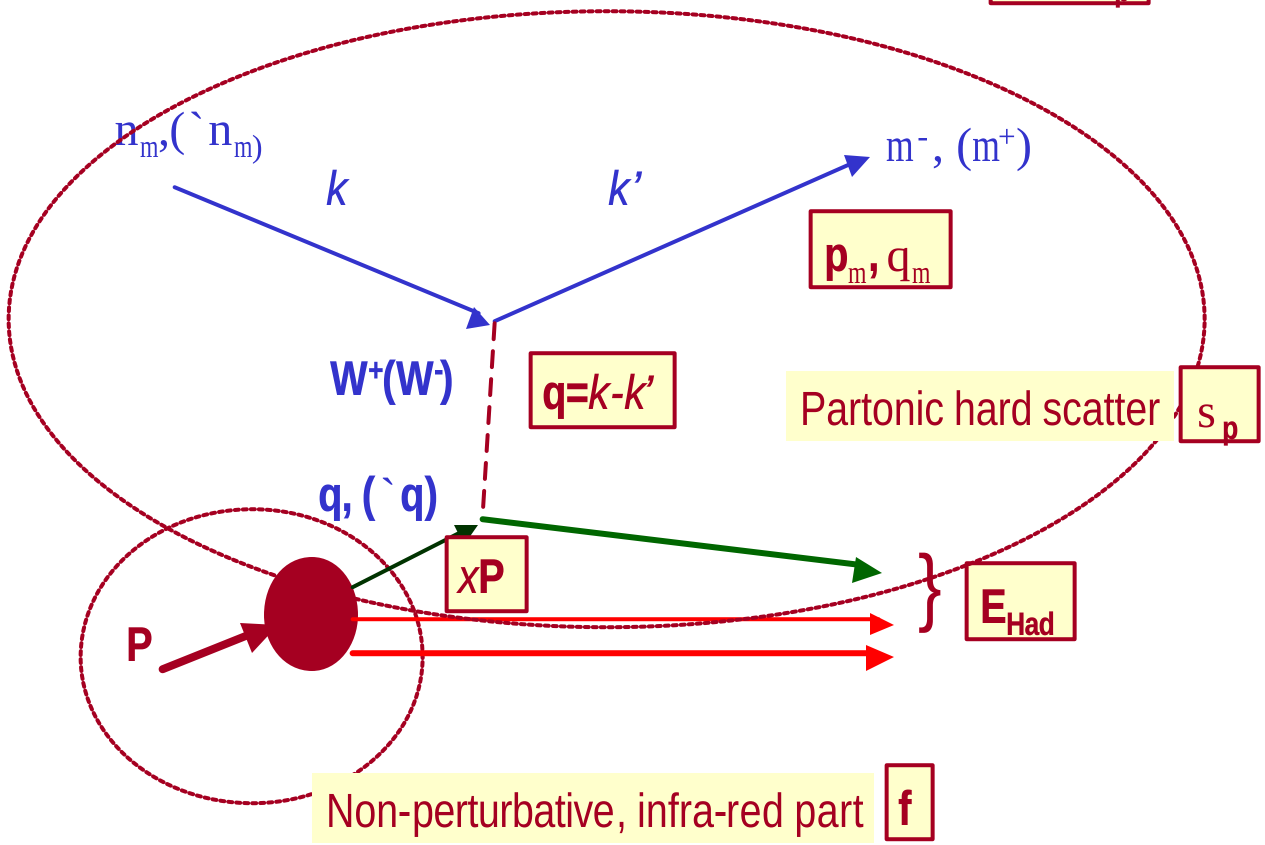
PHYS 5326 – Lecture #5

Wednesday, Jan. 29, 2003
Dr. Jae Yu

1. QCD Evolution of PDFs
2. Measurement of $\sin^2\theta_W$
3. Formalism of $\sin^2\theta_W$ in ν-N DIS
4. Improvements in $\sin^2\theta_W$
5. Interpretation and Link to Higgs

