PHYS 1441 – Section 004
Lecture #6

Monday, Feb. 9, 2004

Dr. Jaehoon Yu

- Chapter three: Motion in two dimension
  - Vector and Scalar
  - Properties of vectors
  - Vector operations
  - Components and unit vectors
  - 2D Kinematic Equations
  - Projectile Motion

Announcements

- There will be a quiz this Wednesday, Feb. 11
  - Will cover
    - Sections A5 – A9
    - Chapter 2
- Everyone is registered to homework system: Good job!!!
- E-mail distribution list (phys1441-004-spring04)
  - 48 of you subscribed as of 11am this morning
  - Test message was sent last night
    - Please reply to ME for verification

Kinematic Equations of Motion on a Straight Line Under Constant Acceleration

$$\boxed{v_{xf}(t) = v_{xi} + a_x t}$$

Velocity as a function of time

$$\boxed{x_f - x_i = \frac{1}{2}\bar{v}_x t = \frac{1}{2}\left(v_{xf} + v_{xi}\right)t}$$

Displacement as a function of velocities and time

$$\boxed{x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2}$$

Displacement as a function of time, velocity, and acceleration

$$\boxed{v_{xf}^2 = v_{xi}^2 + 2a_x\left(x_f - x_i\right)}$$

Velocity as a function of Displacement and acceleration

You may use different forms of Kinetic equations, depending on the information given to you for specific physical problems!!

Vector and Scalar

Vector quantities have both magnitude (size) and direction

Force, gravitational pull, momentum

Normally denoted in **BOLD** letters, $\mathbf{F}$, or a letter with arrow on top $\vec{F}$

Their sizes or magnitudes are denoted with normal letters, F, or absolute values: $\left|\vec{F}\right|$ or $\left|\mathbf{F}\right|$

Scalar quantities have magnitude only

Can be completely specified with a value and its unit

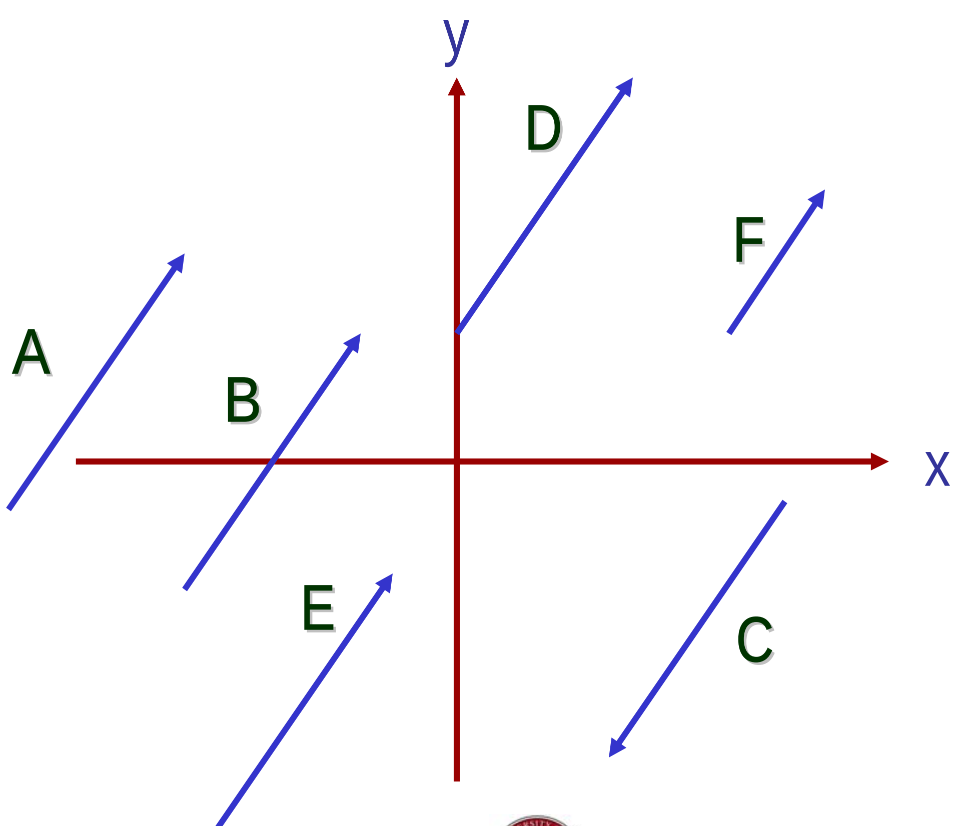
Energy, heat, mass, weight

Normally denoted in normal letters, E

Both have units!!!

