PHYS 5326 – Lecture #4

Monday, Jan. 27, 2003
Dr. Jae Yu

1. Neutrino-Nucleon DIS
2. Formalism of ν-N DIS
3. Proton Structure Functions and PDF

We must meet in a different room Wednesday, Jan. 29!!! Where? SH 129?

Neutrino Nucleon Deep Inelastic Scattering

- DIS (Deep Inelastic Scattering) of lepton-nucleon are traditionally used to probe nucleon structures
- Neutrinos are excellent probes
  - Extremely light
  - Structureless
  - Weak interaction only ➔ Probes helicity
- Nucleons consist of partons
  - Structure of nucleon is described by parton distribution functions (PDF) ➔ Fractional momentum distributions of the constituents
- DIS are viewed as neutrino-parton elastic scattering

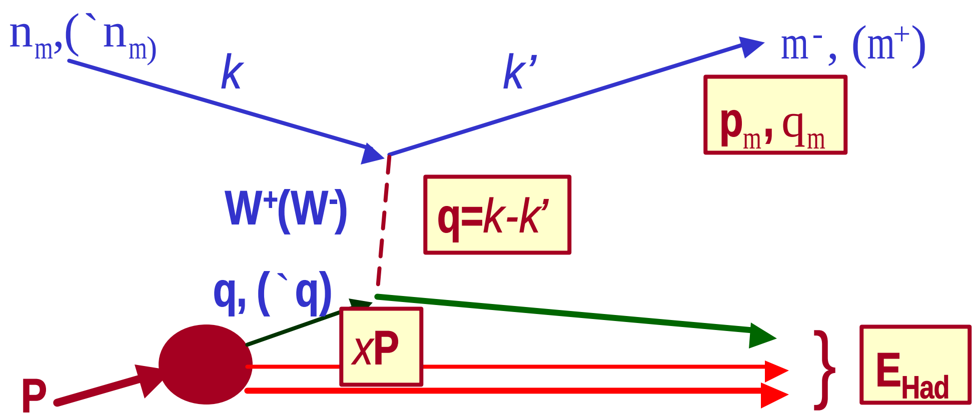
Structure Function Measurements

- A complete set of Lorentz scalars that parameterize the unknown structure of the proton
- Properties of the SF lead to parton model
  - Nucleon is composed of point-like constituents, partons, that elastically scatter with neutrino
- Partons are identified as quarks and gluons of QCD
- QCD does not provide parton distributions within proton
- QCD analysis of SF provides a determination of nucleon's valence and sea quark and gluon distributions (PDF) along with the strong coupling constant, α_s

Kinematics of ν-N CC Interactions

- DIS is a three dimensional problem
- Three kinematic parameters provide full description of a DIS event are
  - p_μ: Muon momentum
  - θ_μ: Angle of outgoing muon
  - E_{Had}: Observed energy of outgoing hadrons
- Neutrino energy becomes
  - $E_\nu = E_{Had} + E_\mu + M_p$

DIS Kinematics



Leptonic System

$$k = (E_n, 0, 0, E_n)$$

$$k' = (E_m, p_m \sin q_m \cos f_m, p_m \sin q_m \sin f_m, p_m \cos q_m)$$

Hadron System

$$p = (M_P, 0, 0, 0)$$

$$p' = p + q = p + (k - k')$$

DIS Lorentz Invariant Variables

CMS Energy $s=(p+k)=M_P^2+2M_P E_n$

Energy Transferred to Hadronic System

$$n=\frac{p\cdot q}{M_P}=E_n-E_m=E_{Had}$$

Four Momentum Transfer of the Interaction

$$Q^2=-q^2=-(k-k^{'})^2=m_m^2+2E_n(E_m-p_m\cos q_m)$$

Invariant Mass of the hadronic system

$$W^2=(p^{'})^2=(p+q)^2=M_P^2+2M_P n-Q^2$$

DIS Lorentz Invariant Variables cont'd

Bjorken Scaling Variable = Fractional Momentum of the Struck parton within the nucleon

$$x = \frac{-q^2}{2p \cdot q} = \frac{Q^2}{2M_p \nu}$$

Inelasticity $$y = \frac{p \cdot q}{p \cdot k} = \frac{E_{Had}}{E_\nu} = \frac{\nu}{E_\nu}$$

$$y \approx 1 - \frac{1}{2}\left(1 + \cos\theta^*\right)$$

where θ* is CMS scattering angle of μ

DIS Formalism

Matrix element for ν-N interaction

$$M = \frac{G_F}{\sqrt{2}} \frac{1}{1+Q^2/M_W^2} \bar{u}_\mu(k',s')\gamma_\alpha(1-\gamma_5)u_\nu(k,s)\langle X|J_{CC}|N(p,s)\rangle$$

Weak Coupling Constant

W (CC) Propagator

Lepton

Hadron

Inclusive Spin-Averaged Cross section

$$\frac{d^2\sigma^{\nu N}}{d\Omega_\mu dE_\mu} = \frac{1}{\left(1+Q^2/M_W^2\right)} \frac{G_F}{2} \frac{m_\nu}{E_\nu} \frac{m_\mu}{E_\mu} \frac{E_\mu^2}{(2\pi)^2} L_{\alpha\beta} W^{\alpha\beta}$$

