1443-501 Spring 2002
Lecture #11

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1. Center of Mass
2. Motion of a System of Particles
3. Angular Displacement, Velocity, and Acceleration
4. Angular Kinematics Under Constant Angular Acceleration

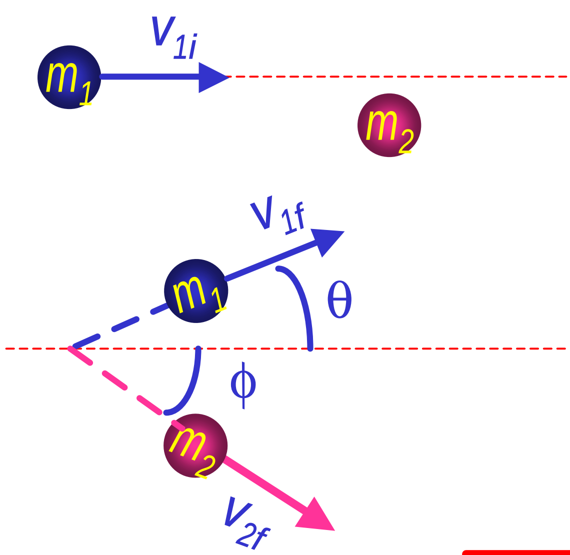
Homework: http://hw.utexas.edu/studentInstructions.html – Do Homework #3.
Unique ID for homework course is **43501**. Sorry….

Homework Assignment Pickup and Submission

1. Go to the URL https://hw.utexas.edu/roster.html and register yourself as a student by providing the information requested. Please keep your username and password well. The unique ID for the class is 43501 and there is no password assigned for this class.
2. Click "OK" button at the bottom of the page
3. Provide your username and password again in the new page brought up.
4. Once 1-3 is done, go to the "Student Login" page at URL https://hw.utexas.edu/index.html and log into the page by providing your username and password again.
5. You can pick up the home work by clicking on the button #2.1.
    1. You need to print it out to work on the homework problems.
    2. Solutions will be available after the due dates (1 week). The button #2.2 is solution pick up.
6. After the completion you can submit the homework by clicking on the button #3.1

Collisions in Two Dimension

In two dimension, one can use components of momentum to apply momentum conservation to solve physical problems.



$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$$

$$m_1 v_{1ix} + m_2 v_{2ix} = m_1 v_{1fx} + m_2 v_{2fx}$$

$$m_1 v_{1iy} + m_2 v_{2iy} = m_1 v_{1fy} + m_2 v_{2fy}$$

Consider a system of two particle collisions and scatters in two dimension as shown in the picture. (This is the case at fixed target accelerator experiments.) The momentum conservation tells us:

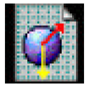
$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1i}$$

$$m_1 v_{1ix} = m_1 v_{1fx} + m_2 v_{2fx} = m_1 v_{1f}\cos\theta + m_2 v_{2f}\cos\phi$$

$$m_1 v_{1iy} = 0 = m_1 v_{1fy} + m_2 v_{2fy} = m_1 v_{1f}\sin\theta - m_2 v_{2f}\sin\phi$$

And for the elastic conservation, the kinetic energy is conserved:

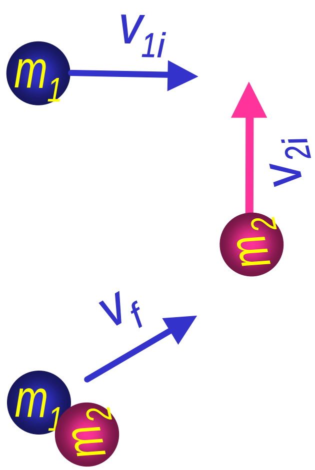
$$\frac{1}{2}m_1 v_{1i}^2 = \frac{1}{2}m_1 v_{1f}^2 + \frac{1}{2}m_2 v_{2f}^2$$



Ex09-09.ip

Example 9.9

A 1500kg car traveling east with a speed of 25.0 m/s collides at an interaction with a 2500kg van traveling north at a speed of 20.0 m/s. After the collision the two cars stuck to each other, and the wreckage is moving together. Determine the velocity of the wreckage after the collision, assuming the vehicles underwent a perfectly inelastic collision.



