PHYS 5326 – Lecture #24

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• Issues with SM picture
• Introduction to SUSY
• Construction of SUSY Lagrangian
• Super-potential
• R-parity conservation
Issues with Higgs

• Haven’t been observed yet.
• Potential is not well known.
• Does not explain why fermion masses are what they are.
• Minimal SM loop corrections give quadratic divergences to the mass of Higgs.
• Other Symmetry breaking models…
• How many Higgs?
• What are the couplings (Yukawa?)? How strong are they?
Issues With SM Picture

- $M_h$ with one loop correction is

$$M_h^2 \approx M_{h_0}^2 + \frac{g_F^2}{4\pi^2} (\Lambda^2 + m_F^2) - \frac{g_S^2}{4\pi^2} (\Lambda^2 + m_S^2)$$

- If $g_S = g_F$, we obtain

$$M_h^2 \approx M_{h_0}^2 + \frac{g_F^2}{4\pi^2} (m_F^2 - m_S^2)$$

- It is acceptable if the difference between $m_F$ and $m_S$ are at the natural level (<1TeV)
- Requires a symmetry in order for this cancellation to persists in all orders of perturbation theory
Introduction of SUSY

• An alternate solution to resolve mass hierarchy issue caused by the quadratic divergences in SM.
• Avoiding the divergences require a tuning at the precision of $10^{16}$ GeV
  – SM EW scale of $\sim 10^3$ GeV
  – No other physics from EW scale to Planck scale of $\sim 10^{19}$ GeV
  – Leading to the adjustment of $10^{16}$ level
  – This is unacceptable
SUSY Lagrangian

• The simplest form of $L_{\text{SUSY}}$ contains a complex scalar, $S$, and two-component Majorana Fermion, $\psi$, where $\psi^c = \psi$.

$$L = -\partial_\mu S^* \partial^\mu S - i \psi \sigma^\mu \partial_\mu \psi - \frac{1}{2} m (\psi \psi + \bar{\psi} \psi)$$

$$-c S \psi \psi - c^2 S^2 \bar{\psi} \psi - \left| m S + c S^2 \right|^2$$

• The $L_{\text{SUSY}}$ is invariant under the transformation between Scalar and Fermions
• Lagrangian contains both a scalar and fermion of equal mass
What is SUSY?

• A symmetry that relates particles of differing spins.
• Particles are combined in superfields which contain fields differing by spin $\frac{1}{2}$.
• Scalars and fermions in superfields have the same coupling to gauge bosons and cause the quadratic divergence to cancel.
• The $L_{\text{SUSY}}$ contains scalar and fermion pairs of equal mass.
  – SUSY connects particles of different spins but all other characteristics the same.
SUSY is a Broken Symmetry

• Since there are no observed candidate of scalar partners of fermions of the same mass vice verse in nature.
• Non-zero mass splitting of the superfields is necessary.
• SUSY is a broken symmetry.
Construction of SUSY

• Start from the supersymmetric version of SM.
• Pick the particles in superfields.
• Two types of superfields relevant for this are:

Chiral Superfields: A complex scalar field, S, and a 2 component Majorana Fermion Field, y

Massless Vector Superfields: a massless gauge field with field strength $F_{\mu\nu}^A$ and a 2-component Majorana Fermion fields, $\lambda_A$, termed a gaugino.
The Particles of the MSSM

Uses the same SU(3) x SU_L(2) x U_Y(1) gauge symmetry as the Standard Model and yields the following list of particles

