1443-501 Spring 2002
Lecture #23

Dr. Jaehoon Yu

1. Superposition and Interference
2. Speed of Waves on Strings
3. Reflection and Transmission
4. Sinusoidal Waves
5. Rate of Energy Transfer by Sinusoidal Waves

Today's Homework Assignments is #12.

Final Exam at 5:30pm, Monday, May 6 (covers Ch 1- 16).

Review on Wednesday, May 1.

Superposition and Interference

If two or more traveling waves are moving through a medium, the resultant wave function at any point is the algebraic sum of the wave functions of the individual waves.

Superposition Principle

The waves that follow this principle are called ***linear waves*** which in general have small amplitudes. The ones that don't are ***nonlinear waves*** with larger amplitudes.

Thus, one can write the resultant wave function as

$$y = y_1 + y_2 + \cdots + y_n = \sum_{i=1}^{n} y_i$$

Two traveling linear waves can pass through each other without being destroyed or altered.

What do you think will happen to the water waves when you throw two stones on the pond?

They will pass right through each other.

What happens to the waves at the point where they meet?

The shape of wave will change➔ Interference

Constructive interference: The amplitude increases when the waves meet

Destructive interference: The amplitude decreases when the waves meet

Speed of Waves on Strings

How do we determine the speed of a transverse pulse traveling on a string?

If a string under tension is pulled sideways and released, the tension is responsible for accelerating a particular segment of the string back to the equilibrium position.

So what happens when the tension increases?

The acceleration of the particular segment increases

Which means?

The speed of the wave increases.

Now what happens when the mass per unit length of the string increases?

For the given tension, acceleration decreases, so the wave speed decreases.

Which law does this hypothesis based on?

Newton's second law of motion

Based on the hypothesis we have laid out above, we can construct a hypothetical formula for the speed of wave

$$v = \sqrt{\frac{T}{m}}$$

T: Tension on the string

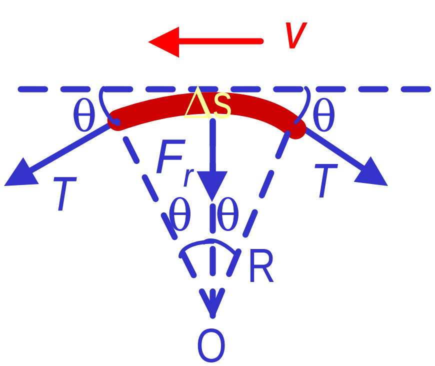
μ: Unit mass per length

Is the above expression dimensionally sound?

$T=[MLT^{-2}]$, $\mu=[ML^{-1}]$

$(T/\mu)^{1/2}=[L^2T^{-2}]^{1/2}=[LT^{-1}]$

Speed of Waves on Strings cont'd



Let's consider a pulse moving to right and look at it in the frame that moves along with the the pulse.

Since in the reference frame moves with the pulse, the segment is moving to the left with the speed *v*, and the centripetal acceleration of the segment is

$$a_r = \frac{v^2}{R}$$

Now what do the force components look in this motion when θ is small?

$$\sum F_t = T\cos\theta - T\cos\theta = 0$$

$$\sum F_r = 2T\sin\theta \approx 2T\theta$$

What is the mass of the segment when the line density of the string is μ?

$$m = \mu\Delta s = \mu R 2\theta = 2\mu R\theta$$

Using the radial force component

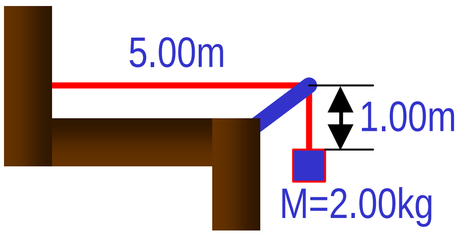
$$\sum F_r = ma = m\frac{v^2}{R} = 2\mu R\theta\frac{v^2}{R} = 2T\theta$$

Therefore the speed of the pulse is

$$v = \sqrt{\frac{T}{\mu}}$$

Example 16.2

A uniform cord has a mass of 0.300kg and a length of 6.00m. The cord passes over a pulley and supports a 2.00kg object. Find the speed of a pulse traveling along this cord.



Since the speed of wave on a string with line density μ and under the tension T is

$$v = \sqrt{\frac{T}{m}}$$

The line density μ is

$$m = \frac{0.300\,kg}{6.00\,m} = 5.00 \times 10^{-2}\,kg/m$$

The tension on the string is provided by the weight of the object. Therefore

$$T = Mg = 2.00 \times 9.80 = 19.6\,kg \cdot m/s^2$$

Thus the speed of the wave is

$$v = \sqrt{\frac{T}{m}} = \sqrt{\frac{19.6}{5.00 \times 10^{-2}}} = 19.8\,m/s$$

Reflection and Transmission

A pulse or a wave undergoes various changes when the medium it travels changes.

