Phys 5326 – Lecture #9 & 10

Friday, Feb. 21, 2003
Dr. Jae Yu

1. Interpretation of $\sin^2 \theta_W$ results
2. The link to Higgs
3. Neutrino Oscillation

• Next makeup class is Friday, Mar. 14, 1-2:30pm, rm 200.
SM Global Fits with New Results

Without NuTeV
\[ \chi^2/\text{dof}=20.5/14: \text{P} = 11.4\% \]

With NuTeV
\[ \chi^2/\text{dof}=29.7/15: \text{P} = 1.3\% \]

Confidence level in upper \( M_{\text{higgs}} \) limit weakens slightly.

LEP EWWG: [http://www.cern.ch/LEPEWWG](http://www.cern.ch/LEPEWWG)
Tree-level Parameters: $\rho_0$ and $\sin^2 \theta_W^{(on-shell)}$

Either $\sin^2 \theta_W^{(on-shell)}$ or $\rho_0$ could agree with SM but both agreeing simultaneously is unlikely.

$$\sin^2 \theta_W = 0.2265 \pm 0.0031$$

$$\rho_0 = 0.9983 \pm 0.040$$

$$d\sin^2 \theta_W^{(on-shell)} = -0.00022 \times \left( \frac{M_t^2 - (175 \text{GeV})^2}{(50 \text{GeV})^2} \right)$$

$$+ 0.00032 \times \ln \left( \frac{M_H}{150 \text{GeV}} \right)$$
Model Independent Analysis

- $R^\nu(\bar{\nu})$ can be expressed in terms of quark couplings:

$$R^\nu(\bar{\nu}) \equiv \frac{\sigma\left(\nu N \to \nu X\right)}{\sigma\left(\nu N \to \ell^{-} X\right)} = g_L^2 + r^{(-)} g_R^2$$

Where

$$r \equiv \frac{s\left(\bar{\nu} N \to \ell^{+} X\right)}{s\left(\bar{\nu} N \to \ell^{-} X\right)} \approx \frac{1}{2}$$

Paschos-Wolfenstein formula can be expressed as

$$R^- = \frac{s^?_{NC} - s^?_{NC}}{s^?_{CC} - s^?_{CC}} = \frac{1}{2} \sin^2 \theta_w \left(\frac{1}{2} - \sin^2 \theta_W\right) = \frac{R^? - rR^?}{1 - r} = g_L^2 - g_R^2$$
Model Independent Analysis

- Performed a fit to quark couplings (and $g_L$ and $g_R$)
  
  - For isoscalar target, the $\nu N$ couplings are

  \[
  g_L^2 = u_L^2 + d_L^2 = \frac{1}{2} - \sin^2\theta_w + \frac{5}{9} \sin^4\theta_w
  \]

  \[
  g_R^2 = u_R^2 + d_R^2 = \frac{5}{9} \sin^4\theta_w
  \]

  - From two parameter fit to $R_v^{\text{exp}}$ and $R_v^{\text{exp}}$

  \[
  g_L^2 = 0.3005 \pm 0.0014 \quad \text{(SM: 0.3042 ←-2.6\sigma deviation)}
  \]

  \[
  g_R^2 = 0.0310 \pm 0.0011 \quad \text{(SM: 0.0301 ← Agreement)}
  \]
Model Independent Analysis

Difficult to explain the disagreement with SM by:
Parton Distribution Function or LO vs NLO or Electroweak Radiative Correction: large $M_{Higgs}$

Vary PDF Sets (LO and NLO)
What is the discrepancy due to (Old Physics)?

- R technique is sensitive to q vs $\bar{q}$ differences and NLO effect
  - Difference in valence quark and anti-quark momentum fraction
- Isospin symmetry assumption might not be entirely correct
  - Expect violation about 1% $\Rightarrow$ NuTeV reduces this effect by using the ratio of $\nu$ and $\bar{\nu}$ cross sections
  $\Rightarrow$ Reducing dependence by a factor of 3
What is the discrepancy due to (Old Physics)?