The initial momentum of the two car system before the collision is

$$\vec{p}_i = m_1 v_{1i}\vec{i} + m_2 v_{2i}\vec{j} = 1500 \times 25.0\vec{i} + 2500 \times 20.0\vec{j}$$
$$= 3.75\times10^4\vec{i} + 5.0\times10^4\vec{j}$$

The final momentum of the two car system after the perfectly inelastic collision is

$$\vec{p}_f = (m_1 + m_2)\left(v_{fx}\vec{i} + v_{fy}\vec{j}\right) = 4.0\times10^3 v_{fx}\vec{i} + 4.0\times10^3 v_{fy}\vec{j}$$

Using momentum conservation

$$\vec{p}_f = \vec{p}_i$$

$$v_{fx} = \frac{3.75\times10^4}{1500 + 2500} = 9.38\ m/s$$

$$v_{fy} = \frac{5.0\times10^4}{1500 + 2500} = 12.5\ m/s$$

$$\vec{v}_f = \left(9.38\vec{i} + 12.5\vec{j}\right)m/s$$

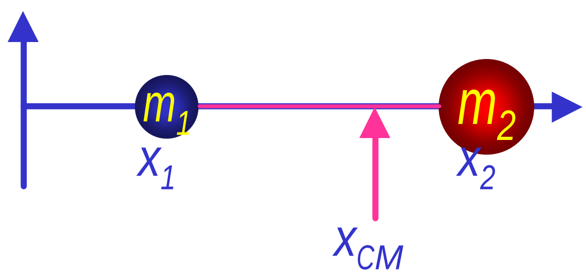
Center of Mass

We've been solving physical problems treating objects as sizeless points with masses, but in realistic situation objects have shapes with masses distributed throughout the body.

Center of mass of a system is the average position of the system's mass and represents the motion of the system as if all the mass is on the point.

What does above statement tell you concerning forces being exerted on the system?

The total external force exerted on the system of total mass M causes the center of mass to move at an acceleration given by $\vec{a} = \sum \vec{F} / M$ *as if all the mass of the system is concentrated on the center of mass.*



Consider a massless rod with two balls attached at either end.

The position of the center of mass of this system is the mass averaged position of the system

$$x_{CM} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

CM is closer to the heavier object

Center of Mass of a Rigid Object

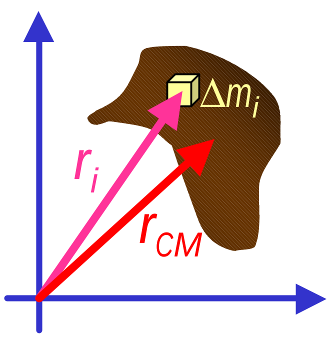
The formula for CM can be expanded to Rigid Object or a system of many particles

$$x_{CM} = \frac{m_1 x_1 + m_2 x_2 + \cdots + m_n x_n}{m_1 + m_2 + \cdots + m_n} = \frac{\sum_i m_i x_i}{\sum_i m_i}$$

$$y_{CM} = \frac{\sum_i m_i y_i}{\sum_i m_i}; \qquad z_{CM} = \frac{\sum_i m_i z_i}{\sum_i m_i}$$

The position vector of the center of mass of a many particle system is

$$\vec{r}_{CM} = x_{CM}\vec{i} + y_{CM}\vec{j} + z_{CM}\vec{k}$$
$$= \frac{\sum_i m_i x_i \vec{i} + \sum_i m_i y_i \vec{j} + \sum_i m_i z_i \vec{k}}{\sum_i m_i} = \frac{\sum_i m_i \vec{r}_i}{M}$$



A rigid body – an object with shape and size with mass spread throughout the body, ordinary objects – can be considered as a group of particles with mass m_i densely spread throughout the given shape of the object

