PHYS 1441 – Section 004
Lecture #13

Wednesday, Mar. 10, 2004

Dr. Jaehoon Yu

- Conservation of Mechanical Energy
- Work Done by Non-conservative forces
- Power
- Energy Loss in Automobile
- Linear Momentum
- Linear Momentum Conservation

Today's homework is homework #8, due 1pm, Wednesday, Mar. 24!!

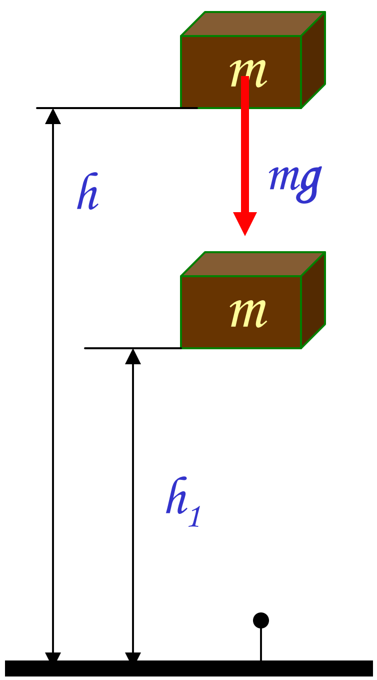
Announcements

- Spring break: Mar. 15 – 19
- Second term exam on Monday, Mar. 29
  - In the class, 1:00 – 2:30pm
  - Sections 5.6 – 8.8
  - Mixture of multiple choices and numeric problems

Conservation of Mechanical Energy

Total mechanical energy is the sum of kinetic and potential energies

$$E \equiv K + U$$



Let's consider a brick of mass *m* at a height *h* from the ground

What is its potential energy?

$$U_g = mgh$$

What happens to the energy as the brick falls to the ground?

$$\Delta U = U_f - U_i$$

The brick gains speed

By how much? $v = gt$

So what? *The brick's kinetic energy increased*

$$K = \frac{1}{2}mv^2 = \frac{1}{2}mg^2t^2$$

And? *The lost potential energy converted to kinetic energy*

$$= mgh$$

What does this mean?

The total mechanical energy of a system remains constant in any isolated system of objects that interacts only through conservative forces:

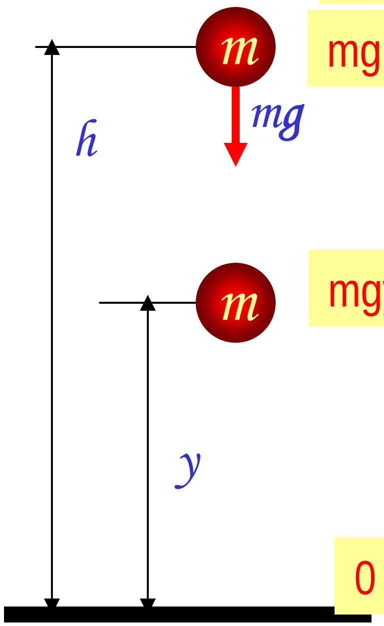
Principle of mechanical energy conservation

$$E_i = E_f$$

$$K_i + \sum U_i = K_f + \sum U_f$$

Example for Mechanical Energy Conservation

A ball of mass m is dropped from a height h above the ground. Neglecting air resistance determine the speed of the ball when it is at a height y above the ground.



PE	KE	
mgh	0	$mv_i^2/2$
mgy	$mv^2/2$	$mv_i^2/2$
0		

Using the principle of mechanical energy conservation

$$K_i + U_i = K_f + U_f \qquad 0 + mgh = \frac{1}{2}mv^2 + mgy$$

$$\frac{1}{2}mv^2 = mg(h-y)$$

$$\therefore v = \sqrt{2g(h-y)}$$

b) Determine the speed of the ball at y if it had initial speed v_i at the time of release at the original height h.

