1443-501 Spring 2002
Lecture #12

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1. Motion of a System of Particles
2. Angular Displacement, Velocity, and Acceleration
3. Angular Kinematics Under Constant Angular Acceleration
4. Relationship Between Angular and Linear Quantities

Remember the mid-term exam on Wednesday, Mar. 13. Will cover Chapters 1-10.

Today's Homework Assignment is the Homework #4!!!

Motion of a Group of Particles

We've learned that the CM of a system can represent the motion of a system. Therefore, for an isolated system of many particles in which the total mass M is preserved, the velocity, total momentum, acceleration of the system are

Velocity of the system

$$\vec{v}_{CM} = \frac{d\vec{r}_{CM}}{dt} = \frac{d}{dt}\left(\frac{1}{M}\sum m_i\vec{r}_i\right) = \frac{1}{M}\sum m_i\frac{d\vec{r}_i}{dt} = \frac{\sum m_i\vec{v}_i}{M}$$

Total Momentum of the system

$$\vec{p}_{tot} = M\vec{v}_{CM} = M\frac{\sum m_i\vec{v}_i}{M} = \sum m_i\vec{v}_i = \sum \vec{p}_i$$

Acceleration of the system

$$\vec{a}_{CM} = \frac{d\vec{v}_{CM}}{dt} = \frac{d}{dt}\left(\frac{1}{M}\sum m_i\vec{v}_i\right) = \frac{1}{M}\sum m_i\frac{d\vec{v}_i}{dt} = \frac{\sum m_i\vec{a}_i}{M}$$

External force exerting on the system

$$\sum \vec{F}_{ext} = M\vec{a}_{CM} = \sum m_i\vec{a}_i = \frac{d\vec{p}_{tot}}{dt}$$

What about the internal forces?

If net external force is 0

$$\sum \vec{F}_{ext} = 0 = \frac{d\vec{p}_{tot}}{dt}; \quad \vec{p}_{tot} = \text{const}$$

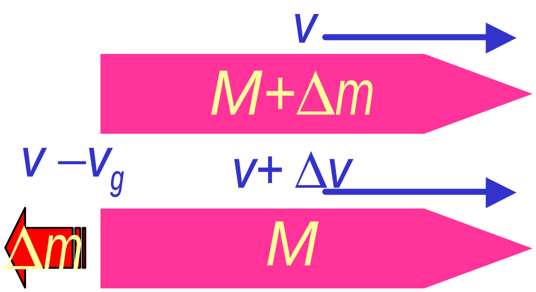
System's momentum is conserved.

Rocket Propulsion

What is the biggest difference between ordinary vehicles and a rocket?

The force that gives propulsion for normal vehicles is the friction between the surface of the road and the tire. The system in this case consists of the tire and the surface of the road. Newton's 3rd law and the momentum conservation of an isolated system.

Since there is no road to push against, the rockets obtain propulsion from momentum conservation in the system consists of the rocket and gas from burnt fuel.



Initial momentum before burning fuel

$$\vec{p} = (M + \Delta m)\vec{v}$$

Final momentum after burning fuel and ejecting the gas

$$\vec{p} = M\left(\vec{v} + \Delta\vec{v}\right) + \Delta m\left(\vec{v} - \vec{v}_g\right)$$

From momentum conservation

$$M\left(\vec{v} + \Delta\vec{v}\right) + \Delta m\left(\vec{v} - \vec{v}_g\right) = M\vec{v} + \Delta m\vec{v}; \quad M\Delta\vec{v} = \Delta m\vec{v}_g$$

Since dm is the same as –dM, one can obtain

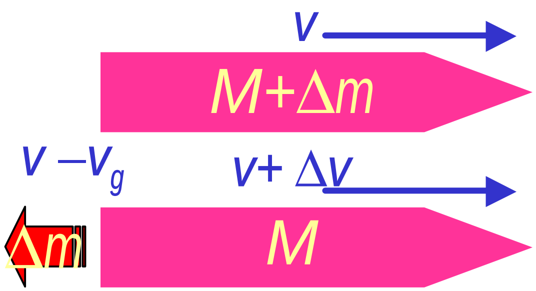
$$d\vec{v} = \frac{dm\vec{v}_g}{M}; \quad \int_i^f d\vec{v} = \vec{v}_f - \vec{v}_i = -\vec{v}_g \int_i^f \frac{dM}{M} = \vec{v}_g \ln\left(\frac{M_i}{M_f}\right)$$

Thrust is the force exerted on the rocket by the ejected gas

$$Thurst = M\frac{dv}{dt} = \left|v_g \frac{dM}{dt}\right|$$

Example 9.18

A rocket moving in free space has a speed of 3.0×10^3 m/s relative to the Earth. Its engines are turned on, and fuel is ejected in a direction opposite the rocket's motion at a speed of 5.0×10^3 m/s relative to rocket. A) What is the speed of the rocket relative to the Earth once its mass is reduced to one-half the mass before ignition?



