceleration of the puck whose mass is 0.30kg and is being pulled by two forces, **F1** and **F2,** as shown in the picture, whose magnitudes of the forces are 8.0 N and 5.0 N, respectively.



Components of F_1

$$F_{1x} = \left|\vec{F}_1\right| \cos\theta_1 = 8.0 \times \cos\left(60^\circ\right) = 4.0\,N$$

$$F_{1y} = \left|\vec{F}_1\right| \sin\theta_1 = 8.0 \times \sin\left(60^\circ\right) = 6.9\,N$$

Components of F_2

$$F_{2x} = \left|\vec{F}_2\right| \cos\theta_2 = 5.0 \times \cos\left(-20^\circ\right) = 4.7\,N$$

$$F_{2y} = \left|\vec{F}_2\right| \sin\theta_2 = 5.0 \times \sin\left(-20^\circ\right) = -1.7\,N$$

Components of total force F

$$F_x = F_{1x} + F_{2x} = 4.0 + 4.7 = 8.7\,N = ma_x$$

$$F_y = F_{1y} + F_{2y} = 6.9 - 1.7 = 5.2\,N = ma_y$$

Magnitude and direction of acceleration a

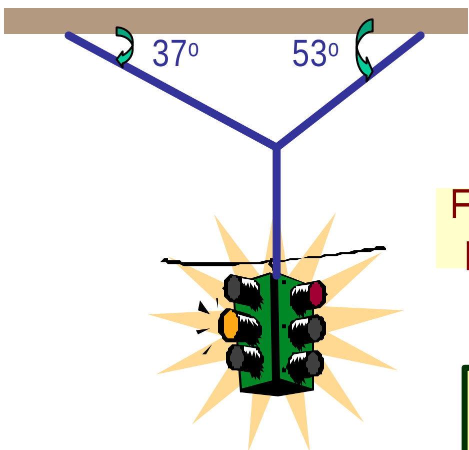
$$a_x = \frac{F_x}{m} = \frac{8.7}{0.3} = 29\,m/s^2, \quad a_y = \frac{F_y}{m} = \frac{5.2}{0.3} = 17\,m/s^2$$

$$\left|\vec{a}\right| = \sqrt{(29)^2 + (17)^2} = 34\,m/s^2, \quad \theta = \tan^{-1}\left(\frac{a_y}{a_x}\right) = \tan^{-1}\left(\frac{17}{29}\right) = 30^\circ$$

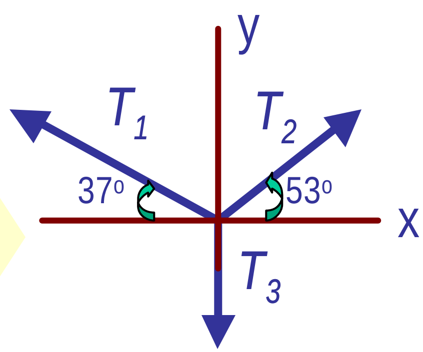
$$\vec{a} = a_x\hat{i} + a_y\hat{j} = \left(29\hat{i} + 17\hat{j}\right)m/s^2$$

Example 5.4

A traffic light weighing 125 N hangs from a cable tied to two other cables fastened to a support. The upper cables make angles of 37.0° and 53.0° with the horizontal. Find the tension in the three cables.



Free-body Diagram



$$\vec{F} = \vec{T_1} + \vec{T_2} + \vec{T_3}; \quad F_x = \sum_{i=1}^{i=3} T_{ix} = 0; \quad F_y = \sum_{i=1}^{i=3} T_{iy} = 0$$

$$T_1 \sin(37^\circ) + T_2 \sin(53^\circ) - mg = 0$$

$$-T_1 \cos(37^\circ) + T_2 \cos(53^\circ) = 0$$

$$\therefore T_1 = \frac{\cos(53^\circ)}{\cos(37^\circ)} T_2 = 0.754\, T_2$$

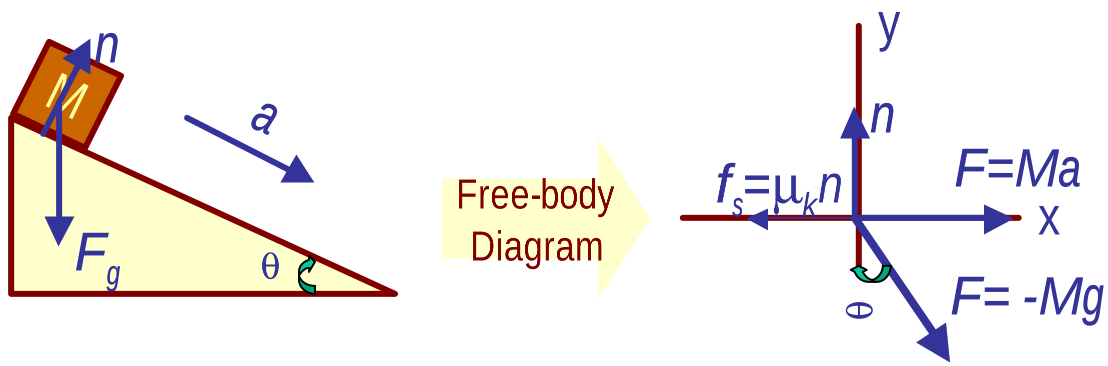
$$T_2 \left[\sin(53^\circ) + 0.754 \times \sin(37^\circ)\right] = 1.25\, T_2 = 125\ N$$

$$T_2 = 100\ N; \quad T_1 = 0.754\, T_2 = 75.4\ N$$

Example 5.12

Ex05-12.ip

Suppose a block is placed on a rough surface inclined relative to the horizontal. The inclination angle is increased till the block starts to move. Show that by measuring this critical angle, θ_c, one can determine coefficient of static friction, μ_s.