Leptonic Tensor

Hadronic Tensor

ν-N DIS Cross Sections for SF Extraction

$$\frac{d^2 s^{n(\bar{n})}}{dxdy} = \frac{2G_F M_P E_n}{p}\left[\left(1 - y - \frac{M_P xy}{2E_n}\right)F_2^{n(\bar{n})}(x,Q^2) + \frac{y^2}{2}2xF_1^{n(\bar{n})}(x,Q^2) \pm y\left(1-\frac{y}{2}\right)xF_3^{n(\bar{n})}(x,Q^2)\right]$$

Using ratio of absorption xsec for longitudinal and transversely polarized boson, R

$$R(x,Q^2) \equiv \frac{s_L}{s_T} = \frac{F_2}{2xF_1}\left(1 - \frac{Q^2}{(2M_P x)^2}\right) - 1$$

$$\frac{d^2 s^{n(\bar{n})}}{dxdy} = \frac{2G_F M_P E_n}{p}\left[\left(1 - y - \frac{M_P xy}{2E_n} + \frac{y^2}{2}\frac{1 + 4M_P^2 x^2 / Q^2}{1 + R(x,Q^2)}\right)F_2^{n(\bar{n})}(x,Q^2) \pm y\left(1-\frac{y}{2}\right)nF_3^{n(\bar{n})}(x,Q^2)\right]$$

Structure Functions and PDF's

- Assuming parton model, n-N cross section can be rewritten in terms of point-like particle interactions

$$\frac{d^2 s^{nT}}{dxdy} = \frac{G_F^2 xs}{p\left(1+Q^2/M_W^2\right)^2}\left[q^{nT}(x)+\left(1-y^2\right)\overline{q}^{nT}+2(1-y)k^{nT}(x)\right]$$

$$\frac{d^2 s^{\bar{n}T}}{dxdy} = \frac{G_F^2 xs}{p\left(1+Q^2/M_W^2\right)^2}\left[\overline{q}^{\bar{n}T}(x)+\left(1-y^2\right)q^{\bar{n}T}+2(1-y)k^{\bar{n}T}(x)\right]$$

Spin 0 partons

- Comparing the parton-neutrino to proton-neutrino SF and PDF's are related as

$$2xF_1^{n(\bar{n})T} = 2\left[xq^{n(\bar{n})T}(x)+x\overline{q}^{n(\bar{n})T}(x)\right]$$

$$F_2^{n(\bar{n})T} = 2\left[xq^{n(\bar{n})T}(x)+x\overline{q}^{n(\bar{n})T}(x)+2xk^{n(\bar{n})T}\right]$$

$$xF_3^{n(\bar{n})T} = 2\left[xq^{n(\bar{n})T}(x)-x\overline{q}^{n(\bar{n})T}(x)\right]$$

Parity violating components

If no spin 0, 2xF1=F2

Linking to Quark Flavors

- ν-N scattering resolves flavor of constituents
  - CC changes the flavor of the struck quark
  - Charge conservation at the vertex constraints
    - Neutrinos to interact with d, s, $\bar{u}$, $\bar{c}$
    - Anti-neutrinos to interact with $\bar{d}$, $\bar{s}$, u, c
- For parton target, the quark densities contribute to SF are

$$q^{\nu p} = d^p(x) + s^p(x)$$
$$\bar{q}^{\nu p} = \bar{u}^p(x) + \bar{c}^p(x)$$
$$q^{\bar{\nu} p} = u^p(x) + c^p(x)$$
$$\bar{q}^{\bar{\nu} p} = \bar{d}^p(x) + \bar{s}^p(x)$$

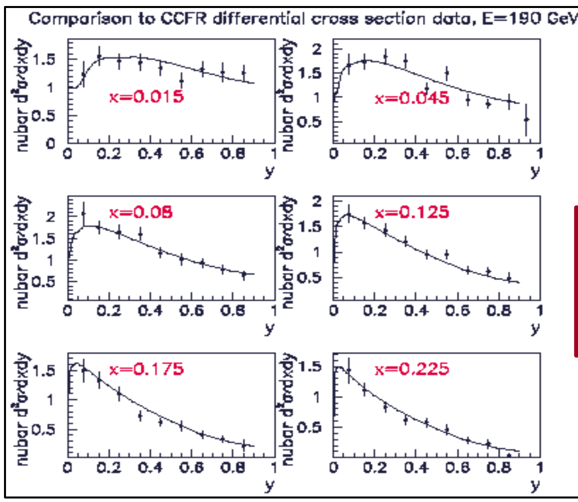
$$2xF_1^{\nu N}(x) = 2xF_1^{\bar{\nu} N}(x)$$
$$= xu(x) + x\bar{u}(x) + xd(x) + x\bar{d}(x)$$
$$+ xs(x) + x\bar{c}(x) + xc(x) + x\bar{c}(x)$$

$$xF_3^{\nu N}(x) = xu_V(x) + xd_V(x) + 2xs(x) - 2xc(x)$$
$$xF_3^{\bar{\nu} N}(x) = xu_V(x) + xd_V(x) - 2xs(x) + 2xc(x)$$