Again using the principle of mechanical energy conservation but with non-zero initial kinetic energy!!!

$$K_i + U_i = K_f + U_f$$

$$\frac{1}{2}mv_i^2 + mgh = \frac{1}{2}mv_f^2 + mgy$$

Reorganize the terms

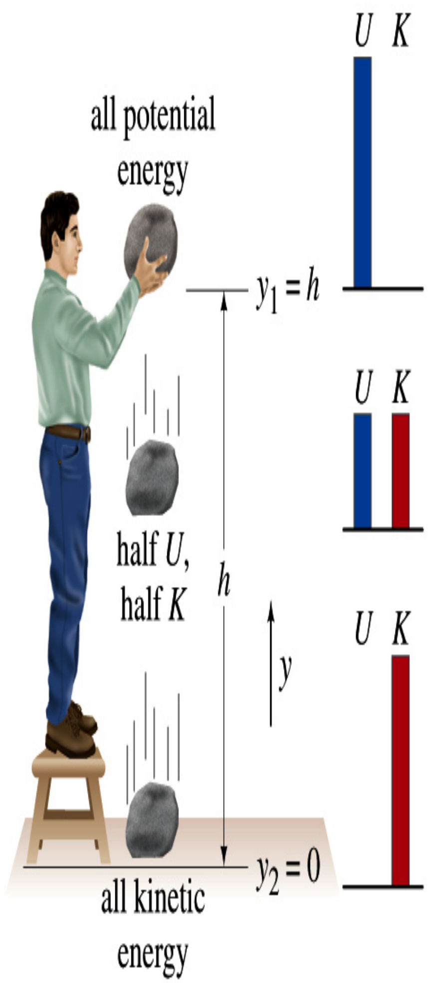
$$\frac{1}{2}m\left(v_f^2 - v_i^2\right) = mg\left(h-y\right)$$

$$\therefore v_f = \sqrt{v_i^2 + 2g(h-y)}$$

This result look very similar to a kinematic expression, doesn't it? Which one is it?

Example 6.8

If the original height of the stone in the figure is y1=h=3.0m, what is the stone's speed when it has fallen 1.0 m above the ground? Ignore air resistance.



At y=3.0m

$$\frac{1}{2}mv_1^2 + mgy_1 = mgh = 3.0mg$$

At y=1.0m

$$\frac{1}{2}mv_2^2 + mgy_2 = \frac{1}{2}mv_2^2 + 1.0mg$$

Since Mechanical Energy is conserved

$$\frac{1}{2}mv_2^2 + 1.0mg = 3.0mg$$

$$\frac{1}{2}mv_2^2 = 2.0mg$$

Cancel m

$$\frac{1}{2}v_2^2 = 2.0g$$

Solve for v

$$v_2 = \sqrt{4.0g} = \sqrt{4.0 \times 9.8} = 6.3m/s$$

Work Done by Non-conservative Forces

Mechanical energy of a system is not conserved when any one of the forces in the system is a non-conservative force.

Two kinds of non-conservative forces:

*Applied forces: Forces that are external to the system. These forces can take away or add energy to the system. So the **mechanical energy of the system is no longer conserved.***

If you were to carry around a ball, the force you apply to the ball is external to the system of ball and the Earth. Therefore, you add kinetic energy to the ball-Earth system.

$$W_{you} + W_g = \Delta K; \quad W_g = -\Delta U$$

$$W_{you} = W_{app} = \Delta K + \Delta U$$

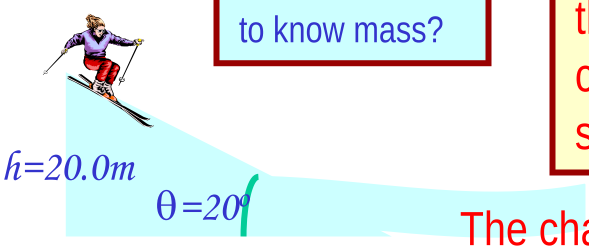
Kinetic Friction: Internal non-conservative force that causes irreversible transformation of energy. The friction force causes the kinetic and potential energy to transfer to internal energy

$$W_{friction} = \Delta K_{friction} = -f_k d$$

$$\Delta E = E_f - E_i = \Delta K + \Delta U = -f_k d$$

Example for Non-Conservative Force

A skier starts from rest at the top of frictionless hill whose vertical height is 20.0m and the inclination angle is 20°. Determine how far the skier can get on the snow at the bottom of the hill with a coefficient of kinetic friction between the ski and the snow is 0.210.