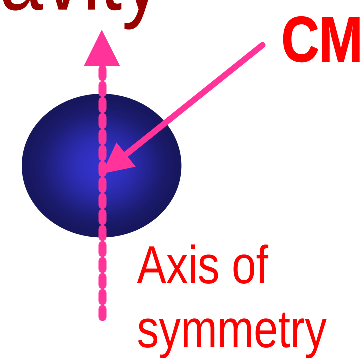
$$x_{CM} \approx \frac{\sum_i \Delta m_i x_i}{M}$$

$$x_{CM} = \lim_{\Delta m_i \to 0} \frac{\sum_i \Delta m_i x_i}{M} = \frac{1}{M}\int x\,dm$$

$$\vec{r}_{CM} = \frac{1}{M}\int \vec{r}\,dm$$

Center of Mass and Center of Gravity

The center of mass of any symmetric object lies on an axis of symmetry and on any plane of symmetry, if object's mass is evenly distributed throughout the body.

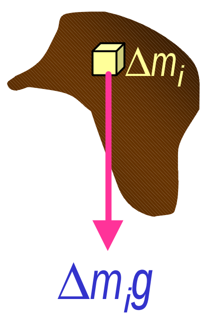


How do you think you can determine the CM of objects that are not symmetric?

One can use gravity to locate CM.

1. Hang the object by one point and draw a vertical line following a plum-bob.
2. Hang the object by another point and do the same.
3. The point where the two lines meet is the CM.

Center of Gravity



Since a rigid object can be considered as collection of small masses, one can see the total gravitational force exerted on the object as

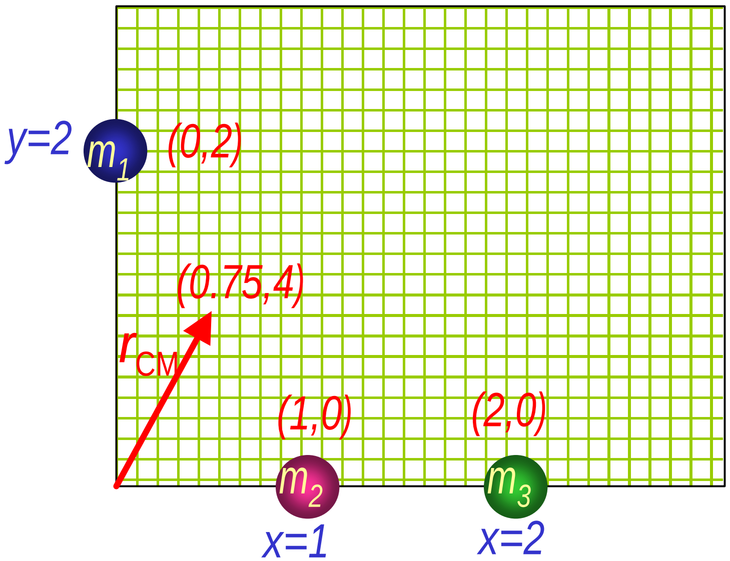
$$\vec{F}_g = \sum_i \vec{F}_i = \sum_i \Delta m_i \vec{g} = M\vec{g}$$

What does this equation tell you?

The net effect of these small gravitational forces is equivalent to a single force acting on a point (Center of Gravity) with mass M.

Example 9.12

A system consists of three particles as shown in the figure. Find the position of the center of mass of this system.



Using the formula for CM for each position vector component

$$x_{CM} = \frac{\sum_i m_i x_i}{\sum_i m_i}; \qquad y_{CM} = \frac{\sum_i m_i y_i}{\sum_i m_i}$$

One obtains

$$x_{CM} = \frac{\sum_i m_i x_i}{\sum_i m_i} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3} = \frac{m_2 + 2m_3}{m_1 + m_2 + m_3}$$

$$y_{CM} = \frac{\sum_i m_i y_i}{\sum_i m_i} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{m_1 + m_2 + m_3} = \frac{2m_1}{m_1 + m_2 + m_3}$$