- **s vs \( \bar{s} \) quark asymmetry**
  - \( s \) and \( \bar{s} \) need to be the same but the momentum could differ
  - A value of \( \Delta s = x_s - x_{\bar{s}} \approx +0.002 \) could shift \( \sin^2 \theta_W \) by \(-0.0026\), explaining \( \frac{1}{2} \) the discrepancy (S. Davison, et. al., hep-ph/0112302)

- **NuTeV di-\( \mu \) measurement** shows that \( \Delta s \approx -0.0027 \pm 0.0013 \)

Use opposite sign di-\( \mu \) events to measure \( s \) and \( \bar{s} \).
What is the discrepancy due to (Old Physics)?

• NLO and PDF effects
  – PDF, $m_c$, Higher Twist effect, etc, are small changes

• Heavy vs light target PDF effect (Kovalenko et al., hep-ph/0207158)
  – Using PDF from light target on Iron target could make up the difference ➞ NuTeV result uses PDF extracted from CCFR (the same target)
$\nu_e \xrightarrow{} \nu_s$ Oscillations with Large $M_\nu$

- LSND result implicate a large $\Delta m^2$ ($\sim 10 - 100 \text{eV}^2$) solution for $\nu_e$ oscillation $\xrightarrow{}$ MiniBooNe at FNAL is running to put the nail on the coffin

- How would this affect NuTeV $\sin^2 \theta_W$?

$$\sin^2 \theta_W = \frac{1}{2} \left( R^? - rR^? \right)$$

$$R^\nu = \frac{N^\nu_{\text{Short}} - N^\nu_{\nu_e}}{N^\nu_{\text{Long}}}$$

If $\nu_e \xrightarrow{} \nu_s$ with $P_{\nu_e \rightarrow \nu_s}$ then $N_{\nu_e} = N^{MC}_{\nu_e} P_{\nu_e \rightarrow \nu_s} = N^{MC}_{\nu_e} (1 - P_{\nu_e \rightarrow \nu_s})$

Thus, MC will subtract more than it is in nature, causing measured $R^\nu$ to be smaller and thereby increasing $\sin^2 \theta_W$
New Physics: Interactions from Extra U(1) – Z’

- Extra U(1) gauge group giving rise to interactions mediated by heavy Z’ boson ($M_{Z'} >> M_Z$)
- While couplings in these groups are arbitrary, E(6) gauge groups can provide mechanism for extra U(1) interaction via heavy Z’.
- Can give rise to $g_R$ but not $g_L$ which is strongly constrained by precision Z measurement.
What other explanations (New Physics)?

- Heavy non-SM vector boson exchange: $Z'$, LQ, etc
  - Suppressed $Z\nu\nu$ (invisible) coupling
  - LL coupling enhanced than LR needed for NuTeV

Both precision data are lower than SM
What other explanations (New Physics)?

- Propagator and coupling corrections
  - Small compared to the effect
- MSSM: Loop corrections wrong sign and small for the effect
- Many other attempts in progress but so far nothing seems to explain the NuTeV results
  - Lepto-quarks
  - Contact interactions with LL coupling (NuTeV wants $m_{Z'} \sim 1.2$ TeV, CDF/DØ: $m_{Z'} > 700$ GeV)
  - Almost sequential $Z'$ with opposite coupling to $\nu$

Langacker et al., Rev. Mod. Phys. 64 87; Cho et al., Nucl. Phys. B531, 65; Zppenfeld and Cheung, hep-ph/9810277; Davidson et al., hep-ph/0112302
Linking $\sin^2\theta_W$ with Higgs through $M_{\text{top}}$ vs $M_W$

One-loop correction to $\sin^2\theta_W$

$$d\sin^2\theta_W^{(\text{on-shell})} = -0.00022 \left( \frac{M_t^2 - (175\text{GeV})^2}{(50\text{GeV})^2} \right) + 0.00032 \ln \left( \frac{M_H}{150\text{GeV}} \right)$$

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PHYS 5326, Spring 2003
Jae Yu
Neutrino Oscillation

