PHYS 1441 – Section 004
Lecture #5

Wednesday, Feb. 4, 2004

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- Chapter two: Motion in one dimension
  - One dimensional motion at constant acceleration
    - Falling Motion
  - Coordinate systems
- Chapter three: Motion in two dimension
  - Vector and Scalar
  - Properties of vectors
  - Vector operations
  - Components and unit vectors

Today's homework is homework #3, due 1pm, next Wednesday!!

Announcements

- There will be a quiz next Wednesday, Feb. 11
  - Will cover
    - Sections A5 – A9
    - Chapter 2
- Homework Registration: 60/61
  - Roster will be locked again at 2:40pm today
  - Come see me if you haven’t registered.
- E-mail distribution list (phys1441-004-spring04)
  - 45 of you subscribed as of 10am this morning
  - Test message will be issued today, Feb. 4

Displacement, Velocity and Speed

Displacement

$$\boxed{\Delta x \equiv x_f - x_i}$$

Average velocity

$$\boxed{v_x \equiv \frac{x_f - x_i}{t_f - t_i} = \frac{\Delta x}{\Delta t}}$$

Average speed

$$\boxed{v \equiv \frac{\text{Total Distance Traveled}}{\text{Total Time Spent}}}$$

Instantaneous velocity

$$\boxed{v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}}$$

Instantaneous speed

$$\boxed{\left|v_x\right| = \left|\lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}\right| = \left|\frac{dx}{dt}\right|}$$

Acceleration

Change of velocity in time (what kind of quantity is this?)

- Average acceleration:

$$a_x \equiv \frac{v_{xf} - v_{xi}}{t_f - t_i} = \frac{\Delta v_x}{\Delta t} \quad \text{analogs to} \quad v_x \equiv \frac{x_f - x_i}{t_f - t_i} = \frac{\Delta x}{\Delta t}$$

- Instantaneous acceleration:

$$a_x \equiv \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2} \quad \text{analogs to} \quad \boxed{v_x \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}}$$

- In calculus terms: A slope (derivative) of velocity with respect to time or change of slopes of position as a function of time

One Dimensional Motion

- Let's start with the simplest case: acceleration is a constant ($a=a_0$)
- Using definitions of average acceleration and velocity, we can derive equations of motion (description of motion, velocity and position as a function of time)

$$\bar{a}_x = a_x = \frac{v_{xf} - v_{xi}}{t_f - t_i}$$ (If $t_f=t$ and $t_i=0$) $$a_x = \frac{v_{xf} - v_{xi}}{t} \Rightarrow v_{xf} = v_{xi} + a_x t$$

For constant acceleration, average velocity is a simple numeric average

$$\bar{v}_x = \frac{v_{xi} + v_{xf}}{2} = \frac{2v_{xi} + a_x t}{2} = v_{xi} + \frac{1}{2}a_x t$$

$$\bar{v}_x = \frac{x_f - x_i}{t_f - t_i}$$ (If $t_f=t$ and $t_i=0$) $$\bar{v}_x = \frac{x_f - x_i}{t} \Rightarrow x_f = x_i + \bar{v}_x t$$

Resulting Equation of Motion becomes

$$x_f = x_i + \bar{v}_x t = x_i + v_{xi} t + \frac{1}{2} a_x t^2$$

One Dimensional Motion cont'd

Average velocity $\bar{v}_x = \frac{v_{xi} + v_{xf}}{2}$ $\quad x_f = x_i + \bar{v}_x t = x_i + \left(\frac{v_{xi} + v_{xf}}{2}\right) t$

Since $a_x = \frac{v_{xf} - v_{xi}}{t}$ Solving for t $t = \frac{v_{xf} - x_{xi}}{a_x}$

Substituting t in the above equation,

$$x_f = x_i + \left(\frac{v_{xf} + v_{xi}}{2}\right)\left(\frac{v_{xf} - v_{xi}}{a_x}\right) = x_i + \frac{v_{xf}^2 - v_{xi}^2}{2a_x}$$

Resulting in

$$v_{xf}^2 = v_{xi}^2 + 2a_x\left(x_f - x_i\right)$$

Kinematic Equations of Motion on a Straight Line Under Constant Acceleration

$$v_{xf}(t) = v_{xi} + a_x t$$

Velocity as a function of time

$$x_f - x_i = \frac{1}{2}\bar{v}_x t = \frac{1}{2}(v_{xf} + v_{xi})t$$

Displacement as a function of velocities and time

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$$

Displacement as a function of time, velocity, and acceleration

$$v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$$

Velocity as a function of Displacement and acceleration

You may use different forms of Kinetic equations, depending on the information given to you for specific physical problems!!