Factorization

$\boxed{s=f^{*}s_{p}}$



DGLAP QCD Evolution Equations

- The evolution equations by **D**okshitzer-**G**ribov-**L**ipatov-**A**ltarelli-**P**arisi provide mechanism to evolve PDF's to any kinematic regime or momentum scale

$$\frac{dq^{NS}(x,M^2)}{d\ln M^2}=\sum_i q^i-\bar{q}^i=u_V+d_V=\frac{\alpha_s(\mu^2)}{2\pi}\int_x^1\frac{dy}{y}q^{NS}(y,M^2)P_{qq}\left(\frac{x}{y}\right)$$

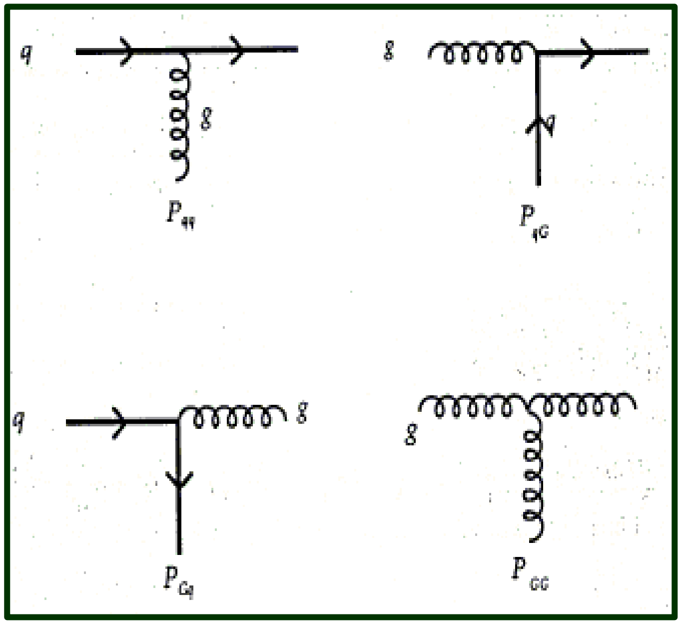
$$\frac{dq^S(x,M^2)}{d\ln M^2}=\sum_i q^i+\bar{q}^i=\frac{\alpha_s(\mu^2)}{2\pi}\int_x^1\frac{dy}{y}\left[q^{NS}(y,M^2)P_{qq}^s\left(\frac{x}{y}\right)+G(y,M^2)P_{qG}^s\left(\frac{x}{y}\right)\right]$$

$$\frac{dG(x,M^2)}{d\ln M^2}=\frac{\alpha_s(\mu^2)}{2\pi}\int_x^1\frac{dy}{y}\left[q^S(y,M^2)P_{Gq}^s\left(\frac{x}{y}\right)+G(y,M^2)P_{GG}\left(\frac{x}{y}\right)\right]$$

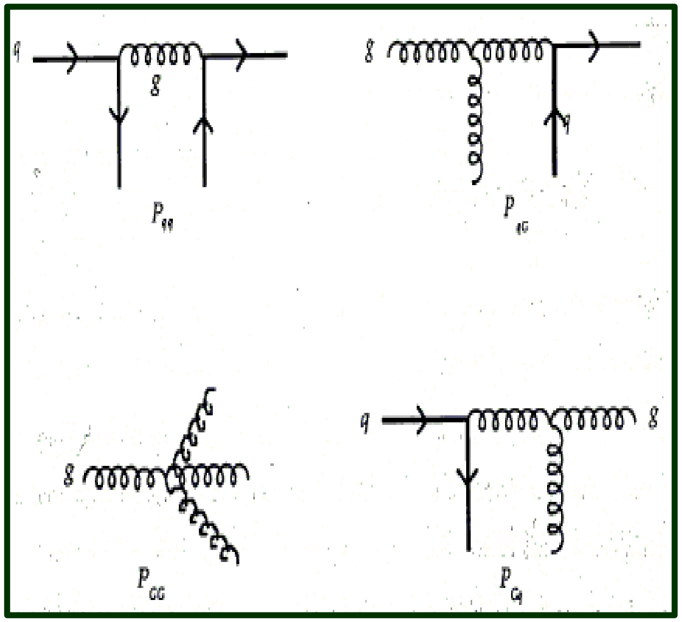
$P_{ij}(x/y)$: Splitting function that is the probability of parton i with momentum y get resolved as parton j with momentum x<y

Feynman Diagrams for Parton Splitting

LO: O(α_s)



