# PHYS 5326 – Lecture #19

*Wednesday, Apr. 2, 2003*
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- Wrapping up the Higgs Mechanism
- Introduction of Super Symmetry

# Introducing Mass Terms

Consider a free Lagrangian for a scalar field, φ:

$$L = \frac{1}{2}\left(\partial_m f\right)\left(\partial^m f\right) + e^{-(af)}$$

No apparent mass terms unless we expand the second term and compare *L* with the Klein-Gordon *L*:

$$L = \frac{1}{2}\left(\partial_m f\right)\left(\partial^m f\right) + 1 - (af)^2 + \frac{1}{2}(af)^4 - \frac{1}{6}(af)^6 + \ldots.$$

$$L_{KG} = \frac{1}{2}\left(\partial_m f\right)\left(\partial^m f\right) - \frac{1}{2}\left(\frac{mc}{\hbar}\right)^2 f^2$$

where $m = \sqrt{2}a\hbar / c$

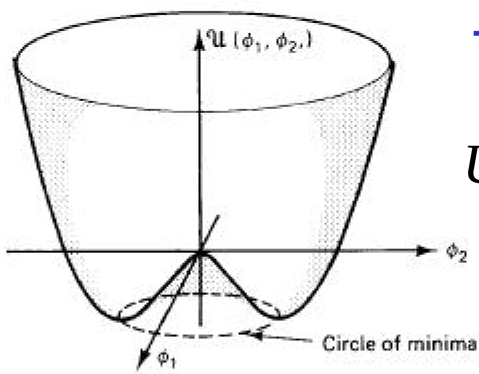
# Spontaneous Breaking of a Continuous Symmetry

A lagrangian, *L*, for two fields, $\phi_1$ and $\phi_2$ can be written

$$L = \frac{1}{2}\left(\partial_m f_1\right)\left(\partial^m f_1\right) + \frac{1}{2}\left(\partial_m f_2\right)\left(\partial^m f_2\right)$$

$$+ \frac{1}{2} m^2 \left(f_1^2 + f_2^2\right) - \frac{1}{4} l^2 \left(f_1^2 + f_2^2\right)^2$$

is even and thus invariant under $\phi_1, \phi_2 \rightarrow -\phi_1, -\phi_2$ .




The potential energy term becomes

$$U = -\frac{1}{2} m^2 \left(f_1^2 + f_2^2\right) + \frac{1}{4} l^2 \left(f_1^2 + f_2^2\right)^2$$

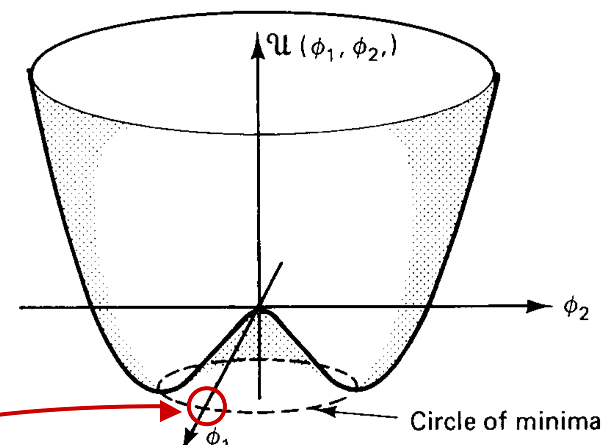
w/ the minima on the circle:

$$f_{1,\min}^2 + f_{2,\min}^2 = m^2 / l^2$$

# Spontaneous Breaking of a Continuous Symmetry

To apply Feynman calculus, we need to expand about a particular ground state (the "vacuum"). Picking

$f_{1,\mathrm{min}} = m/l$ and $f_{2,\mathrm{min}} = 0$




$<\phi_{min}>$=Vacuum expectation values (VEV)

And introduce two new fields, η and ξ, which are fluctuations about the vacuum:

$$h \equiv f_1 - m/l \qquad \text{and} \qquad x \equiv f_2$$

# Spontaneous Breaking of a Continuous Symmetry

## The *L* with broken symmetry becomes

$$L = \left[\frac{1}{2}\left(\partial_m h\right)\left(\partial^m h\right) - m^2 h^2\right] + \left[\frac{1}{2}\left(\partial_m x\right)\left(\partial^m x\right)\right]$$

$$-\left[m l\left(h^3 + h x^2\right) + \frac{l^2}{4}\left(h^4 + x^4 + 2h^2 x^2\right)\right] + \frac{1}{4}\left(m^2 / l\right)^2$$