How do we solve a problem using a kinematic formula for constant acceleration?

- Identify what information is given in the problem.
  - Initial and final velocity?
  - Acceleration?
  - Distance?
  - Time?
- Identify what the problem wants.
- Identify which formula is appropriate and easiest to solve for what the problem wants.
  - Frequently multiple formula can give you the answer for the quantity you are looking for. ➔ Do not just use any formula but use the one that can be easiest to solve.
- Solve the equation for the quantity wanted

Example 2.10

Suppose you want to design an air-bag system that can protect the driver in a head-on collision at a speed 100km/s (~60miles/hr). Estimate how fast the air-bag must inflate to effectively protect the driver. Assume the car crumples upon impact over a distance of about 1m. How does the use of a seat belt help the driver?

How long does it take for the car to come to a full stop? ⇒ As long as it takes for it to crumple.

The initial speed of the car is $$v_{xi} = 100km/h = \frac{100000m}{3600s} = 28m/s$$

We also know that $v_{xf} = 0m/s$ and $x_f - x_i = 1m$

Using the kinematic formula $$v_{xf}^2 = v_{xi}^2 + 2a_x\left(x_f - x_i\right)$$

The acceleration is $$a_x = \frac{v_{xf}^2 - v_{xi}^2}{2\left(x_f - x_i\right)} = \frac{0-\left(28m/s\right)^2}{2\times 1m} = -390m/s^2$$

Thus the time for air-bag to deploy is $$t = \frac{v_{xf} - v_{xi}}{a} = \frac{0-28m/s}{-390m/s^2} = 0.07s$$

Falling Motion

- Falling motion is a motion under the influence of gravitational pull (gravity) only; Which direction is a freely falling object moving?
  - A motion under constant acceleration
  - All kinematic formula we learned can be used to solve for falling motions.
- Gravitational acceleration is inversely proportional to the distance between the object and the center of the earth
- The gravitational acceleration is $g=9.80m/s^2$ on the surface of the earth, most of the time.
- The direction of gravitational acceleration is <u>ALWAYS</u> toward the center of the earth, which we normally call (-y); where up and down direction are indicated as the variable "y"
- Thus the correct denotation of gravitational acceleration on the surface of the earth is $g=-9.80m/s^2$

Example for Using 1D Kinematic Equations on a Falling object

Stone was thrown straight upward at t=0 with +20.0m/s initial velocity on the roof of a 50.0m high building,

What is the acceleration in this motion? $\boxed{g=-9.80m/s^2}$

(a) Find the time the stone reaches at the maximum height.

What is so special about the maximum height? V=0

$$v_f = v_{yi} + a_y t = +20.0 - 9.80t = 0.00m/s$$

Solve for t

$$t = \frac{20.0}{9.80} = 2.04s$$

(b) Find the maximum height.

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2 = 50.0 + 20 \times 2.04 + \frac{1}{2} \times (-9.80) \times (2.04)^2$$

$$= 50.0 + 20.4 = 70.4(m)$$

Example of a Falling Object cnt'd

(c) Find the time the stone reaches back to its original height.

$$t = 2.04 \times 2 = 4.08s$$

(d) Find the velocity of the stone when it reaches its original height.

$$v_{yf} = v_{yi} + a_y t = 20.0 + (-9.80) \times 4.08 = -20.0(m/s)$$

(e) Find the velocity and position of the stone at t=5.00s.

