PHYS 1441 – Section 004
Lecture #7

Wednesday, Feb. 11, 2004

Dr. Jaehoon Yu

- Projectile Motion
- Maximum Range and Height
- Reference Frame and Relative Velocity

• Chapter four: Newton's Laws of Motion

- Newton's First Law of Motion
- Newton's Second Law of Motion
- Gravitational Force
- Newton's Third Law of Motion

Today's homework is homework #4, due 1pm, next Wednesday!!

1st term exam in the class at 1pm, Monday, Feb. 23!!

Displacement, Velocity, and Acceleration in 2-dim

- Displacement: $\Delta\vec{r} = \vec{r}_f - \vec{r}_i$
- Average Velocity: $\vec{v} \equiv \frac{\Delta\vec{r}}{\Delta t} = \frac{\vec{r}_f - \vec{r}_i}{t_f - t_i}$
- Instantaneous Velocity: $\vec{v} \equiv \lim_{\Delta t \to 0} \frac{\Delta\vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$
- Average Acceleration: $\vec{a} \equiv \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i}$
- Instantaneous Acceleration: $\vec{a} \equiv \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left(\frac{d\vec{r}}{dt}\right) = \frac{d^2\vec{r}}{dt^2}$

How is each of these quantities defined in 1-D?

Projectile Motion

- A 2-dim motion of an object under the gravitational acceleration with the assumptions
  - Gravitational cceleration, -*g*, is constant over the range of the motion
  - Air resistance and other effects are negligible
- A motion under constant acceleration!!!! ➔ Superposition of two motions
  - Horizontal motion with constant velocity and
  - Vertical motion under constant acceleration

In a projectile motion, the only acceleration is gravitational one whose direction is always toward the center of the earth (downward).

Show that a projectile motion is a parabola!!!

$$v_{xi} = v_i \cos\theta_i \qquad v_{yi} = v_i \sin\theta_i$$

$$\vec{a} = a_x \vec{i} + a_y \vec{j} \equiv -g\vec{j}$$

$a_x=0$ ➔

$$x_f = v_{xi} t = v_i \cos\theta_i t \qquad t = \frac{x_f}{v_i \cos\theta_i}$$

$$y_f = v_{yi} t + \frac{1}{2}(-g)\, t^2$$

$$= v_i \sin\theta_i t - \frac{1}{2} g t^2$$

Plug in the t above ➔

$$y_f = v_i \sin\theta_i \left(\frac{x_f}{v_i \cos\theta_i}\right) - \frac{1}{2} g \left(\frac{x_f}{v_i \cos\theta_i}\right)^2$$

$$y_f = x_f \tan\theta_i - \left(\frac{g}{2 v_i^2 \cos^2\theta_i}\right) x_f^2$$

What kind of parabola is this?

Example for Projectile Motion

A ball is thrown with an initial velocity **v**=(20**i**+40**j**)m/s. Estimate the time of flight and the distance the ball is from the original position when landed.

Which component determines the flight time and the distance?

Flight time is determined by y component, because the ball stops moving when it is on the ground after the flight.

$$y_f = 40t + \frac{1}{2}(-g)t^2 = 0m$$

$$t(80 - gt) = 0$$

$$\therefore t = 0 \text{ or } t = \frac{80}{g} \approx 8\,\text{sec}$$

$$\therefore \ t \approx 8\,\text{sec}$$

Distance is determined by x component in 2-dim, because the ball is at $y=0$ position when it completed it's flight.

$$x_f = v_{xi}t = 20 \times 8 = 160(m)$$

Horizontal Range and Max Height

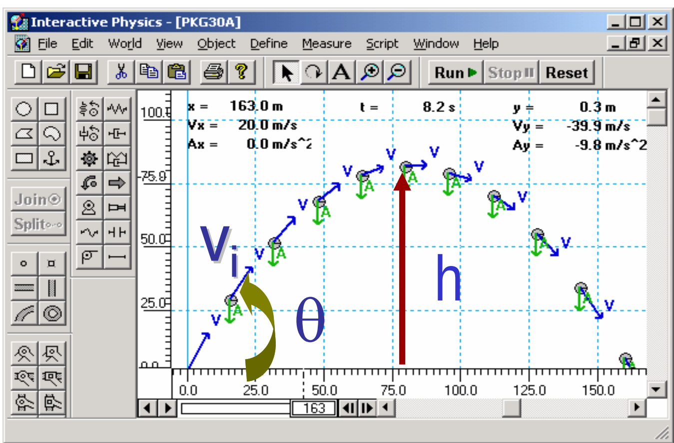
- Based on what we have learned in the previous pages, one can analyze a projectile motion in more detail
  - Maximum height an object can reach
  - Maximum range

What happens at the maximum height?

At the maximum height the object's vertical motion stops to turn around!!

$$v_{yf} = v_{yi} + a_y t = v_i \sin\theta_i - g t_A = 0$$

$$\therefore t_A = \frac{v_i \sin\theta_i}{g}$$



$$y_f = h = v_{yi} t + \frac{1}{2}(-g)t^2$$

$$y_f = v_i \sin\theta_i \left(\frac{v_i \sin\theta_i}{g}\right) - \frac{1}{2} g \left(\frac{v_i \sin\theta_i}{g}\right)^2$$

$$\boxed{y_f = \left(\frac{v_i^2 \sin^2\theta_i}{2g}\right)}$$

Since no acceleration in x, it still flies even if $v_y=0$

$$R = v_{xi} 2 t_A = 2 v_i \cos\theta_i \left(\frac{v_i \sin\theta_i}{g}\right)$$

$$\boxed{R = \left(\frac{v_i^2 \sin 2\theta_i}{g}\right)}$$

Maximum Range and Height

- What are the conditions that give maximum height and range of a projectile motion?

$$h = \left(\frac{v_i^2 \sin^2 \theta_i}{2g} \right)$$

This formula tells us that the maximum height can be achieved when $\theta_i = 90^o$!!!

$$R = \left(\frac{v_i^2 \sin 2\theta_i}{g} \right)$$

This formula tells us that the maximum range can be achieved when $2\theta_i = 90^o$, i.e., $\theta_i = 45^o$!!!

Example for Projectile Motion

- A stone was thrown upward from the top of a building at an angle of 30º to horizontal with initial speed of 20.0m/s. If the height of the building is 45.0m, how long is it before the stone hits the ground?

$$v_{xi} = v_i \cos\theta_i = 20.0 \times \cos 30^\circ = 17.3 m/s$$

$$v_{yi} = v_i \sin\theta_i = 20.0 \times \sin 30^\circ = 10.0 m/s$$

$$y_f = -45.0 = v_{yi}t - \frac{1}{2}gt^2 \qquad gt^2 - 20.0t - 90.0 = 9.80t^2 - 20.0t - 90.0 = 0$$

$$t = \frac{20.0 \pm \sqrt{(-20)^2 - 4 \times 9.80 \times (-90)}}{2 \times 9.80}$$

$$t = -2.18 s \quad \text{or} \quad t = 4.22 s$$

$$t = 4.22 s$$

Only physical answer

- What is the speed of the stone just before it hits the ground?

$$v_{xf} = v_{xi} = v_i \cos\theta_i = 20.0 \times \cos 30^\circ = 17.3 m/s$$

$$v_{yf} = v_{yi} - gt = v_i \sin\theta_i - gt = 10.0 - 9.80 \times 4.22 = -31.4 m/s$$

$$|v| = \sqrt{v_{xf}^2 + v_{yf}^2} = \sqrt{17.3^2 + (-31.4)^2} = 35.9 m/s$$