Depending on how rigid the support is, two radically different reflection patterns can be observed.

1. The support is rigidly fixed: The reflected pulse will be inverted to the original due to the force exerted on to the string by the support in reaction to the force on the support due to the pulse on the string.
2. The support is freely moving: The reflected pulse will maintain the original shape but moving in the reverse direction.

If the boundary is intermediate between the above two extremes, part of the pulse reflects, and the other undergoes transmission, passing through the boundary and propagating in the new medium.

When a wave pulse travels from medium A to B:

- $v_A > v_B$ (or $\mu_A < \mu_B$), the pulse is inverted upon reflection.
- $v_A < v_B$ (or $\mu_A > \mu_B$), the pulse is not inverted upon reflection.

Sinusoidal Waves

Equation of motion of a simple harmonic oscillation is a sine function.

But it does not travel. Now how does wave form look like when the wave travels?

The function describing the position of particles, located at x, of the medium through which the sinusoidal wave is traveling can be written at t=0

$$y = A\sin\left(\frac{2p}{l}x\right)$$

Amplitude

Wave Length

The wave form of the wave traveling at the speed v in +x at any given time t becomes

$$y = A\sin\left(\frac{2p}{l}(x - vt)\right)$$

By definition, the speed of wave in terms of wave length and period T is

$$v = \frac{l}{T}$$

Thus the wave form can be rewritten

$$y = A\sin\left[2p\left(\frac{x}{l} - \frac{t}{T}\right)\right]$$

Defining, angular wave number *k* and angular frequency ω,

$$k \equiv \frac{2p}{l};\; w = \frac{2p}{T}$$

The wave form becomes

$$y = A\sin(kx - wt)$$

Frequency, *f*,

$$f = \frac{1}{T}$$

Wave speed, *v*

$$v = \frac{l}{T} = \frac{w}{k}$$

General wave form

$$y = A\sin(kx - wt + f)$$

Example 16.3

A sinusoidal wave traveling in the positive x direction has an amplitude of 15.0cm, a wavelength of 40.0cm, and a frequency of 8.00Hz. The vertical displacement of the medium at t=0 and x=0 is also 15.0cm. a) Find the angular wave number k, period T, angular frequency ω, and speed v of the wave.

Using the definition, angular wave number k is

$$k = \frac{2p}{l} = \frac{2p}{0.40} = 5.00p = 15.7 rad / m$$

Period is

$$T = \frac{1}{f} = \frac{1}{8.00} = 0.125 \sec$$

Angular frequency is

$$w = \frac{2p}{T} = 2pf = 50.3 rad / s$$

Using period and wave length, the wave speed is

$$v = \frac{l}{T} = lf = 0.400 \times 8.00 = 3.2 m / s$$

b) Determine the phase constant φ, and write a general expression of the wave function.

At x=0 and t=0, y=15.0cm, therefore the phase φ becomes

$$y = 0.150 \sin(f) = 0.150$$

$$\sin f = 1; \quad f = \frac{p}{2}$$

Thus the general wave function is

$$y = A\sin(kx - wt + f) = 0.150 \sin\left(15.7x - 50.3t + \frac{p}{2}\right)$$

Sinusoidal Waves on Strings

Let's consider the case where a string is attached to an arm undergoing a simple harmonic oscillation. The trains of waves generated by the motion will travel through the string, causing the particles in the string to undergo simple harmonic motion on y-axis.

If the wave at t=0 is $y = A\sin\left(\frac{2p}{l}x\right)$ What does this mean? $f = 0$

The wave function can be written $y = A\sin(kx - wt)$

This wave function describes the vertical motion of any point on the string at any time t. Therefore, we can use this function to obtain transverse speed, v_y, and acceleration, a_y.

$$v_y = \left.\frac{dy}{dt}\right|_{xconst} = \frac{\partial y}{\partial t} = -wA\cos(kx - wt)$$

$$a_y = \left.\frac{dv_y}{dt}\right|_{xconst} = \frac{\partial v_y}{\partial t} = -w^2 A\sin(kx - wt)$$

These are the speed and acceleration of the particle in the medium not of the wave.

The maximum speed and the acceleration of the particle in the medium at position x at time t are

$$v_{y,\max} = wA$$

$$a_{y,\max} = w^2 A$$

How do these look for simple harmonic motion?

Example 16.4

A string is driven at a frequency of 5.00Hz. The amplitude of the motion is 12.0cm, and the wave speed is 20.0m/s. Determine the angular frequency ω and angular wave number k for this wave, and write and expression for the wave function.