<table>
<thead>
<tr>
<th>Superfield</th>
<th>SU(3)</th>
<th>SU(2)_L</th>
<th>U(1)_Y</th>
<th>Particle Content</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q</td>
<td>3</td>
<td>2</td>
<td>1/6</td>
<td>(u_L, d_L), (u_R, d_R)</td>
</tr>
<tr>
<td>Q^c</td>
<td>3</td>
<td>1</td>
<td>-2/3</td>
<td>(\bar{u}_R), (\bar{d}_R)</td>
</tr>
<tr>
<td>D^c</td>
<td>3</td>
<td>1</td>
<td>2/3</td>
<td>(\bar{d}_R), (\bar{D}_R)</td>
</tr>
<tr>
<td>L</td>
<td>1</td>
<td>2</td>
<td>-1/2</td>
<td>((\nu_L, \nu_R), ((\bar{\nu}_L, \bar{\nu}_R))</td>
</tr>
<tr>
<td>L^c</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>(\nu_L, \bar{\nu}_L)</td>
</tr>
<tr>
<td>(\hat{H}_1)</td>
<td>1</td>
<td>2</td>
<td>-1/2</td>
<td>(H_1, \bar{H}_1)</td>
</tr>
<tr>
<td>(\hat{H}_2)</td>
<td>1</td>
<td>2</td>
<td>1/2</td>
<td>(H_2, \bar{H}_2)</td>
</tr>
</tbody>
</table>

**Chiral Superfields**

<table>
<thead>
<tr>
<th>Superfield</th>
<th>SU(3)</th>
<th>SU(2)_L</th>
<th>U(1)_Y</th>
<th>Particle Content</th>
</tr>
</thead>
<tbody>
<tr>
<td>G^a</td>
<td>8</td>
<td>1</td>
<td>0</td>
<td>g, (\bar{g})</td>
</tr>
<tr>
<td>(\tilde{W})</td>
<td>1</td>
<td>3</td>
<td>0</td>
<td>(W, \bar{W})</td>
</tr>
<tr>
<td>(\tilde{B})</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>B, (\bar{B})</td>
</tr>
</tbody>
</table>

**Vector Superfield**

Scalar and vector particle names have suffix -inos.

Majorana fermion partners
Higgs Doublets in the MSSM

• In SM contains a single SU(2)_L scalar doublet, the Higgs Doublet.
• In MSSM this doublet acquires a SUSY partner which is a SU(2)L double of Majorana Fermion fields, \(^{-}h_1\), the higgsinos
• These Higgsinos contribute to the triangle SU(2) and U(1) gauge anomalies
  • The SM fermions have exactly the right QN to cancel these anomalies
  • The SUSY fermionic partners contribute to the anomaly
• Simplest solution of adding a second doublet, H2, with precisely the opposite QN to the first doublet and its fermionic partner
• Theory becomes anomaly free and sensible
SUSY Higgs Mechanism

Before the EW symmetry was broken, the two complex SU(2)$_L$ Higgs doublets had 8 DoF of which three have been are observed to give masses to W and Z gauge bosons, leaving five physical DoF.

These remaining DoF are two charged Higgs bosons ($H^+/-$), a CP-odd neutral Higgs boson, $A^0$, and 2 CP-even neutral higgs bosons, $h^0$ and $H^0$.

It is a general prediction of supersymmetric models to expand physical Higgs sectors.
The MSSM

• SUSY’s predictive power comes from the limited ways of interaction between ordinary particles and their super-partners
• However, there is nothing that can stop us from adding more particles to the list of particles given previously, as long as any additional contributions to gauge anomalies cancel among themselves.
• Some models add an additional gauge singlet superfield to the spectrum
• But we stick with the simplest case here
Construction of MSSM L

- The SUSY associates each 2-component Majorana fermion with a complex scalar, while the massive fermions in SM are Dirac fermions with four components.
- The canonical kinetic energy of the field of MSSM is

\[ L_{KE} = \sum_i \left\{ (\partial_\mu S_i^*) (\partial^\mu S_i) + i \bar{\psi}_i D \psi_i \right\} + \sum_A \left\{ -\frac{1}{4} F_{\mu\nu}^A F^{\mu\nu A} + i \bar{\lambda}_A D \lambda_A \right\} \]

Where \( D \) is the SU(3)\( \times \)SU(2)\( _L \times \)U(1)\( _Y \) gauge invariant derivative.
\( \Sigma_i \) is for all fermion fields of the SM, \( \psi_i \), and their scalar partners, \( \Sigma_i \).
\( \Sigma_A \) is over the SU(3), SU(2)\( _L \) and U(1)\( _Y \) gauge fields with their fermion partners, the gauginos.
\textbf{L for Interactions between Chiral superfields and gauginos}