Precisely the case we've discussed in the previous slide.

$$v_f = v_i + v_g \ln\left(\frac{M_i}{M_f}\right) = 3.0\times10^3 + 5.0\times10^3 \times \ln(2) = 6.5\times10^3 m/s$$

Find the thrust on the rocket if it burns fuel at the rate of 50kg/s?

Since the thrust is given proportional to the rate of mass change or the rate the fuel burns as given in the formula

$$Thurst = M\frac{dv}{dt} = \left|v_g \frac{dM}{dt}\right|$$

One can obtain

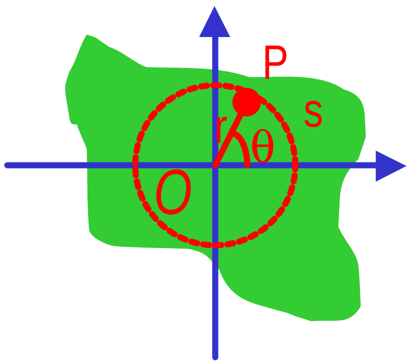
$$Thurst = \left|v_g \frac{dM}{dt}\right| = 5.0\times10^5 m/s \times 50kg/s = 2.5\times10^7 N$$

Fundamentals on Rotation

Linear motions can be described as the motion of the center of mass with all the mass of the object concentrated on it.

Is this still true for rotational motions?

No, because different parts of the object have different linear velocities and accelerations.



Consider a motion of a rigid body – an object that does not change its shape – rotating about the axis protruding out of the slide.

The arc length, or sergita, is $s = r\theta$

Therefore the angle, θ, is $\theta = \frac{s}{r}$*. And the unit of the angle is in radian.*

One radian is the angle swept by an arc length equal to the radius of the arc.

Since the circumference of a circle is 2πr, $360^\circ = 2\pi r / r = 2\pi$

The relationship between radian and degrees is $1\,\mathrm{rad} = 360^\circ / 2\pi = 180^\circ / \pi$

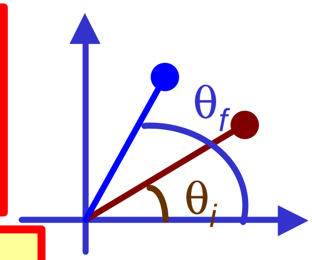
Angular Displacement, Velocity, and Acceleration

Using what we have learned in the previous slide, how would you define the angular displacement?

$$\Delta q = q_f - q_i$$

How about the average angular speed?

$$\overline{w} \equiv \frac{q_f - q_i}{t_f - t_i} = \frac{\Delta q}{\Delta t}$$



And the instantaneous angular speed?

$$w \equiv \lim_{\Delta t \to 0} \frac{\Delta q}{\Delta t} = \frac{dq}{dt}$$

By the same token, the average angular acceleration

$$\overline{a} \equiv \frac{w_f - w_i}{t_f - t_i} = \frac{\Delta w}{\Delta t}$$

And the instantaneous angular acceleration?

$$a \equiv \lim_{\Delta t \to 0} \frac{\Delta w}{\Delta t} = \frac{dw}{dt}$$

When rotating about a fixed axis, every particle on a rigid object rotates through the same angle and has the same angular speed and angular acceleration.

Rotational Kinematics

The first type of motion we have learned in linear kinematics was under a constant acceleration. We will learn about the rotational motion under constant acceleration, because these are the simplest motions in both cases.

Just like the case in linear motion, one can obtain

Angular Speed under constant angular acceleration:

$$w_f = w_i + at$$

Angular displacement under constant angular acceleration:

$$q_f = q_i + w_i t + \frac{1}{2} a t^2$$

One can also obtain

$$w_f^2 = w_i^2 + 2a\left(q_f - q_i\right)$$

Example 10.1

A wheel rotates with a constant angular acceleration pf 3.50 rad/s^2. If the angular speed of the wheel is 2.00 rad/s at t_i=0, a) through what angle does the wheel rotate in 2.00s?

Using the angular displacement formula in the previous slide, one gets

$$q_f - q_i = wt + \frac{1}{2}at^2$$

$$= 2.00 \times 2.00 + \frac{1}{2}3.50 \times (2.00)^2$$

$$= 11.0 rad = \frac{11.0}{2p} rev. = 1.75 rev.$$

What is the angular speed at t=2.00s?