• First suggestion of neutrino mixing by B. Pontecorvo at the K0, K0-bar mixing in 1957

• Solar neutrino deficit in 1969 by Ray Davis in Homestake Mine in SD. ➔ Called MSW effect

• Caused by the two different eigenstates for mass and weak

• Neutrinos change their flavor as they travel ➔ Neutrino flavor mixing

• Oscillation probability depends on
  – Distance between the source and the observation point
  – Energy of the neutrinos
  – Difference in square of the masses
Neutrino Oscillation Formalism

• Two neutrino mixing case:

\[
\begin{pmatrix}
|v_e\rangle \\
|v_\mu\rangle
\end{pmatrix} =
\begin{pmatrix}
\cos \theta & \sin \theta \\
-\sin \theta & \cos \theta
\end{pmatrix}
\begin{pmatrix}
|v_1\rangle \\
|v_2\rangle
\end{pmatrix}
\]

OR

\[
|v_e\rangle = \cos \theta |v_1\rangle + \sin \theta |v_2\rangle
\]

\[
|v_\mu\rangle = -\sin \theta |v_1\rangle + \cos \theta |v_2\rangle
\]

where \( |v_e\rangle \) and \( |v_\mu\rangle \) are weak eigenstates, while \( |v_1\rangle \) and \( |v_2\rangle \) are mass eigenstates, and \( \theta \) is the mixing angle that gives the extent of mass eigenstate mixture, analogous to Cabbio angle.
Oscillation Probability

• Let $\nu_\mu$ at time $t=0$ be the linear combination of $\nu_1$ and $\nu_2$ with masses $m_1$ and $m_1$, the wave function becomes:

$$|\nu_\mu (t = 0)\rangle = -\sin \theta |\nu_1\rangle + \cos \theta |\nu_2\rangle$$

• Then later time $t$ the $\nu_\mu$ wave function becomes:

$$|\nu_\mu (t)\rangle = -\sin \theta \exp\left[-i\left(\frac{E_1}{\hbar}\right)t\right]|\nu_1\rangle + \cos \theta \exp\left[-i\left(\frac{E_2}{\hbar}\right)t\right]|\nu_2\rangle$$

• For relativistic neutrinos ($E_\nu >> m_i$), the energies of the mass eigenstates are:

$$E_k = \sqrt{p^2 + m_k^2} \approx p + \frac{m_k^2}{2p}$$
Oscillation Probability

- Substituting the energies into the wave function:

\[
|\nu_\mu(t)| = \exp \left[ -it \left( p + \frac{m_1^2}{2E_\nu} \right) \right] \left[ -\sin \theta |\nu_1\rangle + \cos \theta |\nu_2\rangle \right] \exp \left[ i\Delta m^2 t \frac{l}{2E_\nu} \right]
\]

where \(\Delta m^2 \equiv m_1^2 - m_2^2\) and \(E_\nu \equiv p\).

- Since the \(\nu\)'s move at the speed of light, \(t=x/c\), where \(x\) is the distance to the source of \(\nu_\mu\).

- The probability for \(\nu_\mu\) with energy \(E_\nu\) oscillates to \(\nu_e\) at the distance \(L\) from the source becomes

\[
P(\nu_\mu \rightarrow \nu_e) = \sin^2 2\theta \sin^2 \left( \frac{1.27 \Delta m^2 L}{E_\nu} \right)
\]
Homework Assignments

• Produce an electron $E_T$ spectrum of the highest $E_T$ electrons in your samples
  – Due Wednesday, Feb. 26
• Complete the derivation of the probability for nm of energy $E_\nu$ to oscillate to $\nu_e$ at the distance $L$ away from the source of $\nu_\mu$.
• Draw the oscillation probability distributions as a function of
  – Distance $L$ for a fixed neutrino beam energy $E_\nu$ (=5, 50, 150 GeV)
  – $E_\nu$ for a detector at a distance $L$ (=1.5, 735, 2200km) away from the source
• Due Wednesday, Mar. 5