NLO: O(α_s^2)



Electroweak Threory

- Standard Model unifies Weak and EM to SU(2)xU(1) gauge theory
  - Weak neutral current interaction
  - Measured physical parameters related to mixing parameters for the couplings

$$g' = g\tan ?_W, e = g\sin ?_W, G_F = \frac{g^2\sqrt{2}}{8M_W^2}, \frac{M_W}{M_Z} = \cos ?_W$$

- Neutrinos in this picture are unique because they only interact through left-handed weak interactions ➔ Probe weak sector only
  - Less complication in some measurements, such as proton structure

$\sin^2\theta_W$ and ν-N scattering

- In the electroweak sector of the Standard Model, it is not known *a priori* what the mixture of electrically neutral electomagnetic and weak mediator is ➔ This fractional mixture is given by the mixing angle
- Within the on-shell renormalization scheme, $\sin^2\theta_W$ is:

$$\sin^2\theta_W^{On-Shell} = 1 - \frac{M_W^2}{\rho_0 M_Z^2}$$

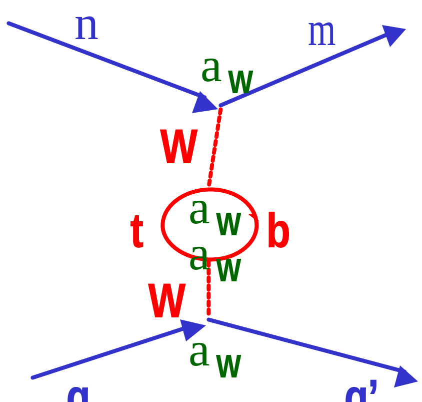
•Provides independent measurement of M_W & information to pin down M_{Higgs} via higher order loop corrections, in comparable uncertainty to direct measurements

•Measures light quark couplings ➔ Sensitive to other types (anomalous) of couplings

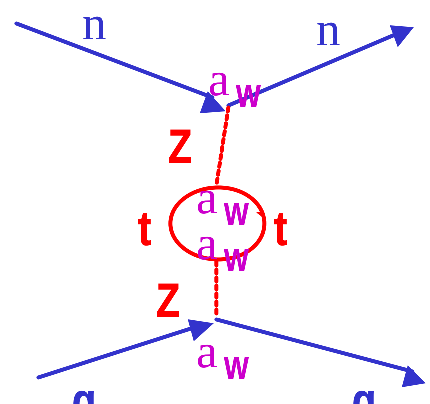
•In other words, sensitive to physics beyond SM ➔ New vector bosons, compositeness, ν-oscillations, etc

Higher Order Corrections

- LO GSW requires three parameters: α, G_F and M_Z
- Higher order corrections bring in dependence two additional parameters: M_{Top} and M_{Higgs}

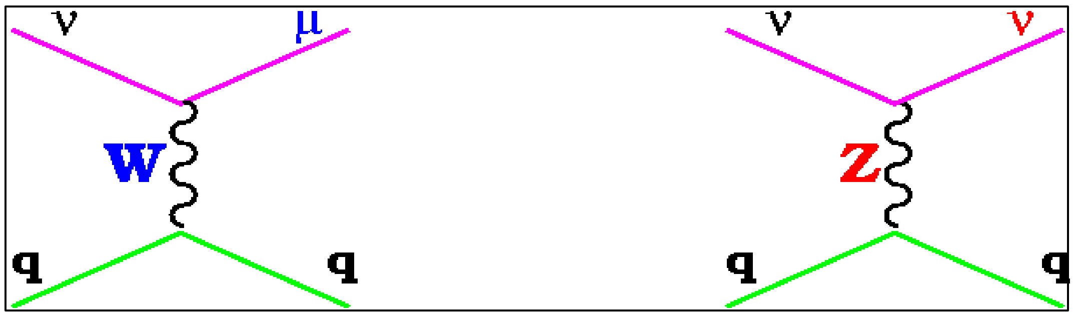


$$\sigma \propto O\left(\alpha_W^4\right)$$



$$\sigma \propto O\left(\alpha_W^4\right)$$

How is sin^{2}q$_W$ measured?