Using the angular speed and acceleration relationship

$$w_f = w_i + at$$

$$= 2.00 + 3.50 \times 2.00$$

$$= 9.00 rad / s$$

Find the angle through which the wheel rotates between t=2.00 s and t=3.00 s.

$$q_2 = 11.0 rad$$

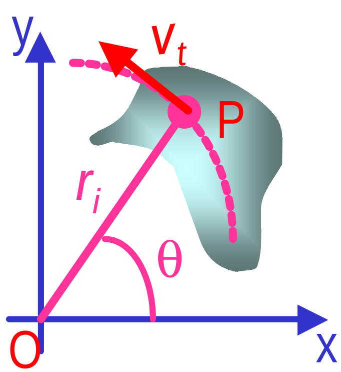
$$q_3 = 2.00 \times 3.00 + \frac{1}{2}3.50 \times (3.00)^2 = 21.8 rad$$

$$\Delta q = q_3 - q_2 = 10.8 rad = \frac{10.8}{2p} rev. = 1.72 rev.$$

Relationship Between Angular and Linear Quantities

What do we know about a rigid object that rotates about a fixed axis of rotation?

Every particle (or masslet) in the object moves in a circle centered at the axis of rotation.



When a point rotates, it has both the linear and angular motion components in its motion.

What is the linear component of the motion you see?

Linear velocity along the tangential direction.

The direction of ω follows a right-hand rule.

How do we related this linear component of the motion with angular component?

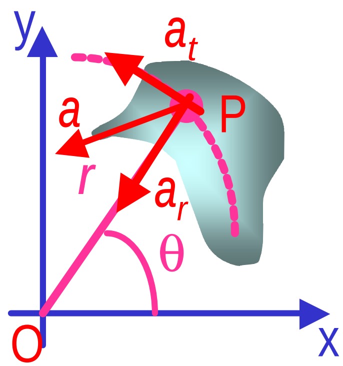
The arc-length is $s = rq$ So the tangential speed v_t is $v_t = \frac{ds}{dt} = \frac{d}{dt}(rq) = r\frac{dq}{dt} = rw$

What does this relationship tell you about the tangential speed of the points in the object and their angular speed?:

Although every particles in the object has the same angular speed, its tangential speed differs proportional to its distance from the axis of rotation.

The farther away the particle is from the center of rotation, the higher the tangential speed.

How about the Accelerations?



How many different linear accelerations do you see in a circular motion and what are they? Two

Tangential, a_t, and the radial acceleration, a_r.

Since the tangential speed v_t is $v_t = r\omega$

The magnitude of tangential acceleration a_t is

$$a_t = \frac{dv_t}{dt} = \frac{d}{dt}(r\omega) = r\frac{d\omega}{dt} = r\alpha$$

What does this relationship tell you?

Although every particle in the object has the same angular acceleration, its tangential acceleration differs proportional to its distance from the axis of rotation.

The radial or centripetal acceleration a_r is

$$a_r = \frac{v^2}{r} = \frac{(r\omega)^2}{r} = r\omega^2$$

What does this tell you?

The father away the particle from the rotation axis the more radial acceleration it receives. In other words, it receives more centripetal force.

Total linear acceleration is

$$a = \sqrt{a_t^2 + a_r^2} = \sqrt{(r\alpha)^2 + (r\omega^2)^2} = r\sqrt{\alpha^2 + \omega^4}$$

Example 10.2

Audio information on compact discs are transmitted digitally through the readout system consisting of laser and lenses. The digital information on the disc are stored by the pits and flat areas on the track. Since the speed of readout system is constant, it reads out the same number of pits and flats at the same time interval. In other words, the linear speed is the same no matter which track is played. a) Assuming the linear speed is 1.3 m/s, find the anfular speed of the disc in revolutions per minute when the inner most (r=23mm) and outer most track (r=58mm) are read.

Using the relationship between angular and tangential speed

$$v = rw; \quad w = \frac{v}{r}$$

$$w = \frac{v}{r} = \frac{1.3m/s}{23mm} = \frac{1.3}{23\times10^{-3}} = 56.5 rad/s = 9.00 rev/s = 5.4\times10^{2} rev/\min$$

$$w = \frac{1.3m/s}{58mm} = \frac{1.3}{58\times10^{-3}} = 22.4 rad/s = 2.1\times10^{2} rev/\min$$

b) The maximum playing time of a standard music CD is 74 minutes and 33 seconds. How many revolutions does the disk make during that time?

$$\bar{w} = \frac{(w_i + w_f)}{2} = \frac{(540 + 210)rev/\min}{2} = 375 rev/\min$$

$$q_f = q_i + \bar{w}t = 0 + \frac{375}{60} rev/s \times 4473 s = 2.8\times10^{4} rev$$

c) What is the total length of the track past through the readout mechanism?

$$l = v_t \Delta t = 1.3m/s \times 4473 s$$
$$= 5.8\times10^{3} m$$

d) What is the angular acceleration of the CD over the 4473s time interval, assuming constant α?

$$a = \frac{(w_f - w_i)}{\Delta t} = \frac{(22.4 - 56.5)rad/s}{4473 s}$$
$$= 7.6\times10^{-3} rad/s^{2}$$