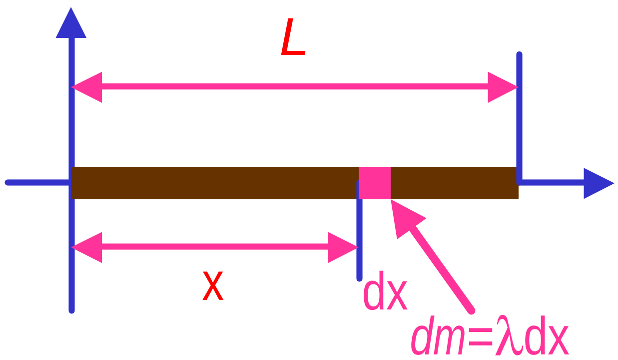
$$\vec{r}_{CM} = \frac{(m_2 + 2m_3)\vec{i} + 2m_1\vec{j}}{m_1 + m_2 + m_3}$$

If $m_1 = 2kg; m_2 = m_3 = 1kg$

$$\vec{r}_{CM} = \frac{3\vec{i} + 4\vec{j}}{4} = 0.75\vec{i} + \vec{j}$$

Example 9.13

Show that the center of mass of a rod of mass *M* and length *L* lies in midway between its ends, assuming the rod has a uniform mass per unit length.



The formula for CM of a continuous object is

$$x_{CM} = \frac{1}{M}\int_{x=0}^{x=L} x\,dm$$

Since the density of the rod is constant, one can write

$$dm = \lambda\,dx;\ \text{where}\ \ \lambda = M/L$$

Therefore

$$x_{CM} = \frac{1}{M}\int_{x=0}^{x=L} \lambda x\,dx = \frac{1}{M}\left[\frac{1}{2}\lambda x^2\right]_{x=0}^{x=L} = \frac{1}{M}\left(\frac{1}{2}\lambda L^2\right) = \frac{1}{M}\left(\frac{1}{2}ML\right) = \frac{L}{2}$$

Find the CM when the density of the rod non-uniform but varies linearly as a function of x, λ=α x

$$M = \int_{x=0}^{x=L} \lambda\,dx = \int_{x=0}^{x=L} \alpha x\,dx$$

$$= \left[\frac{1}{2}\alpha x^2\right]_{x=0}^{x=L} = \frac{1}{2}\alpha L^2$$

$$x_{CM} = \frac{1}{M}\int_{x=0}^{x=L} \lambda x\,dx = \frac{1}{M}\int_{x=0}^{x=L} \alpha x^2\,dx = \frac{1}{M}\left[\frac{1}{3}\alpha x^3\right]_{x=0}^{x=L}$$

$$= \frac{1}{M}\left(\frac{1}{3}\alpha L^3\right) = \frac{1}{M}\left(\frac{2}{3}ML\right) = \frac{2L}{3}$$

Motion of a Group of Particles

We've learned that the CM of a system can represent the motion of a system. Therefore, for an isolated system of many particles in which the total mass M is preserved, the velocity, total momentum, acceleration of the system are

Velocity of the system

$$\vec{v}_{CM}=\frac{d\vec{r}_{CM}}{dt}=\frac{d}{dt}\left(\frac{1}{M}\sum m_i\vec{r}_i\right)=\frac{1}{M}\sum m_i\frac{d\vec{r}_i}{dt}=\frac{\sum m_i\vec{v}_i}{M}$$

Total Momentum of the system

$$\vec{p}_{tot}=M\vec{v}_{CM}=M\frac{\sum m_i\vec{v}_i}{M}=\sum m_i\vec{v}_i=\sum\vec{p}_i$$

Acceleration of the system

$$\vec{a}_{CM}=\frac{d\vec{v}_{CM}}{dt}=\frac{d}{dt}\left(\frac{1}{M}\sum m_i\vec{v}_i\right)=\frac{1}{M}\sum m_i\frac{d\vec{v}_i}{dt}=\frac{\sum m_i\vec{a}_i}{M}$$

External force exerting on the system

$$\sum\vec{F}_{ext}=M\vec{a}_{CM}=\sum m_i\vec{a}_i=\frac{d\vec{p}_{tot}}{dt}$$

What about the internal forces?