Don't we need to know mass?

Compute the speed at the bottom of the hill, using the mechanical energy conservation on the hill before friction starts working at the bottom

$$ME = mgh \quad = \frac{1}{2}mv^2$$

$$v = \sqrt{2gh}$$

$$v = \sqrt{2\times 9.8\times 20.0} = 19.8m/s$$

The change of kinetic energy is the same as the work done by kinetic friction.

What does this mean in this problem?

$$\Delta K = K_f - K_i = -f_k d$$

Since we are interested in the distance the skier can get to before stopping, the friction must do as much work as the available kinetic energy.

Since $K_f = 0$

$$-K_i = -f_k d; \quad f_k d = K_i$$

$$f_k = \mu_k n \quad = \mu_k mg$$

$$d = \frac{K_i}{\mu_k mg} = \frac{\frac{1}{2}mv^2}{\mu_k mg} = \frac{v^2}{2\mu_k g} = \frac{(19.8)^2}{2\times 0.210\times 9.80} = 95.2m$$

Well, it turns out we don't need to know mass.

What does this mean?

No matter how heavy the skier is he will get as far as anyone else has gotten.

Energy Diagram and the Equilibrium of a System

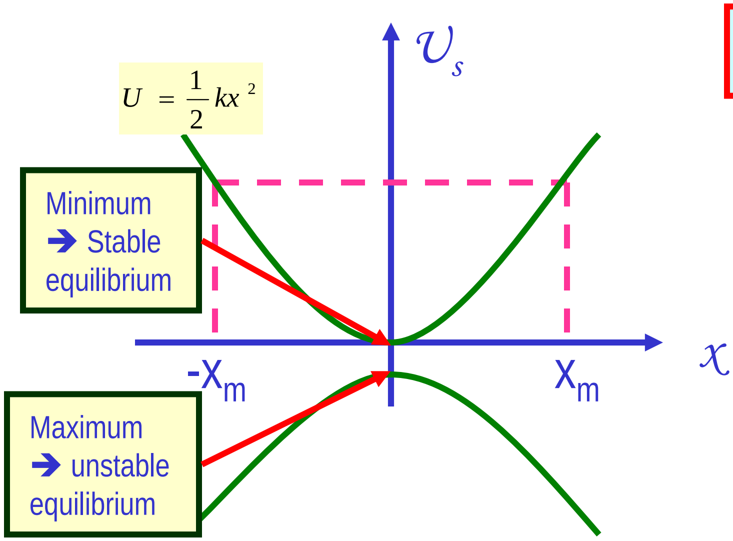
One can draw potential energy as a function of position ➔ Energy Diagram

Let's consider potential energy of a spring-ball system

$$U_s = \frac{1}{2}kx^2$$

What shape would this diagram be?

A Parabola



What does this energy diagram tell you?

1. *Potential energy for this system is the same independent of the sign of the position.*
2. *The force is 0 when the slope of the potential energy curve is 0 at the position.*
3. *x=0 is one of the stable or equilibrium of this system where the potential energy is minimum.*

Position of a stable equilibrium corresponds to points where potential energy is at a minimum.

Position of an unstable equilibrium corresponds to points where potential energy is a maximum.

General Energy Conservation and Mass-Energy Equivalence

General Principle of Energy Conservation

The total energy of an isolated system is conserved as long as all forms of energy are taken into account.

What about friction?

Friction is a non-conservative force and causes mechanical energy to change to other forms of energy.

However, if you add the new form of energy altogether, the system as a whole did not lose any energy, as long as it is self-contained or isolated.