Properties of Vectors

- Two vectors are the same if their sizes and the directions are the same, no matter where they are on a coordinate system.



Which ones are the same vectors?

A=B=E=D

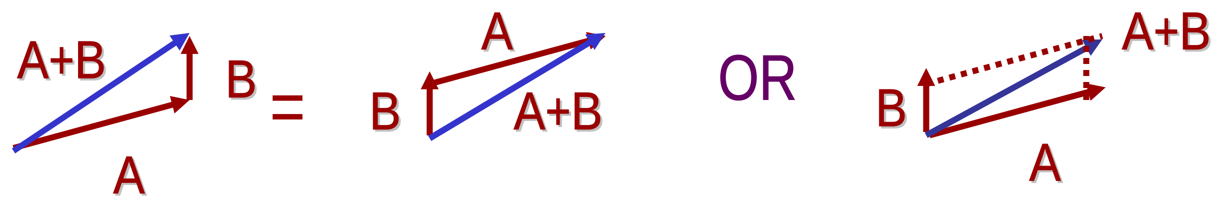
Why aren't the others?

C: The same magnitude but opposite direction: **C=-A:**A negative vector

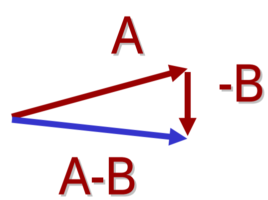
F: The same direction but different magnitude

Vector Operations

- Addition:
  - Triangular Method: One can add vectors by connecting the head of one vector to the tail of the other (head-to-tail)
  - Parallelogram method: Connect the tails of the two vectors and extend
  - Addition is commutative: Changing order of operation does not affect the results A+B=B+A, A+B+C+D+E=E+C+A+B+D



- Subtraction:
  - The same as adding a negative vector: A - B = A + (-B)



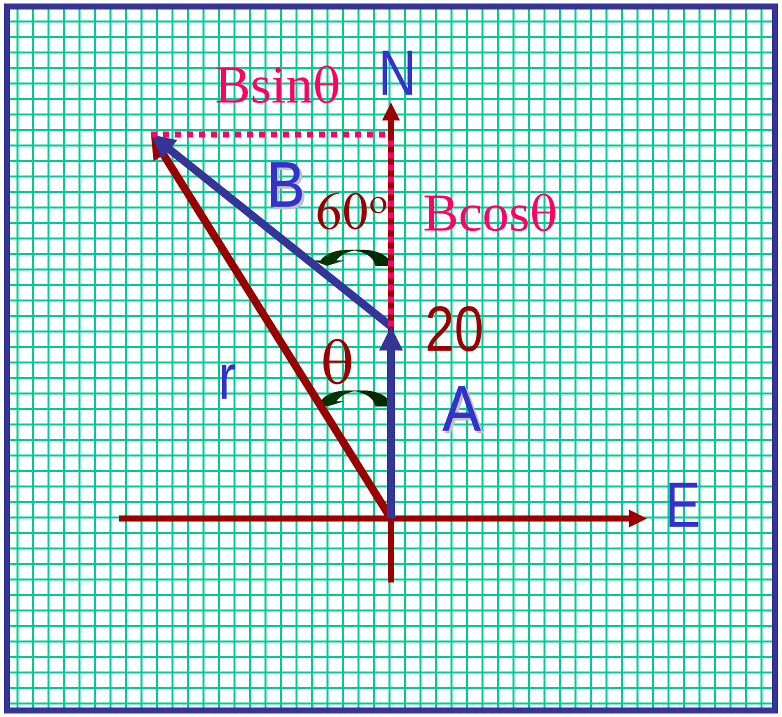
Since subtraction is the equivalent to adding a negative vector, subtraction is also commutative!!!

- Multiplication by a scalar is increasing the magnitude A, B=2A

$$\boxed{|\mathcal{B}| = 2|\mathcal{A}|}$$

Example of Vector Addition

A car travels 20.0km due north followed by 35.0km in a direction 60.0º west of north. Find the magnitude and direction of resultant displacement.



$$r = \sqrt{(A + B\cos\theta)^2 + (B\sin\theta)^2}$$

$$= \sqrt{A^2 + B^2\left(\cos^2\theta + \sin^2\theta\right) + 2AB\cos\theta}$$

$$= \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

$$= \sqrt{(20.0)^2 + (35.0)^2 + 2\times 20.0\times 35.0\cos 60}$$

$$= \sqrt{2325} = 48.2(km)$$

$$\theta = \tan^{-1}\frac{\left|\vec{B}\right|\sin 60}{\left|\vec{A}\right| + \left|\vec{B}\right|\cos 60}$$