Using frequency, the angular frequency is

$$w = \frac{2p}{\mathrm{T}} = 2pf = 2p \cdot 5.00 = 31.4 rad \ / \ s$$

Angular wave number k is

$$k = \frac{2p}{l} = \frac{2p}{vT} = \frac{2pf}{v} = \frac{w}{v} = \frac{31.4}{20.0} = 1.57 rad \ / \ m$$

Thus the general expression of the wave function is

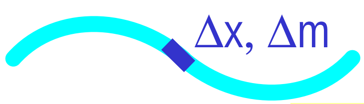
$$y = A\sin(kx - wt) = 0.120\sin(1.57x - 31.4t)$$

Rate of Energy Transfer by Sinusoidal Waves on Strings

Waves traveling through medium carries energy.

When an external source performs work on the string, the energy enters into the string and propagates through the medium as wave.

What is the potential energy of one wave length of a traveling wave?



Elastic potential energy of a particle in a simple harmonic motion $U = \frac{1}{2}ky^2$

Since $\omega^2 = k/m$ $U = \frac{1}{2}mw^2y^2$ The energy ΔU of the segment Δm is $\Delta U = \frac{1}{2}\Delta m w^2 y^2 = \frac{1}{2}m\Delta x w^2 y^2$

As Δx→0, the energy ΔU becomes $dU = \frac{1}{2}mw^2y^2dx$

Using the wave function,the energy is $dU = \frac{1}{2}mw^2A^2\sin^2(kx - wt)dx$

For the wave at t=0, the potential energy in one wave length, λ, is

$$U_l = \frac{1}{2}mw^2A^2\int_{x=0}^{x=l}\sin^2 kx dx = \frac{1}{2}mw^2A^2\int_{x=0}^{x=l}\frac{1-\cos 2kx}{2}dx$$

$$= \frac{1}{2}mw^2A^2\left[\frac{1}{2}x - \frac{1}{4k}\sin\frac{4px}{l}\right]_{x=0}^{x=l} = \frac{1}{4}mw^2A^2l$$

Recall $k=2\pi/\lambda$

Rate of Energy Transfer by Sinusoidal Waves cont'd

How does the kinetic energy of each segment of the string in the wave look?

Since the vertical speed of the particle is

$$v_y = -wA\cos(kx - wt)$$

The kinetic energy, ΔK, of the segment Δm is

$$\Delta K = \frac{1}{2}\Delta m v_y^2 = \frac{1}{2} m\Delta x w^2 A^2 \cos^2(kx - wt)$$

As Δx→0, the energy ΔK becomes

$$dK = \frac{1}{2} m w^2 A^2 \cos^2(kx - wt)dx$$

For the wave at t=0, the kinetic energy in one wave length, λ, is

$$K_l = \frac{1}{2} m w^2 A^2 \int_{x=0}^{x=l} \cos^2 kx dx = \frac{1}{2} m w^2 A^2 \int_{x=0}^{x=l} \frac{1+\cos 2kx}{2} dx$$

$$= \frac{1}{2} m w^2 A^2 \left[\frac{1}{2}x + \frac{1}{4k}\sin\frac{4px}{l}\right]_{x=0}^{x=l} = \frac{1}{4} m w^2 A^2 l$$

Recall $k=2\pi/\lambda$

Just like harmonic oscillation, the total mechanical energy in one wave length, λ, is

$$E_l = U_l + K_l = \frac{1}{2} m w^2 A^2 l$$

As the wave moves along the string, the amount of energy passes by a given point changes during one period. So the power, the rate of energy transfer becomes

$$P = \frac{E_l}{\Delta t} = \frac{1}{2} m w^2 A^2 \frac{l}{T}$$

$$= \frac{1}{2} m w^2 A^2 v$$

P of any sinusoidal wave is proportion to the square of angular frequency, the square of amplitude, density of medium, and wave speed.

Example 16.5

A taut string for which μ=5.00x10^{-2} kg/m is under a tension of 80.0N. How much power must be supplied to the string to generate sinusoidal waves at a frequency of 60.0Hz and an amplitude of 6.00cm?

The speed of the wave is

$$v = \sqrt{\frac{T}{m}} = \sqrt{\frac{80.0}{5.00 \times 10^{-2}}} = 40.0 m/s$$

Using the frequency, angular frequency ω is

$$w = \frac{2p}{\mathrm{T}} = 2pf = 2p \cdot 60.0 = 377\, rad/s$$

Since the rate of energy transfer is

$$P = \frac{E_l}{\Delta t} = \frac{1}{2} m w^2 A^2 v$$

$$= \frac{1}{2} \times 5.00 \times 10^{-2} \times (377)^2 \times (0.06)^2 \times (40.0) = 512W$$