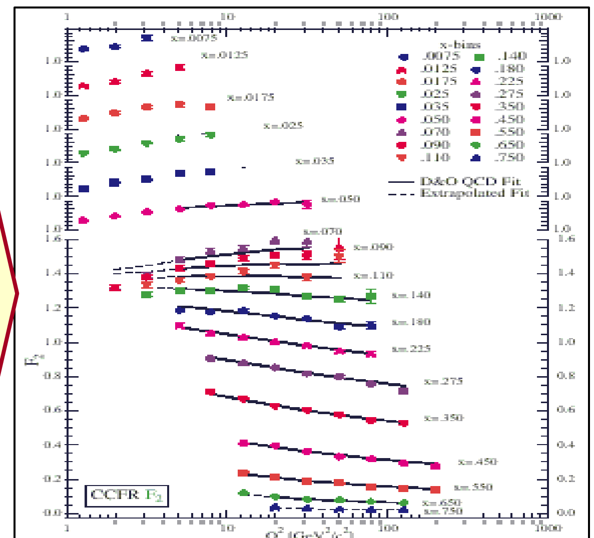
$$u_V(x) \equiv u - \bar{u}; \quad d_V(x) \equiv d - \bar{d}$$

How Are PDFs Determined?

- Measure ν-N differential cross sections, correcting for target
- Compare them with theoretical x-sec
- Fit SF's to measured x-sec
- Extract PDF's from the SF fits➔
  - Different QCD models could generate different sets of PDF's
  - CTEQ, MRST, GRV, etc



Fit to Data for SF

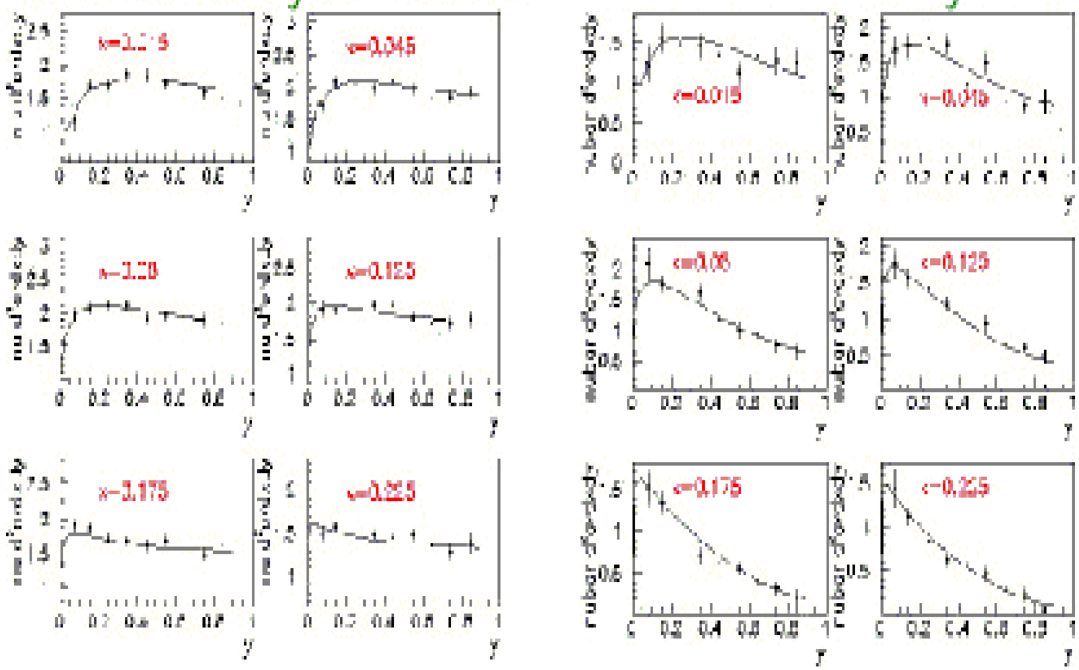


Comparisons of Neutrino and Antineutrino cross sections

Neutrino xsec vs y at 190 GeV

Antineutrino xsec vs y at 190 GeV

CCFR Data



What can PDF's depend on?

- Different functional forms of PDF and SF's
- Order of QCD calculations
  - Higher order (NLO) calculations require higher order PDF's
- Different assumptions in the protons
  - No intrinsic sea quarks
  - Fixed flavors only
- Approximation at non-perturbative regime
  - Different method of approximating low x behavior

Homework Assignments

- Provide a method to measure the average valence quark distributions in a ν-N scattering experiment
  - Due: One week from today, Mon., Feb. 3
- Derive the Lorentz invariant variables of n-N scattering, s, Q^2, W^2, x and y on pages 6 and 7 of this lecture.
  - Due: One week from today, Mon., Feb. 3