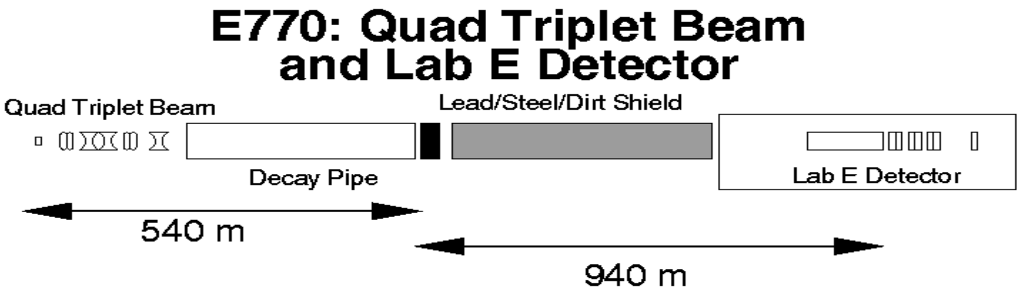


$$coupling \propto I^{(3)}_{weak} \qquad\qquad coupling \propto I^{(3)}_{weak} - Q_{EM}\sin^2 q_W$$

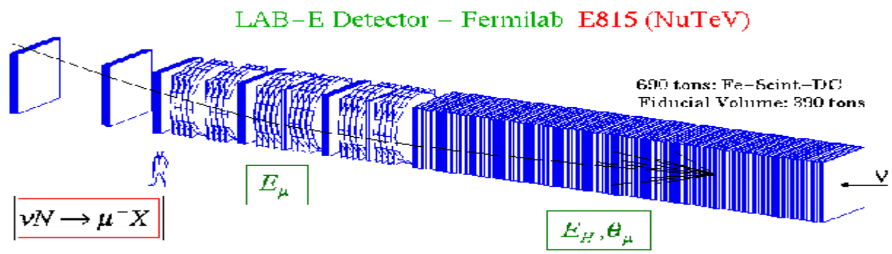
- Cross section ratios between NC and CC proportional to sin$^2\theta_W$
- Llewellyn Smith Formula:

$$\mathbf{R}^{n\,(\bar{n})} = \frac{\mathbf{s}^{\,n\,(\bar{n})}_{\mathbf{NC}}}{\mathbf{s}^{\,n\,(\bar{n})}_{\mathbf{CC}}} = \mathbf{?}^{2}\left(\frac{1}{2} - \sin^{2}\mathbf{?}_{W} + \frac{5}{9}\sin^{4}\mathbf{?}_{W}\left(1 + \frac{\mathbf{s}^{\,\bar{n}\,(n)}_{\mathbf{CC}}}{\mathbf{s}^{\,n\,(\bar{n})}_{\mathbf{CC}}}\right)\right)$$

Original Experiments



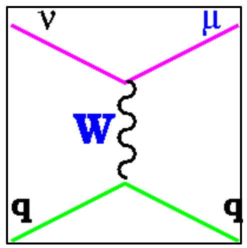
- Conventional neutrino beam from π/k decays
- Focus all signs of π/k for neutrinos and antineutrinos
- Only ν_μ in the beam (NC events are mixed)

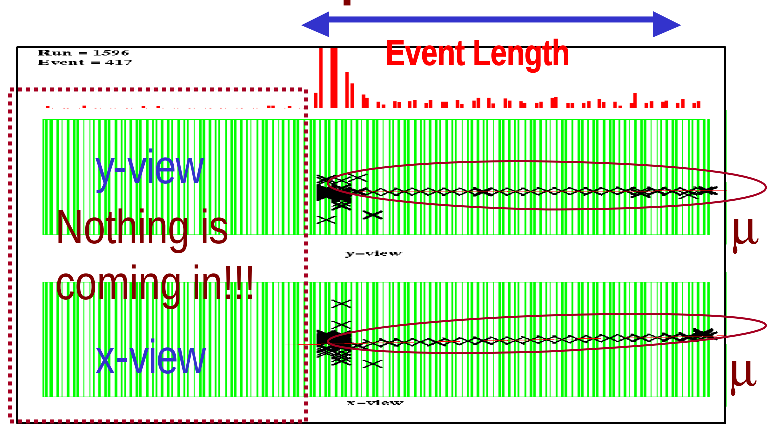


- Very small cross section ➔ Heavy neutrino target
- ν_e are the killers (CC events look the same as NC events)

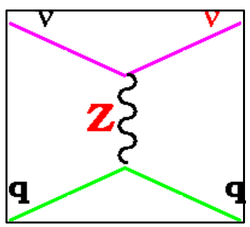
How Can Events be Separated?

