1443-501 Spring 2002
Lecture #9

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1. Conservative Forces and Potential Energy
2. Conservation of Mechanical Energy
3. Work done by non-conservative forces
4. How are conservative forces and potential energy related?
5. Equilibrium of a system
6. General Energy Conservation
7. Mass-Energy Equivalence

Conservative Forces and Potential Energy

The work done on an object by a conservative force is equal to the decrease in the potential energy of the system

$$W_c = \int_{x_i}^{x_f} F_x dx = -\Delta U$$

What else does this statement tell you?

The work done by a conservative force is equal to the negative of the change of the potential energy associated with that force.

Only the changes in potential energy of a system is physically meaningful!!

We can rewrite the above equation in terms of potential energy U

$$\Delta U = U_f - U_i = -\int_{x_i}^{x_f} F_x dx$$

So the potential energy associated with a conservative force at any given position becomes

$$U_f(x) = -\int_{x_i}^{x_f} F_x dx + U_i$$

Potential energy function

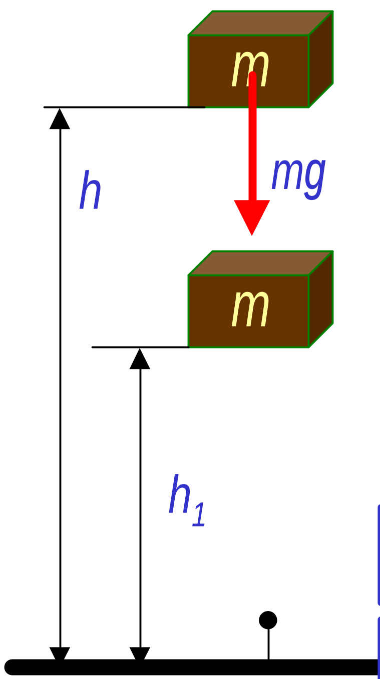
What can you tell from the potential energy function above?

Since U_i is a constant, it only shifts the resulting $U_f(x)$ by a constant amount. One can always change the initial potential so that U_i can be 0.

Conservation of Mechanical Energy

Total mechanical energy is the sum of kinetic and potential energies

$$E \equiv K + U$$



Let's consider a brick of mass *m* at a height *h* from the ground

What is its potential energy?

$$U_g = mgh$$

What happens to the energy as the brick falls to the ground?

$$\Delta U = U_f - U_i = -\int_{x_i}^{x_f} F_x dx$$

The brick gains speed

By how much?

$$v = gt$$

So what?

The brick's kinetic energy increased

$$K = \frac{1}{2}mv^2 = \frac{1}{2}mg^2t^2$$

And?

The lost potential energy converted to kinetic energy

What does this mean?

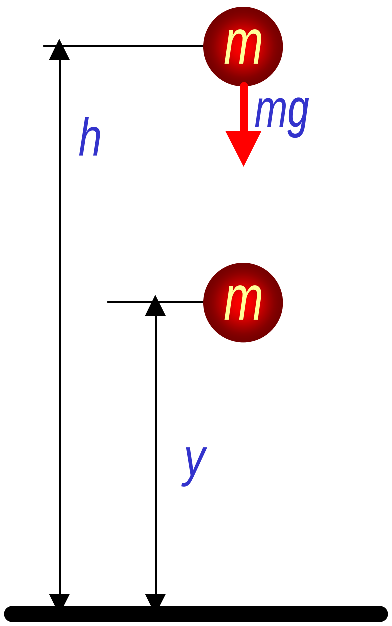
The total mechanical energy of a system remains constant in any isolated system of objects that interacts only through conservative forces: ***Principle of mechanical energy conservation***

$$E_i = E_f$$

$$K_i + \sum U_i = K_f + \sum U_f$$

Example 8.2

A ball of mass *m* is dropped from a height *h* above the ground. Neglecting air resistance determine the speed of the ball when it is at a height *y* above the ground.



Using the principle of mechanical energy conservation

$$K_i + U_i = K_f + U_f$$

$$0 + mgh = \frac{1}{2}mv^2 + mgy$$

$$\frac{1}{2}mv^2 = mg(h-y)$$

$$\therefore \; v = \sqrt{2g(h-y)}$$

b) Determine the speed of the ball at *y* if it had initial speed v_i at the time of release at the original height *h*.

Again using the principle of mechanical energy conservation but with non-zero initial kinetic energy!!!

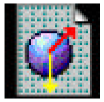
$$K_i + U_i = K_f + U_f$$

$$\frac{1}{2}mv_i^2 + mgh = \frac{1}{2}mv_f^2 + mgy$$

$$\frac{1}{2}m\left(v_f^2 - v_i^2\right) = mg(h-y)$$

$$\therefore \; v_f = \sqrt{v_i^2 + 2g(h-y)}$$

This result look very similar to a kinematic expression, doesn't it? Which one is it?



