# PHYS 5326 – Lecture #18

*Monday, Mar. 26, 2003*
*Dr. Jae Yu*

- Mass Terms in Lagrangians
- Spontaneous Symmetry Breaking
- Higgs Mechanism

# Announcements

- Prepare a semester project progress for next Monday, Mar. 31
  - Prepare slides for improved cuts, explanations, plots, interpretations and the plans to complete the project
  - The presentation can be up to 10 minutes each.
  - Remember that you need to submit a report.
- No written final exams
  - Will keep the mid-term proportion at 20%.
  - Homework takes 20%.
  - Semester project
    - Presentation 30%
    - Note 30%

# Semester Projects

- DØ Data Analysis
  - Need to setup DØ Data Analysis systems
    - See http://www-d0.fnal.gov/computing/algorithms/howto/tutorial.html for tutorial
- Consists of
  - A >=10 page report (must become a UTA-HEP note)
  - A 30 minute presentation
- Topics
  - Number of events vs Number of jets for W and Z events

# Introducing Mass Terms

Consider a free Lagrangian for a scalar field, φ:

$$L = \frac{1}{2}\left(\partial_m f\right)\left(\partial^m f\right) + e^{-(af)}$$

No apparent mass terms unless we expand the second term and compare *L* with the Klein-Gordon *L*:

$$L = \frac{1}{2}\left(\partial_m f\right)\left(\partial^m f\right) + 1 - (af)^2 + \frac{1}{2}(af)^4 - \frac{1}{6}(af)^6 + \ldots.$$

$$L_{KG} = \frac{1}{2}\left(\partial_m f\right)\left(\partial^m f\right) - \frac{1}{2}\left(\frac{mc}{\hbar}\right)^2 f^2$$

where $m = \sqrt{2}a\hbar / c$

# Introducing Mass Terms in Potential

Consider a Lagrangian for a scalar field, $\phi$, in a potential:

$$\mathcal{L} = \frac{1}{2}\left(\partial_{\mu}\phi\right)\left(\partial^{\mu}\phi\right) + \frac{1}{2}\mu^{2}\phi^{2} - \frac{1}{4}\lambda^{2}\phi^{4}$$

Mass term ($\phi^2$ term) has the wrong sign unless mass is imaginary. How do we interpret this $\mathcal{L}$?

In Feynman calculus, the fields are fluctuation (perturbation) from the ground state (vacuum).

Expressing $\mathcal{L}$ =T-U, the potential energy U is

$$U = -\frac{1}{2}\mu^{2}\phi^{2} + \frac{1}{4}\lambda^{2}\phi^{4}$$

# Introducing Mass Terms in Interactions

The field that minimizes U is $f = \pm m / l$

To shift the ground state to occur at 0, we introduce the new variable, η: $h \equiv f \pm m / l$

Replacing field, φ, with the new field, η, the *L* becomes

$$L = \frac{1}{2}\left(\partial_m h\right)\left(\partial^m h\right) - m^2 h^2 \pm m l h^3 - \frac{1}{4} l^2 h^4 + \frac{1}{4}\left(m^2 / l\right)^2$$

Mass term

3point vtx

4point vtx

Comparing the modified *L* with $L_{KG}$, the mass is $m = \sqrt{2} m \hbar / c$

# Spontaneous Symmetry Breaking

The original lagrangian, *L*,

$$L = \frac{1}{2}\left(\partial_m f\right)\left(\partial^m f\right) + \frac{1}{2}m^2 f^2 - \frac{1}{4}l^2 f^4$$

is even and thus invariant under $\phi \rightarrow -\phi$.

However, the new *L*

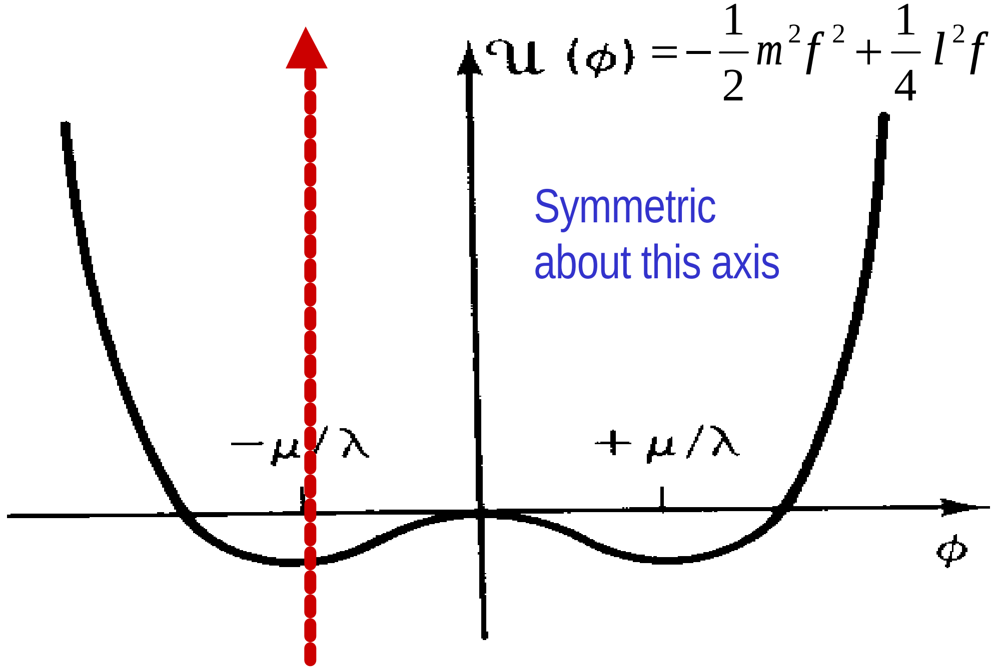
$$L = \frac{1}{2}\left(\partial_m h\right)\left(\partial^m h\right) - m^2 h^2 \pm m l h^3 - \frac{1}{4}l^2 h^4 + \frac{1}{4}\left(m^2 / l\right)^2$$