If net external force is 0

$$\sum\vec{F}_{ext}=0=\frac{d\vec{p}_{tot}}{dt};\quad \vec{p}_{tot}=\text{const}$$

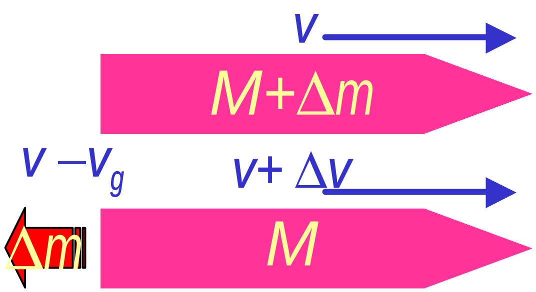
System's momentum is conserved.

Rocket Propulsion

What is the biggest difference between ordinary vehicles and a rocket?

The force that gives propulsion for normal vehicles is the friction between the surface of the road and the tire. The system in this case consists of the tire and the surface of the road. Newton's 3rd law and the momentum conservation of an isolated system.

Since there is no road to push against, the rockets obtain propulsion from momentum conservation in the system consists of the rocket and gas from burnt fuel.



Initial momentum before burning fuel

$$\vec{p} = (M + \Delta m)\vec{v}$$

Initial momentum after burning fuel and ejecting the gas

$$\vec{p} = M\left(\vec{v} + \Delta\vec{v}\right) - \Delta m\vec{v}_g$$

From momentum conservation

$$M\left(\vec{v} + \Delta\vec{v}\right) + \Delta m\left(\vec{v} - \vec{v}_g\right) = M\vec{v} + \Delta m\vec{v}; \quad M\Delta\vec{v} = \Delta m\vec{v}_g$$

Since dm is the same as –dM, one can obtain

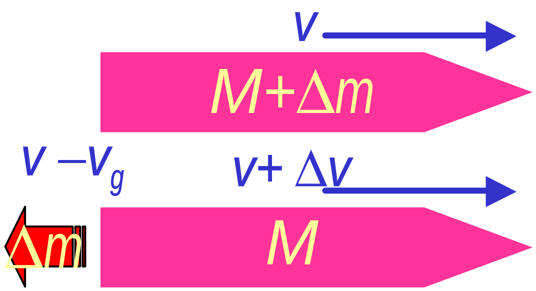
$$d\vec{v} = \frac{dm\vec{v}_g}{M}; \quad \int_i^f d\vec{v} = \vec{v}_f - \vec{v}_i = -\vec{v}_g\int_i^f \frac{dM}{M} = \vec{v}_g \ln\left(\frac{M_i}{M_f}\right)$$

Thrust is the force exerted on the rocket by the ejected gas

$$Thurst = M\frac{dv}{dt} = \left|v_g\frac{dM}{dt}\right|$$

Example 9.18

A rocket moving in free space has a speed of 3.0×10^3 m/s relative to the Earth. Its engines are turned on, and fuel is ejected in a direction opposite the rocket's motion at a speed of 5.0×10^3 m/s relative to rocket. A) What is the speed of the rocket relative to the Earth once its mass is reduced to one-half the mass before ignition?



Precisely the case we've discussed in the previous slide.

$$v_f = v_i + v_g \ln\left(\frac{M_i}{M_f}\right) = 3.0\times10^3 + 5.0\times10^3 \times \ln(2) = 6.5\times10^3\, m/s$$

Find the thrust on the rocket if it burns fuel at the rate of 50kg/s?

Since the thrust is given proportional to the rate of mass change or the rate the fuel burns as given in the formula

$$Thurst = M\frac{dv}{dt} = \left| v_g \frac{dM}{dt} \right|$$

One can obtain

$$Thurst = \left| v_g \frac{dM}{dt} \right| = 5.0\times10^5\, m/s \times 50 kg/s = 2.5\times10^7\, N$$