Velocity

$$v_{yf} = v_{yi} + a_y t = 20.0 + (-9.80) \times 5.00 = -29.0(m/s)$$

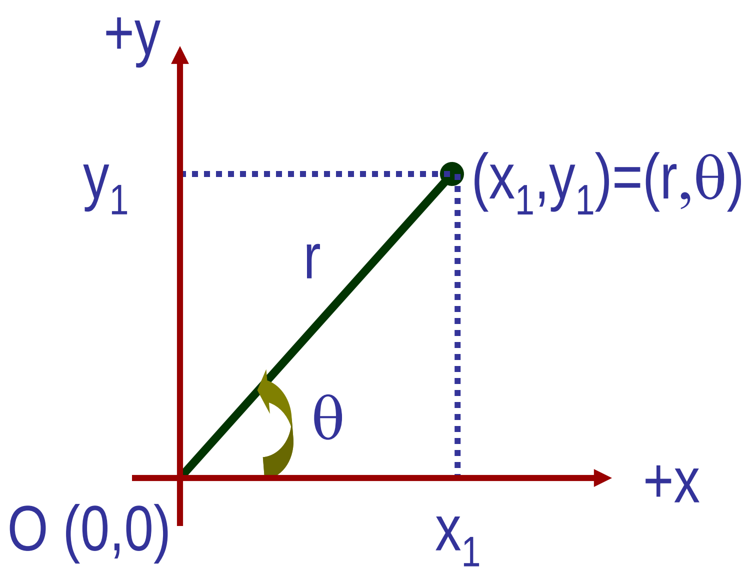
Position

$$y_f = y_i + v_{yi} t + \frac{1}{2} a_y t^2$$

$$= 50.0 + 20.0 \times 5.00 + \frac{1}{2} \times (-9.80) \times (5.00)^2 = +27.5(m)$$

Coordinate Systems

- Makes it easy to express locations or positions
- Two commonly used systems, depending on convenience
  - Cartesian (Rectangular) Coordinate System
    - Coordinates are expressed in (x,y)
  - Polar Coordinate System
    - Coordinates are expressed in (r,θ)
- Vectors become a lot easier to express and compute



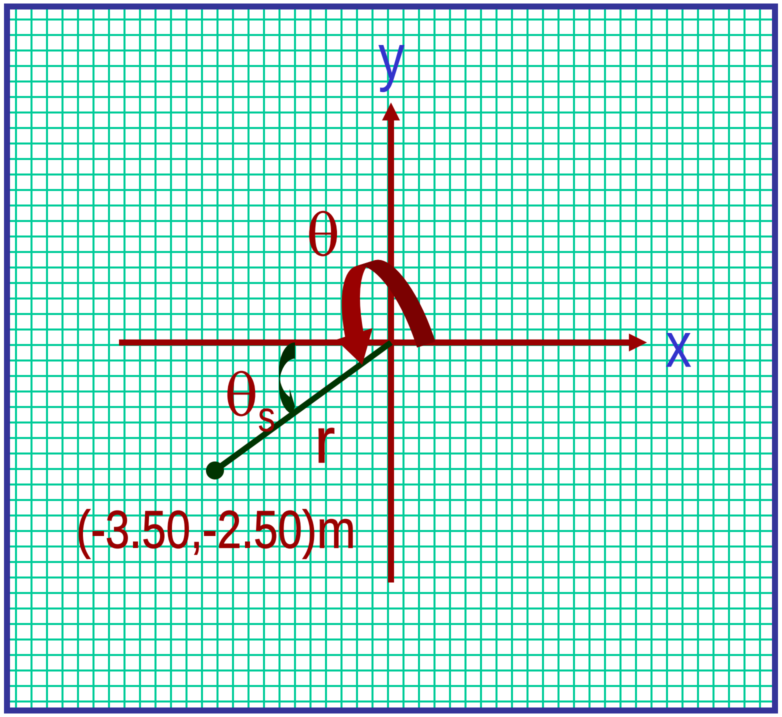
How are Cartesian and Polar coordinates related?

$$x_1 = r\cos\theta \qquad r = \sqrt{\left(x_1^2 + y_1^2\right)}$$

$$y_1 = r\sin\theta \qquad \tan\theta = \frac{y_1}{x_1}$$

Example

Cartesian Coordinate of a point in the xy plane are (x,y)= (-3.50,-2.50)m. Find the polar coordinates of this point.



$$r = \sqrt{\left(x^2 + y^2\right)}$$

$$= \sqrt{\left((-3.50)^2 + (-2.50)^2\right)}$$

$$= \sqrt{18.5} = 4.30(m)$$

$$\theta = 180 + \theta_s$$

$$\tan\theta_s = \frac{-2.50}{-3.50} = \frac{5}{7}$$

$$\theta_s = \tan^{-1}\left(\frac{5}{7}\right) = 35.5^\circ$$

$$\therefore \theta = 180 + \theta_s = 180^\circ + 35.5^\circ = 216^\circ$$

Vector and Scalar

Vector quantities have both magnitude (size) and direction

Force, gravitational pull, momentum

Normally denoted in **BOLD** letters, ***F***, or a letter with arrow on top $\vec{F}$

Their sizes or magnitudes are denoted with normal letters, *F*, or absolute values: $\left|\vec{F}\right|$ or $|\mathbf{F}|$

Scalar quantities have magnitude only

Can be completely specified with a value and its unit

Normally denoted in normal letters, *E*

Energy, heat, mass, weight

Both have units!!!