has an odd term that causes this symmetry to break since any one of the ground states (vacuum) does not share the same symmetry as *L*.

# Potential and Symmetry Breaking

Not symmetric
about this axis

$$\mathcal{U}(\phi) = -\frac{1}{2} m^2 f^2 + \frac{1}{4} l^2 f^4$$



# Spontaneous Symmetry Breaking

While the collection of ground states does preserve the symmetry in *L*, the Feynman formalism allows to work with only one of the ground states. ➔ Causes the symmetry to break.

This is called "spontaneous" symmetry breaking, because symmetry breaking is not externally caused.

The true symmetry of the system is hidden by an arbitrary choice of a particular ground state. This is a case of discrete symmetry w/ 2 ground states.

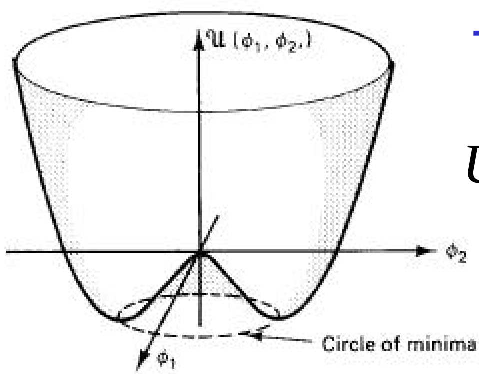
# Spontaneous Breaking of a Continuous Symmetry

A lagrangian, *L*, for two fields, $\phi_1$ and $\phi_2$ can be written

$$L = \frac{1}{2}\left(\partial_m f_1\right)\left(\partial^m f_1\right) + \frac{1}{2}\left(\partial_m f_2\right)\left(\partial^m f_2\right)$$

$$+ \frac{1}{2} m^2 \left(f_1^2 + f_2^2\right) - \frac{1}{4} l^2 \left(f_1^2 + f_2^2\right)^2$$

is even and thus invariant under $\phi_1, \phi_2 \rightarrow -\phi_1, -\phi_2$ .




The potential energy term becomes

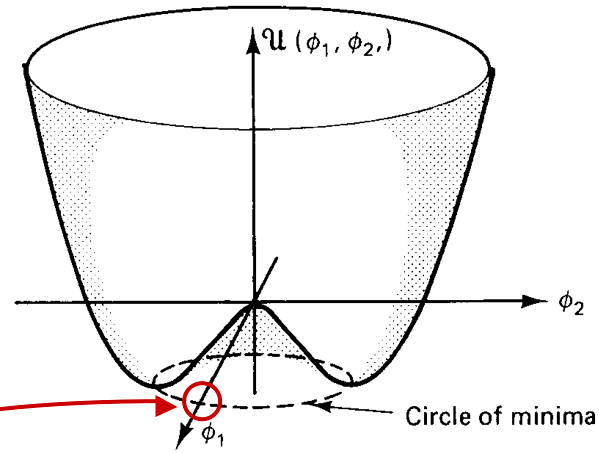
$$U = -\frac{1}{2} m^2 \left(f_1^2 + f_2^2\right) + \frac{1}{4} l^2 \left(f_1^2 + f_2^2\right)^2$$

w/ the minima on the circle:

$$f_{1,\min}^2 + f_{2,\min}^2 = m^2 / l^2$$

# Spontaneous Breaking of a Continuous Symmetry

To apply Feynman calculus, we need to expand about a particular ground state (the "vacuum"). Picking




$f_{1,min} = m/l$ and $f_{2,min} = 0$

And introduce two new fields, $\eta$ and $\zeta$, which are fluctuations about the vacuum:

$$h \equiv f_1 - m/l \qquad \text{and} \qquad x \equiv f_2$$

# Spontaneous Breaking of a Continuous Symmetry

The new $L$ becomes

$$L=\left[\frac{1}{2}\left(\partial_\mu \eta\right)\left(\partial^\mu \eta\right)-\mu^2\eta^2\right]+\left[\frac{1}{2}\left(\partial_\mu \xi\right)\left(\partial^\mu \xi\right)\right]$$

$$-\left[\mu\lambda\left(\eta^3+\eta\xi^2\right)+\frac{\lambda^2}{4}\left(\eta^4+\xi^4+2\eta^2\xi^2\right)\right]+\frac{1}{4}\left(\mu^2/\lambda\right)^2$$

3point vtx

$L_{KG}$ of the field η w/ the mass $m_\eta=\sqrt{2}\mu\hbar/c$

$L_{free}$ of the massless field ξ

4point vtx

# Spontaneous Breaking of Continuous Global Symmetry

One of the fields is automatically massless.