\( \text{L} \) is completely specified by the gauge symmetries and by supersymmetry as follows:

\[
\mathcal{L}_{\text{int}} = -\sqrt{2} \sum_{i,A} g_A \left[ S_i^* T^A \bar{\psi}_i \lambda_A + h.c. \right] - \frac{1}{2} \sum_A \left( \sum_i g_A T^A S_i \right)
\]

Where \( \psi_L = \frac{1}{2} \left( \bar{\psi} \cdot \gamma_5 \right) \psi \)

- \( g_A \) is the relevant gauge coupling constant.
- The interaction strengths are fixed in terms of these constants.
  - E.g., the interaction between a quark, the squark and the gluino is governed by the strong coupling constant, \( g_s \)
The Super-potential

• The only freedom left in constructing the SUSY L is in a function called super-potential, W.

• W is a function of the chiral superfields only
  • Terms in W with more than 3 chiral superfields would yield non-renormalizable interactions in L.
  • W is not allowed to contain derivative interactions.

• Corresponding lagrangian that contains W with scalar potential and the Yukawa interactions of fermions and scalars is

$$L_w = -\sum_i \left( \frac{\partial W}{\partial z_i} \right)^2 + \frac{1}{2} \sum_{i,j} \left[ \bar{\psi}_{iL} \frac{\partial^2 W}{\partial z_i \partial z_j} \psi_j + h.c. \right]$$

Where z is a chiral superfield.
This form of L is dictated by SUSY and by the renomalizability.
Form of the Super-potential

- The most general SU(3) x SU(2)_L x U(1)_Y invariant superpotential with arbitrary coefficients for interactions is

\[ W = \varepsilon_{i,j}^C u \begin{pmatrix} H_1 & H_2 \end{pmatrix} + \varepsilon_{i,j} [\lambda_{ij} H_1 \begin{pmatrix} L^c & E \end{pmatrix} + \lambda_D H_1 \begin{pmatrix} Q^c & D \end{pmatrix} + \lambda_U H_2 \begin{pmatrix} Q^c & U \end{pmatrix}] + \lambda_{ij} \begin{pmatrix} L^c & D \end{pmatrix} \]

- The coefficients are determined in terms of fermion masses and vacuum expectation values of the neutral members of the scalar components of the Higgs double and are not free parameters.

- L so far does not provide masses to all the particles, fermions, scalars and gauge fields yet.

- Terms violating baryon number and mediate proton decay

- Yukawa interactions between fermions and Higgs bosons
R-Parity

• To eliminate the undesirable lepton and baryon number violation terms, a symmetry require these to be forbidden.
• If they are forbidden, they will not reappear in higher order perturbation theory.
• The symmetry to do this is called R-Parity.
• R-parity can be defined as a multiplicative quantum number such that all SM particles have R-parity +1 (even) while their super partners are −1 (odd).
• For particle of spin s, it can also be defined as:

\[ R \equiv (-1)^{3(B-L)+2s} \]
R-Parity Conservation

• Requiring R-parity to conserve has consequences
• Since R-parity is a multiplicative QN, the number of SUSY partners in a given interaction is always conserved modulo 2.
  ➤ SUSY Partners can only be pair produced from SM particles
• SUSY particles will decay in a chain until the lightest SUSY particle, LSP, which must be absolutely stable to conserve R-parity, is produced
  ➤ A theory with R-parity conservation will have a lightest SUSY particle which is stable
• LSP must be stable and neutral (not detectable) since it only interacts only by the exchange of a heavy virtual SUSY particle
  ➤ LSP will interact very weakly w/ ordinary matter
  ➤ Direct signature of R-parity conserving SUSY is missing $E_T$
• There is no LSP for R-parity violating SUSY theories
Suggested Readings

• S. Dawson, “SUSY and Such,” hep-ph/9612229