3point vtx

$L_{KG}$ of the field η w/ the mass $m_h = \sqrt{2} m\hbar / c$

4point vtx

$L_{free}$ of the massless field ξ

# Higgs Mechanism

*L* can be simplified by combining two real fields into one complex field $f \equiv f_1 + i f_2; \;\; f^* f = f_1^2 + f_2^2$

Using this new form of the field, the *L* looks exactly like that of a single scalar field

$$L = \frac{1}{2}\left(\partial_m f\right)^* \left(\partial^m f\right) + \frac{1}{2} m^2 f^* f - \frac{1}{4} l^2 \left(f^* f\right)^2$$

Now the rotational symmetry becomes invariance under U(1) gauge transformation, $f \to e^{iq} f$ .

# Higgs Mechanism

*L* can be made invariant under local gauge transformation by introducing a vector field, $A^{\mu}$, and replacing the partial derivatives with covariant ones.

The new *L* then becomes

$$L = \frac{1}{2}\left[\left(\partial_m - \frac{iq}{\hbar c}A_m\right)f^*\right]\left[\left(\partial^m + \frac{iq}{\hbar c}A^m\right)f\right]$$

$$+\frac{1}{2}m^2 f^* f - \frac{1}{4}l^2\left(f^* f\right)^2 - \frac{1}{16p}F^{mn}F_{mn}$$

We then can break the symmetry as we did before by introducing new fields $h \equiv f_1 - m/l$ and $x \equiv f_2$.

# Higgs Mechanism

The new *L* is written in terms of η and ξ as

$$L=\left[\frac{1}{2}\left(\partial_m h\right)\left(\partial^m h\right)-m^2h^2\right]+\left[\frac{1}{2}\left(\partial_m x\right)\left(\partial^m x\right)\right]$$
$$+\left[-\frac{1}{16p}F^{mn}F_{mn}+\frac{1}{2}\left(\frac{iq}{\hbar c}\frac{m}{l}\right)^2A^mA_m\right]-2i\left(\frac{q}{\hbar c}\frac{m}{l}\right)\left(\partial_m x\right)A^m$$
$$+\left\{\frac{q}{\hbar c}\left[h\left(\partial_m x\right)-x\left(\partial_m h\right)\right]A^m+\left(\frac{q}{\hbar c}\right)^2\left[\frac{m}{l}h+\frac{1}{2}\left(x^2+h^2\right)\right]\left(A^mA_m\right)\right.$$
$$\left.-ml\left(h^3+hx^2\right)-\frac{l^2}{4}\left(h^4+x^4+2h^2x^2\right)\right\}+\frac{1}{4}\left(m^2/l\right)^2$$

$L_{free}$ of the massless field ξ

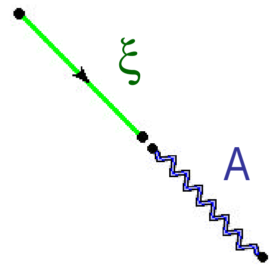
$L_{KG}$ of the field η w/ the mass $m_h=\sqrt{2}m\hbar/c$

$L_{proca}$ of the free gauge field ξ w/ the mass $m_A=2\sqrt{p}\,qm/lc$

# Higgs Mechanism

Issues with the new *L* are the unwanted Goldstone boson ξ and the term

$$-2i\left(\frac{q}{\hbar c}\frac{\mu}{\lambda}\right)\left(\partial_{\mu}\xi\right)A^{\mu}$$




which can be interpreted as one point vertex interaction between scalar field ξ and vector field $A^{\mu}$.

This kind of terms indicate that the fundamental particles in the theory are identified incorrectly. Both problems can be resolved exploiting gauge invariance of *L*.