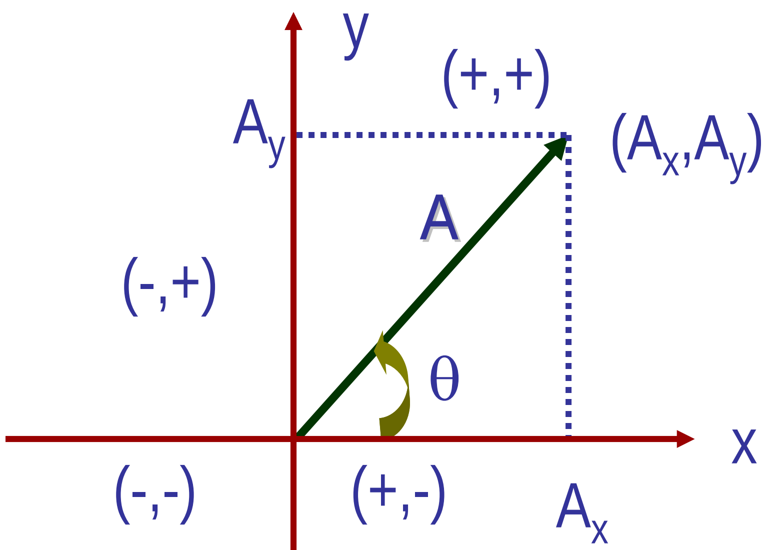
$$= \tan^{-1}\frac{35.0\sin 60}{20.0 + 35.0\cos 60}$$

$$= \tan^{-1}\frac{30.3}{37.5} = 38.9^{\circ} \text{ to W wrt N}$$

Find other ways to solve this problem…

Components and Unit Vectors

Coordinate systems are useful in expressing vectors in their components



$$\boxed{A_x = \left|\vec{A}\right| \cos\theta} \quad \boxed{A_y = \left|\vec{A}\right| \sin\theta} \quad \text{Components}$$

$$\boxed{\left|\vec{A}\right| = \sqrt{A_x^2 + A_y^2}} \quad \text{Magnitude}$$

$$\left|\vec{A}\right| = \sqrt{\left(\left|\vec{A}\right|\cos\theta\right)^2 + \left(\left|\vec{A}\right|\sin\theta\right)^2}$$

$$= \sqrt{\left|\vec{A}\right|^2\left(\cos^2\theta + \sin^2\theta\right)} = \left|\vec{A}\right|$$

- Unit vectors are dimensionless vectors whose magnitude are exactly 1
  - Unit vectors are usually expressed in **i, j, k** or $\vec{i}\,,\ \vec{j}\,,\ \vec{k}$
  - Vectors can be expressed using components and unit vectors

So the above vector **A** can be written as

$$\vec{A} = A_x\vec{i} + A_y\vec{j} = \left|\vec{A}\right|\cos\theta\,\vec{i} + \left|\vec{A}\right|\sin\theta\,\vec{j}$$

Examples of Vector Operations

Find the resultant vector which is the sum of **A**=(2.0**i**+2.0**j**) and **B** =(2.0**i**-4.0**j**)

$$\vec{C} = \vec{A} + \vec{B} = \left(2.0\vec{i} + 2.0\vec{j}\right) + \left(2.0\vec{i} - 4.0\vec{j}\right)$$

$$= \left(2.0+2.0\right)\vec{i} \quad + \left(2.0-4.0\right)\vec{j} \qquad = 4.0\vec{i} \; -2.0\vec{j}\left(m\right)$$

$$\left|\vec{C}\right| = \sqrt{\left(4.0\right)^2 + \left(-2.0\right)^2}$$

$$= \sqrt{16+4.0} = \sqrt{20} = 4.5(m)$$

$$\theta = \tan^{-1}\frac{C_y}{C_x} = \tan^{-1}\frac{-2.0}{4.0} = -27^\circ$$

Find the resultant displacement of three consecutive displacements: **d**$_1$=(15**i**+30**j** +12**k**)cm, **d**$_2$=(23**i**+14**j** -5.0**k**)cm, and **d**$_3$=(-13**i**+15**j**)cm

$$\vec{D} = \vec{d}_1 + \vec{d}_2 + \vec{d}_3 = \left(15\vec{i} + 30\vec{j} + 12\vec{k}\right) + \left(23\vec{i} + 14\vec{j} - 5.0\vec{k}\right) + \left(-13\vec{i} + 15\vec{j}\right)$$

$$= \left(15 + 23 - 13\right)\vec{i} + \left(30 + 14 + 15\right)\vec{j} \; + \left(12 - 5.0\right)\vec{k} = 25\vec{i} + 59\vec{j} + 7.0\vec{k}(cm)$$