In the grand scale of the universe, no energy can be destroyed or created but just transformed or transferred from one place to another. ***Total energy of universe is constant.***

Principle of Conservation of Mass

In any physical or chemical process, mass is neither created nor destroyed. Mass before a process is identical to the mass after the process.

Einstein's Mass-Energy equality.

$$\boxed{E_R = mc^2}$$

How many joules does your body correspond to?

Power

- Rate at which work is done
  - What is the difference for the same car with two different engines (4 cylinder and 8 cylinder) climbing the same hill? ➔ 8 cylinder car climbs up faster

Is the total amount of work done by the engines different? NO

Then what is different? The rate at which the same amount of work performed is higher for 8 cylinder than 4.

Average power $\boxed{\overline{P} = \frac{\Delta W}{\Delta t}}$

Instantaneous power $P \equiv \lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t} = \lim_{\Delta t \to 0} \left|\sum \vec{F}\right| \left|\frac{\Delta \vec{s}}{\Delta t}\right| \cos\theta = \left|\sum \vec{F}\right| \left|\vec{v}\right| \cos\theta$

Unit? $\boxed{J/s = Watts}$ $\boxed{1HP = 746\,Watts}$

What do power companies sell? $1kWH = 1000Watts \times 3600s = 3.6 \times 10^6 J$

Energy

Energy Loss in Automobile

Automobile uses only at 13% of its fuel to propel the vehicle.

Why?

67% in the engine:

1. *Incomplete burning*
2. *Heat*
3. *Sound*

16% in friction in mechanical parts

4% in operating other crucial parts such as oil and fuel pumps, etc

13% used for balancing energy loss related to moving vehicle, like air resistance and road friction to tire, etc

Two frictional forces involved in moving vehicles

$$m_{car} = 1450kg \quad Weight = mg = 14200N$$

Coefficient of Rolling Friction; μ=0.016

$$\mu n = \mu mg = 227N$$

Air Drag

$$f_a = \frac{1}{2}D\rho A v^2 = \frac{1}{2}\times 0.5\times 1.293\times 2v^2 = 0.647v^2$$

Total Resistance

$$f_t = f_r + f_a$$

Total power to keep speed v=26.8m/s=60mi/h

$$P = f_t v = (691N)\cdot 26.8 = 18.5kW$$

Power to overcome each component of resistance

$$P_r = f_r v = (227)\cdot 26.8 = 6.08kW$$

$$P_a = f_a v = (464.7)\cdot 26.8 = 12.5kW$$

Linear Momentum

The principle of energy conservation can be used to solve problems that are harder to solve just using Newton's laws. It is used to describe motion of an object or a system of objects.

A new concept of linear momentum can also be used to solve physical problems, especially the problems involving collisions of objects.

Linear momentum of an object whose mass is m and is moving at a velocity of **v** *is defined as*

$$\vec{p} = m\vec{v}$$

What can you tell from this definition about momentum?

1. *Momentum is a vector quantity.*
2. *The heavier the object the higher the momentum*
3. *The higher the velocity the higher the momentum*
4. *Its unit is kg.m/s*

What else can use see from the definition? Do you see force?

The change of momentum in a given time interval

$$\frac{\Delta\vec{p}}{\Delta t} = \frac{m\vec{v} - m\vec{v}_0}{\Delta t} = \frac{m\left(\vec{v} - \vec{v}_0\right)}{\Delta t} = m\frac{\Delta\vec{v}}{\Delta t} = m\vec{a} = \sum\vec{F}$$

Linear Momentum and Forces

$$\sum \vec{F} = \frac{\Delta \vec{p}}{\Delta t}$$

What can we learn from this Force-momentum relationship?

- The rate of the change of particle's momentum is the same as the net force exerted on it.
- When net force is 0, the particle's linear momentum is constant as a function of time.
- If a particle is isolated, the particle experiences no net force, therefore its momentum does not change and is conserved.

Something else we can do with this relationship. What do you think it is?

The relationship can be used to study the case where the mass changes as a function of time.

Can you think of a few cases like this?

Motion of a meteorite

Motion of a rocket