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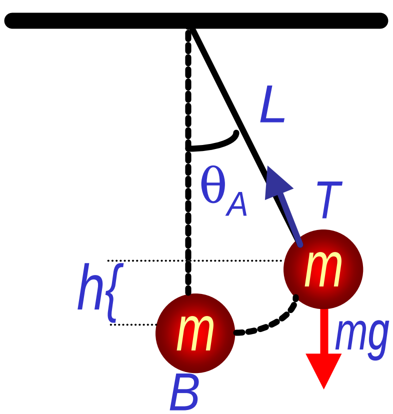
Example 8.3

A ball of mass *m* is attached to a light cord of length L, making up a pendulum. The ball is released from rest when the cord makes an angle θ_A with the vertical, and the pivoting point P is frictionless. Find the speed of the ball when it is at the lowest point, B.



Compute the potential energy at the maximum height, *h*. Remember where the 0 is.

$$h = L - L\cos\theta_A$$
$$U_i = mgL\left(1-\cos\theta_A\right)$$

Using the principle of mechanical energy conservation

$$K_i + U_i = K_f + U_f$$
$$mgh = mgL\left(1-\cos\theta_A\right) = \frac{1}{2}mv^2$$
$$v^2 = 2gL\left(1-\cos\theta_A\right)$$
$$\therefore v = \sqrt{2gL\left(1-\cos\theta_A\right)}$$

b) Determine tension T at the point B.

Recall the centripetal acceleration of a circular motion

$$F_r = ma_r = T - mg = m\frac{v^2}{L}$$
$$T = m\left(g + \frac{v^2}{L}\right) = m\left(g + \frac{2gL\left(1-\cos\theta_A\right)}{L}\right)$$
$$\therefore T = mg\left(3 - 2\cos\theta_A\right)$$

Cross check the result in a simple situation. What happens when the initial angle θ_A is 0? $T = mg$

Work Done by Non-conserve Forces

Mechanical energy of a system is not conserved when any one of the forces in the system is a non-conservative force.

Two kinds of non-conservative forces:

Applied forces: Forces that are external to the system. These forces can take away or add energy to the system. So the mechanical energy of the system is no longer conserved.

If you were to carry around a ball, the force you apply to the ball is external to the system of ball and the Earth. Therefore, you add kinetic energy to the ball-Earth system.

$$W_{you} + W_g = \Delta K; \quad W_g = -\Delta U$$

$$W_{you} = W_{app} = \Delta K + \Delta U$$

Kinetic Friction: Internal non-conservative force that causes irreversible transformation of energy. The friction force causes the kinetic and potential energy to transfer to internal energy

$$W_{friction} = \Delta K_{friction} = -f_k d$$

$$\Delta E = E_f - E_i = \Delta K + \Delta U = -f_k d$$

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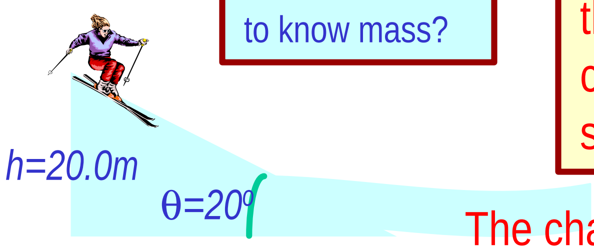
Example 8.6

A skier starts from rest at the top of frictionless hill whose vertical height is 20.0*m* and the inclination angle is 20º. Determine how far the skier can get on the snow at the bottom of the hill with a coefficient of kinetic friction between the ski and the snow is 0.210.



Don't we need to know mass?

Compute the speed at the bottom of the hill, using the mechanical energy conservation on the hill before friction starts working at the bottom

$$mgh = \frac{1}{2}mv^2; \quad v = \sqrt{2gh}$$

$$v = \sqrt{2 \times 9.8 \times 20.0} = 19.8 m/s$$

The change of kinetic energy is the same as the work done by kinetic friction.

What does this mean in this problem?

$$\Delta K = K_f - K_i = -f_k d$$

Since we are interested in the distance the skier can get to before stopping, the friction must do as much work as the available kinetic energy.

Since $K_f = 0$

$$-K_i = -f_k d; \quad f_k d = K_i$$

$$f_k = \mu_k n = \mu_k mg$$

$$d = \frac{K_i}{\mu_k mg} = \frac{\frac{1}{2}mv^2}{\mu_k mg} = \frac{v^2}{2\mu_k g}$$

$$= \frac{(19.8)^2}{2 \times 0.210 \times 9.80} = 95.2\ m$$

Well, it turns out we don't need to know mass.