Goldstone's theorem says that breaking of continuous global symmetry is always accompanied by one or more massless scalar (spin=0) bosons, called Goldstone Bosons.

This again poses a problem because the effort to introduce mass to weak gauge fields introduces a massless scalar boson which has not been observed.

This problem can be addressed if spontaneous SB is applied to the case of local gauge invariance.

# Higgs Mechanism

*L* in page 10 can be simplified by combining two real fields into one complex field $f \equiv f_1 + i f_2;\ f^* f = f_1^2 + f_2^2$

Using this new form of the field, the *L* looks exactly like that of a single scalar field

$$L = \frac{1}{2}\left(\partial_m f\right)^*\left(\partial^m f\right) + \frac{1}{2} m^2 f^* f - \frac{1}{4} l^2 \left(f^* f\right)^2$$

Now the rotational symmetry becomes invariance under U(1) gauge transformation, $f \rightarrow e^{iq} f$ .

# Higgs Mechanism

*L* can be made invariant under local gauge transformation by introducing a vector field, $A^{\mu}$, and replacing the partial derivatives with covariant ones. The new *L* then becomes

$$L = \frac{1}{2}\left[\left(\partial_{m} - \frac{iq}{\hbar c}A_{m}\right)f^{*}\right]\left[\left(\partial^{m} + \frac{iq}{\hbar c}A^{m}\right)f\right]$$

$$+\frac{1}{2}m^{2}f^{*}f - \frac{1}{4}l^{2}\left(f^{*}f\right)^{2} - \frac{1}{16p}F^{mn}F_{mn}$$

We then can break the symmetry as we did before by introducing new fields $h \equiv f_{1} - m/l$ and $x \equiv f_{2}$.

# Higgs Mechanism

The new *L* is written in terms of η and ξ as

$$L=\left[\frac{1}{2}\left(\partial_m h\right)\left(\partial^m h\right)-m^2h^2\right]+\left[\frac{1}{2}\left(\partial_m x\right)\left(\partial^m x\right)\right]$$

$$+\left[-\frac{1}{16p}F^{mn}F_{mn}+\frac{1}{2}\left(\frac{iq}{\hbar c}\frac{m}{l}\right)^2A^mA_m\right]-2i\left(\frac{q}{\hbar c}\frac{m}{l}\right)\left(\partial_m x\right)A^m$$

$$+\left\{\frac{q}{\hbar c}\left[h\left(\partial_m x\right)-x\left(\partial_m h\right)\right]A^m+\left(\frac{q}{\hbar c}\right)^2\left[\frac{m}{l}h+\frac{1}{2}\left(x^2+h^2\right)\right]\left(A^mA_m\right)\right.$$

$$\left.-ml\left(h^3+hx^2\right)-\frac{l^2}{4}\left(h^4+x^4+2h^2x^2\right)\right\}+\frac{1}{4}\left(m^2/l\right)^2$$

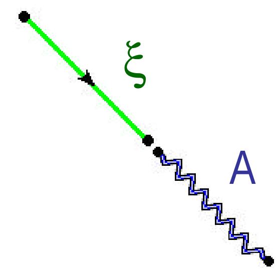
$L_{\text{free}}$ of the massless field ξ

$L_{\text{KG}}$ of the field η w/ the mass $m_h=\sqrt{2}m\hbar/c$

$L_{\text{proca}}$ of the free gauge field ξ w/ the mass $m_A=2\sqrt{p}\,qm/lc$

# Higgs Mechanism

Issues with the new *L* are the unwanted Goldstone boson ξ and the term




$$-2i\left(\frac{q}{\hbar c}\frac{m}{l}\right)(\partial_m x)A^m$$

which can be interpreted as one point vertex interaction between scalar field ξ and vector field $A^\mu$.

This kind of terms indicate that the fundamental particles in the theory are identified incorrectly. Both problems can be resolved exploiting gauge invariance of *L*.

# Homework

- Derive the new L in page 7 (everyone other than BS).
- Derive the new L for two fields in page 12 (BS only).
- Show that one of the two scalar fields could be massless when the choice of minima were made at

$$f_1 = \frac{m}{\sqrt{2}\, l};\ f_2 = -\frac{m}{\sqrt{2}\, l}$$

- Derive the new L in page 16.
- Due Monday, Apr. 7.