$$\vec{F} = \vec{F_g} + \vec{n} + f_s$$

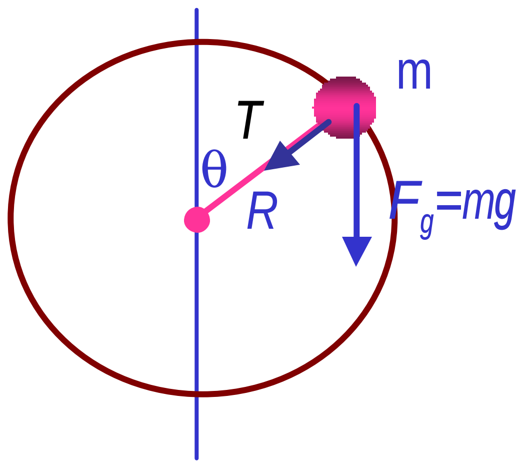
$$F_y = Ma_y = n + F_{gy} = 0; \quad n = -F_{gy} = Mg\cos\theta_c$$

$$F_x = F_{gx} - f_s = 0; \quad f_s = \mu_s n = Mg\sin\theta_c$$

$$\mu_s = \frac{Mg\sin\theta_c}{n} = \frac{Mg\sin\theta_c}{Mg\cos\theta_c} = \tan\theta_c$$

Example 6.8

A ball of mass m is attached to the end of a cord of length R. The ball is moving in a vertical circle. Determine the tension of the cord at any instant when the speed of the ball is *v* and the cord makes an angle θ with vertical.



What are the forces involved in this motion?

The gravitational force F_g and the radial force, T, providing tension.

$$\sum F_t = ma_t = mg \sin q$$

$$a_t = g \sin q$$

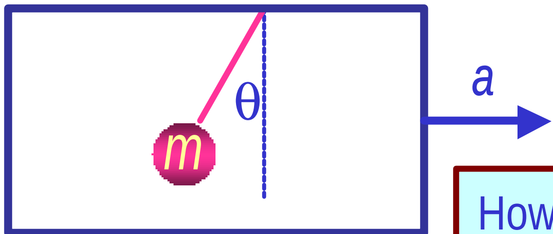
$$\sum F_r = T - mg \cos q = ma_r = m \frac{v^2}{R}$$

$$T = m\left(\frac{v^2}{R} + g \cos q\right)$$

At what angles the tension becomes maximum and minimum. What are the tension?

Example 6.9

A ball of mass m is is hung by a cord to the ceiling of a boxcar that is moving with an acceleration *a*. What do the inertial observer at rest and the non-inertial observer traveling inside the car conclude? How do they differ?

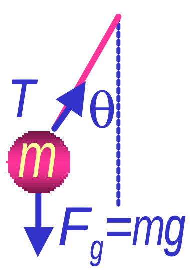


This is how the ball looks like no matter which frame you are in.

How do the free-body diagrams look for two frames?

How do the motions interpreted in these two frames? Any differences?

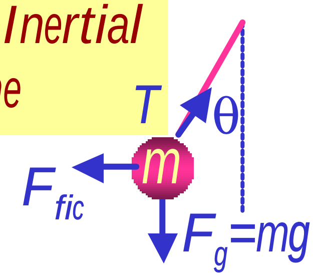
Inertial Frame



$$\sum \vec{F} = \vec{F}_g + \vec{T}$$
$$\sum F_x = ma_x = ma_c = T\sin\theta$$
$$\sum F_y = T\cos\theta - mg = 0$$
$$T = \frac{mg}{\cos\theta};\quad a_c = g\tan\theta$$

For an inertial frame observer, the forces being exerted on the ball are only **T** and $\mathbf{F_g}$. The acceleration of the ball is the same as that of the box car and is provided by the x component of the tension force.

Non-Inertial Frame



$$\sum \vec{F} = \vec{F}_g + \vec{T} + \vec{F}_{fic}$$
$$\sum F_x = T\sin\theta - F_{fic} = 0;\ F_{fic} = ma_{fic} = T\sin\theta$$
$$\sum F_y = T\cos\theta - mg = 0$$
$$T = \frac{mg}{\cos\theta};\quad a_{fic} = g\tan\theta$$

In the non-inertial frame observer, the forces being exerted on the ball are **T**, $\mathbf{F_g}$, and $\mathbf{F_{fic}}$.

For some reason the ball is under a force, $\mathbf{F_{fic}}$, that provides acceleration to the ball.

While the mathematical expression of the acceleration of the ball is identical to that of inertial frame observer's, the cause of the force is dramatically different.