Magnitude

$$\left|\vec{D}\right| = \sqrt{\left(25\right)^2 + \left(59\right)^2 + \left(7.0\right)^2} = 65(cm)$$

Displacement, Velocity, and Acceleration in 2-dim

- Displacement: $$\Delta\vec{r} = \vec{r}_f - \vec{r}_i$$
- Average Velocity: $$\vec{v} \equiv \frac{\Delta\vec{r}}{\Delta t} = \frac{\vec{r}_f - \vec{r}_i}{t_f - t_i}$$
- Instantaneous Velocity: $$\vec{v} \equiv \lim_{\Delta t \to 0} \frac{\Delta\vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$
- Average Acceleration $$\vec{a} \equiv \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i}$$
- Instantaneous Acceleration: $$\vec{a} \equiv \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left(\frac{d\vec{r}}{dt}\right) = \frac{d^2\vec{r}}{dt^2}$$

How is each of these quantities defined in 1-D?

2-dim Motion Under Constant Acceleration

- Position vectors in x-y plane: $\vec{r}_i = x_i\vec{i} + y_i\vec{j}$ $\quad \vec{r}_f = x_f\vec{i} + y_f\vec{j}$
- Velocity vectors in x-y plane: $\vec{v}_i = v_{xi}\vec{i} + v_{yi}\vec{j}$ $\quad \vec{v}_f = v_{xf}\vec{i} + v_{yf}\vec{j}$

Velocity vectors in terms of acceleration vector

$$v_{xf} = v_{xi} + a_x t \qquad v_{yf} = v_{yi} + a_y t$$

$$\vec{v}_f = \left(v_{xi} + a_x t\right)\vec{i} + \left(v_{yi} + a_y t\right)\vec{j} = \vec{v}_i + \vec{a}t$$

- How are the position vectors written in acceleration vectors?

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2 \qquad y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$

$$\vec{r}_f = x_f\vec{i} + y_f\vec{j} = \left(x_i + v_{xi}t + \frac{1}{2}a_x t^2\right)\vec{i} + \left(y_i + v_{yi}t + \frac{1}{2}a_y t^2\right)\vec{j}$$

$$= \left(x_i\vec{i} + y_i\vec{j}\right) + \left(v_{xi}\vec{i} + v_{yi}\vec{j}\right)t + \frac{1}{2}\left(a_x\vec{i} + a_y\vec{j}\right)t^2 = \vec{r}_i + \vec{v}_i t + \frac{1}{2}\vec{a}t^2$$

Example for 2-D Kinematic Equations

A particle starts at origin when t=0 with an initial velocity **v**=(20**i**-15**j**)m/s. The particle moves in the xy plane with $a_x=4.0m/s^2$. Determine the components of velocity vector at any time, t.

$$v_{xf}=v_{xi}+a_x t=20+4.0t\,(m/s) \quad v_{yf}=v_{yi}+a_y t=-15+0t=-15\,(m/s)$$

Velocity vector

$$\vec{v}(t)=v_x(t)\vec{i}+v_y(t)\vec{j}=(20+4.0t)\vec{i}-15\vec{j}\,(m/s)$$

Compute the velocity and speed of the particle at t=5.0 s.

$$\vec{v}_{t=5}=v_{x,t=5}\vec{i}+v_{y,t=5}\vec{j}=(20+4.0\times 5.0)\vec{i}-15\vec{j}=\left(40\vec{i}-15\vec{j}\right)\,m/s$$

$$speed=\left|\vec{v}\right|=\sqrt{(v_x)^2+(v_y)^2}=\sqrt{(40)^2+(-15)^2}=43m/s$$

Example for 2-D Kinematic Eq. Cnt'd

Angle of the Velocity vector

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{-15}{40}\right) = \tan^{-1}\left(\frac{-3}{8}\right) = -21^\circ$$

Determine the x and y components of the particle at t=5.0 s.

$$x_f = v_{xi}t + \frac{1}{2}a_x t^2 = 20\times 5 + \frac{1}{2}\times 4\times 5^2 = 150(m)$$

$$y_f = v_{yi}t = -15\times 5 = -75\,(m)$$

Can you write down the position vector at t=5.0s?

$$\vec{r}_f = x_f\vec{i} + y_f\vec{j} = 150\vec{i} - 75\vec{j}\,(m)$$