# Higgs Mechanism

Since the complex field in *L* is $f \equiv f_1 + i f_2$ ,

The U(1) gauge transformed field is written

$$f \rightarrow f^{'} = e^{iq} f = (\cos q + i \sin q)(f_1 + i f_2)$$
$$= (f_1 \cos q - f_2 \sin q) + i(f_2 \cos q + f_1 \sin q)$$

The transformed field, φ', becomes real by picking θ that makes the complex term 0.

$$f_2 \cos q + f_1 \sin q = 0 \;\Rightarrow q = -\tan^{-1}(f_2 / f_1)$$

Under this condition, since $f^{'} \equiv f_1^{'} + i f_2^{'}$, $\phi'_2$ is 0, making the transformed ξ become 0, eliminating it from *L*.

# Higgs Mechanism

In this gauge, the new *L* becomes

$$L = \left[\frac{1}{2}\left(\partial_\mu \eta\right)\left(\partial^\mu \eta\right) - \mu^2\eta^2\right] + \left[-\frac{1}{16\pi}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}\left(\frac{iq}{\hbar c}\frac{\mu}{\lambda}\right)^2 A^\mu A_\mu\right]$$

$$+\left\{\left(\frac{q}{\hbar c}\right)^2\left[\frac{\mu}{\lambda}\eta + \frac{1}{2}\eta^2\right]\left(A^\mu A_\mu\right) - \mu\lambda\eta^3 - \frac{\lambda^2}{4}\eta^4\right\}$$

$$+\frac{1}{4}\left(\mu^2/\lambda\right)^2$$

$L_{KG}$ of the field η w/ the mass $m_\eta = \sqrt{2}\mu\hbar/c$

$L_{proca}$ of the free gauge Boson field w/ the mass $m_A = 2\sqrt{\pi}\, q\mu/\lambda c$

This *L* has all the necessary properties

1. No massless Goldstone boson & no bad term
2. Massive gauge field
3. A single massive scalar boson (Higgs boson)

# Higgs Mechanism

- The new *L* describes the same physical system
  - A particular gauge has been selected
  - Keep the Feynman calculus still valid
- Symmetry has been spontaneously broken, giving masses to gauge field and producing a massive scalar boson
  - The massive gauge vector boson picks up another degree of freedom, the longitudinal polarization, compared to two transverse ones originally.
  - The longitudinal polarization came from Goldstone boson
  - The original ghost Goldstone boson has been eaten by the gauge boson, giving mass to the gauge vector boson and the longitudinal polarization.
- In SM, the Higgs boson is responsible for **mass of weak vector bosons, leptons and quarks**

The Higgs Mechanism

# Issues with Higgs

- Haven't been observed yet.
- Potential is not well known.
- Does not explain why fermion masses are what they are.
- Minimal SM loop corrections give quadratic divergences to the mass of Higgs.
- Other Symmetry breaking models…
- How many Higgs?
- What are the couplings (Yukawa?)? How strong are they?

# Introduction to Super Symmetry

- An alternate solution to resolve mass hierarchy issue caused by the quadratic divergences.
- A symmetry that relates particles of differing spins
- Particles are combined in superfields which contain fields differing by spin ½.
- Scalars and fermions in superfields have the same coupling to gauge bosons and cause the quadratic divergence to cancel
- The $L_{SUSY}$ contains scalar and fermion pairs of equal mass
  - SUSY connects particles of different spins but all other characteristics the same
- SUSY is a broken symmetry because there is no partner of particles with the same mass but different spin➔ non-zero mass splitting between partners is an indication of broken symmetry

# Homework Reminder

- Derive the new *L* in page 7 (everyone except BS) in lecture #18.
- Derive the new *L* for two fields in page 12 (BS only) in lecture #18.
- Show that one of the two scalar fields could be massless when the choice of minima were made at

$$f_1 = \frac{m}{\sqrt{2}\,l};\, f_2 = -\frac{m}{\sqrt{2}\,l}$$

- Derive the new *L* in page 16 of lecture #18.
- Due Monday, Apr. 7.

# Suggested Reading

- S. Dawson, “Introduction to Electroweak Symmetry Breaking,” hep-ph/9901280: sections 10 & 11.
- Higgs search at TeVatron: http://fnth37.fnal.gov/higgs/higgs.html
- J. Gunion, H. Haber, G. Kane, S. Dawson, “The Higgs Hunter’s Guide,” Perseus Publishing: Ch. 4.