What does this mean?

No matter how heavy the skier is he will get as far as anyone else has gotten.

How are Conserve Forces related to Potential Energy?

Work done by a force component on an object through a displacement Δx is

$$W = F_x \Delta x = -\Delta U$$

For an infinitesimal displacement Δx

$$\lim_{\Delta x \to 0} \Delta U = \lim_{\Delta x \to 0} F_x \Delta x$$

$$dU = -F_x dx$$

Results in the conservative force-potential relationship

$$F_x = -\frac{dU}{dx}$$

This relationship says that any conservative forces acting on an object within a given system is the same as the negative derivative of the potential energy of the system with respect to position.

Does this statement make sense?

1. spring-ball system:

$$F_s = -\frac{dU_s}{dx} = -\frac{d}{dx}\left(\frac{1}{2}kx^2\right) = -kx$$

2. Earth-ball system:

$$F_g = -\frac{dU_g}{dy} = -\frac{d}{dy}\left(mgy\right) = -mg$$

The relationship works in both the conservative force cases we have learned!!!

Energy Diagram and the Equilibrium of a System

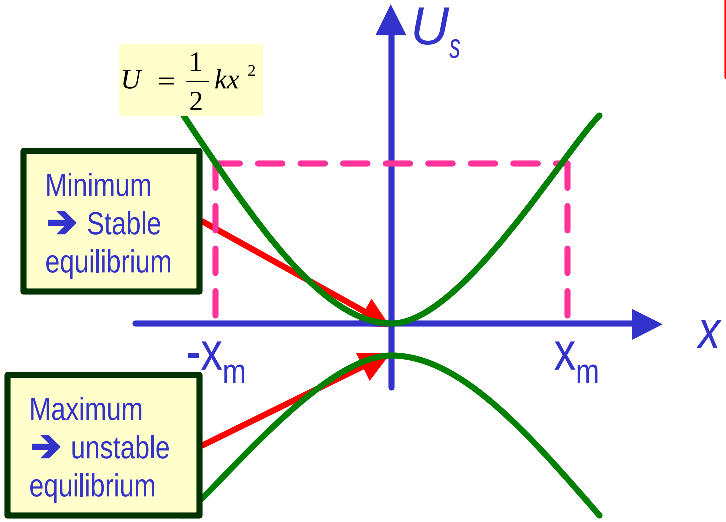
One can draw potential energy as a function of position ➔ *Energy Diagram*

Let's consider potential energy of a spring-ball system

$$U = \frac{1}{2}kx^2$$

What shape would this diagram be?

A Parabola



What does this energy diagram tell you?

1. *Potential energy for this system is the same independent of the sign of the position.*
2. *The force is 0 when the slope of the potential energy curve is 0 with respect to position.*
3. *x=0 is one of the stable or equilibrium of this system when the potential energy is minimum.*

Position of a stable equilibrium corresponds to points where potential energy is at a minimum.

Position of an unstable equilibrium corresponds to points where potential energy is a maximum.

General Energy Conservation and Mass-Energy Equivalence

General Principle of Energy Conservation

The total energy of an isolated system is conserved as long as all forms of energy are taken into account.

What about friction?

Friction is a non-conservative force and causes mechanical energy to change to other forms of energy.

However, if you add the new form of energy altogether the system as a whole did not lose any energy, as long as it is self-contained or isolated.

In the grand scale of the universe, no energy can be destroyed or created but just transformed or transferred from one place to another. Total energy of universe is constant.

Principle of Conservation of Mass

In any physical or chemical process, mass is neither created nor destroyed. Mass before a process is identical to the mass after the process.

Einstein's Mass-Energy equality.

$$\boxed{E_R = mc^2}$$

How many joules does your body correspond to?

Example 8.12

The sun converts 4.19x10^{9}kg of mass into energy per second. What is the power output of the sun?

Using Einstein's mass-energy equality

$$E_R = mc^2$$
$$= 4.19\times10^9 \times \left(3\times10^8\right)^2$$
$$= 37.7\times10^{25} J$$

Since the sun gives out this amount of energy per second the power is simply

$$P = 37.7\times10^{25} W$$

How many 60 W bulbs does this corresponds to? If the cost for electricity is 9c/kWh, how much does an 8 hour worth of sun's energy cost?

$$N_{60W} = \frac{P}{60} = 6.28\times10^{24} W$$

$$E = P \times t = 37.7 \times 10^{25} \times 8$$
$$= 3.02 \times 10^{27}\ kWh$$
$$Cost = \$3.02 \times 10^{27} \times 0.09$$
$$= \$2.72 \times 10^{26}$$