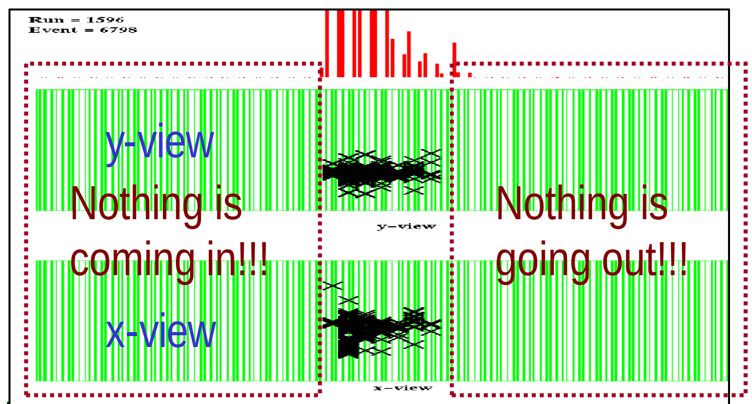
Charged Current Events





Neutral Current Events

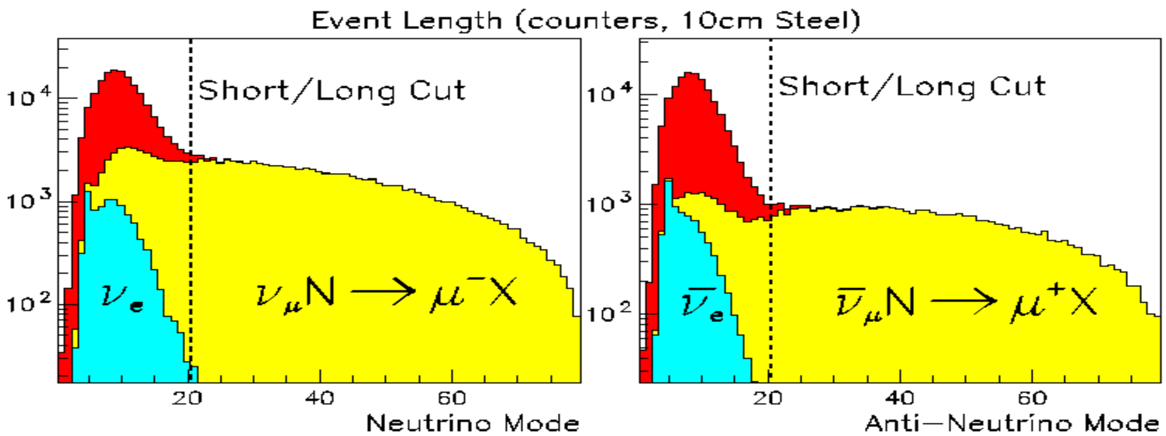




Experimental Variable

Define an Experimental Length variable

➔Distinguishes CC from NC experimentally in statistical manner



Compare experimentally measured ratio

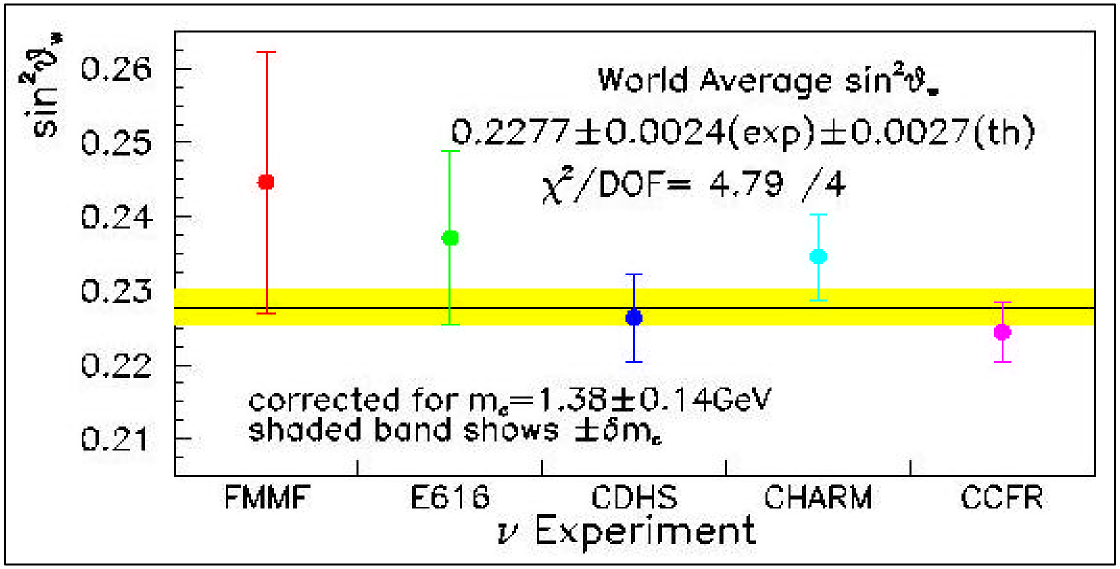
$$R_{Exp} = \frac{N_{Short}}{N_{Long}} = \frac{L < L_{Cut}}{L > L_{Cut}} = \frac{N_{NC\ \ Candidates}}{N_{CC\ \ Candidates}}$$

to theoretical prediction of R^{ν}

Past Experimental Results

$$\sin^2 ?_W^{On-Shell} = 1 - \frac{M_W^2}{M_Z^2} = 0.2277 \pm 0.0036$$

$$\Rightarrow M_W^{On-Shell} = 80.14 \pm 0.19 GeV/c^2$$



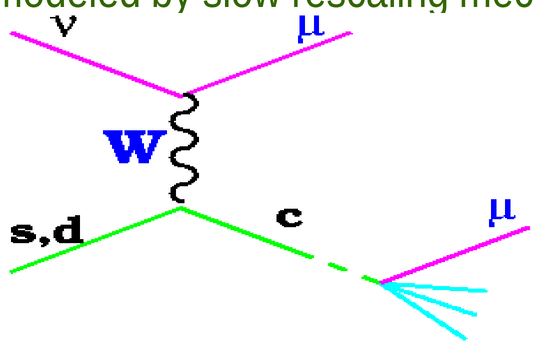
The yellow band represents a correlated uncertainty!!

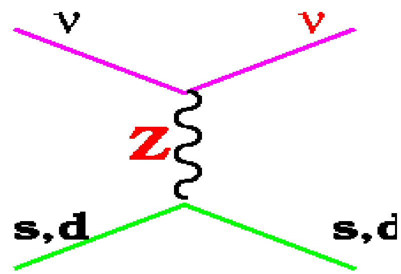
Improvements on Measurements

- Asses the uncertainties from previous measurements
- Determine what the sources of largest theoretical and experimental uncertainties are
- Provide new methods to reduce large uncertainties

$\sin^2\theta_W$ Theoretical Uncertainty

- Significant correlated error from CC production of charm quark (m_c) modeled by slow rescaling mechanism





- Suggestion by Paschos-Wolfenstein by separating ν and $\bar{\nu}$ beams:

$$\mathbf{R}^{-} = \frac{\mathbf{s}^{n}_{\mathbf{NC}} - \mathbf{s}^{\bar{n}}_{\mathbf{NC}}}{\mathbf{s}^{n}_{\mathbf{CC}} - \mathbf{s}^{\bar{n}}_{\mathbf{CC}}} = ?^{2}\left(\frac{1}{2} - \sin^{2}?_{W}\right) = \frac{\mathbf{R}^{n} - \mathbf{R}^{\bar{n}}}{1 - \mathbf{r}}$$

➔Reduce charm CC production error by subtracting sea quark contributions

➔Only valence u, d, and s contributes while sea quark contributions cancel out

➔Massive quark production through Cabbio suppressed d_v quarks only

Homework Assignments

- Draw a few additional Feynman diagrams for higher order GSW corrections to ν-N scattering at the same order as those on pg 7 in this lecture
  - Due: One week from today, Wed., Feb. 5