PHYS 1441 – Section 004
Lecture #19

Monday, Apr. 12, 2004

Dr. Jaehoon Yu

- Examples for Equilibrium Problems
- Elasticity
- Density and Specific Gravity
- Fluid and Pressure
- Variation of Pressure vs Depth
- Pascal's Principle and Hydraulics

Quiz this Wednesday, Apr. 14, 2004!!

Announcements

- Quiz at 1pm this Wednesday, Apr. 14
  - Covers sections 8.3 – 9.3
- Mid-term grade one-on-one discussion
  - I had only 12 students so far.
  - In my office, RM 242-A, SH
  - During office hours: 2:30 – 3:30 pm, Mondays and Wednesdays
    - Today: Last name starts with A – M
    - Wednesday: Last name starts with N – Z

Similarity Between Linear and Rotational Motions

All physical quantities in linear and rotational motions show striking similarity.

Quantities	Linear	Rotational
Mass	Mass M	Moment of Inertia $I = mr^2$
Length of motion	Distance L	Angle θ (Radian)
Speed	$v = \frac{\Delta r}{\Delta t}$	$\omega = \frac{\Delta \theta}{\Delta t}$
Acceleration	$a = \frac{\Delta v}{\Delta t}$	$\alpha = \frac{\Delta \omega}{\Delta t}$
Force	Force $\vec{F} = m\vec{a}$	Torque $\vec{\tau} = I\vec{\alpha}$
Work	Work $W = Fd\cos\theta$	Work $W = \tau\theta$
Power	$P = Fv\cos\theta$	$P = \tau\omega$
Momentum	$\vec{p} = m\vec{v}$	$\vec{L} = I\vec{\omega}$
Kinetic Energy	Kinetic $K = \frac{1}{2}mv^2$	Rotational $K_R = \frac{1}{2}I\omega^2$

More on Conditions for Equilibrium

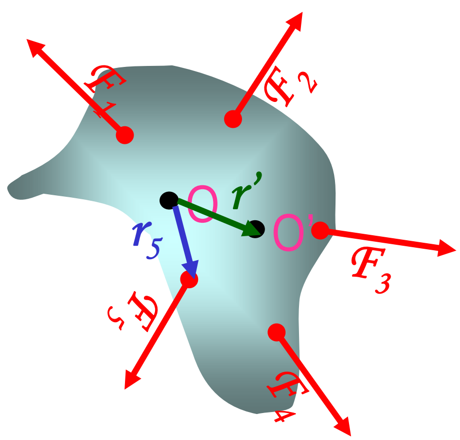
To simplify the problem, we will only deal with forces acting on x-y plane, giving torque only along z-axis. What do you think the conditions for equilibrium be in this case?

The six possible equations from the two vector equations turns to three equations.

$$\sum \vec{F} = 0 \qquad \sum F_x = 0 \qquad \sum F_y = 0$$

$$\sum \vec{\tau} = 0 \qquad \sum \tau_z = 0$$

What happens if there are many forces exerting on the object?



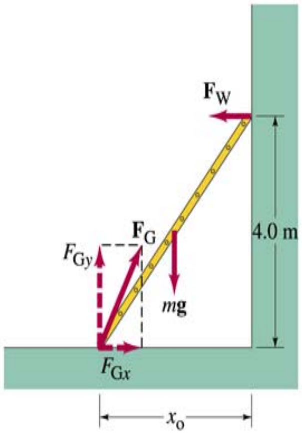
If an object is at its translational static equilibrium, and if the net torque acting on the object is 0 about one axis, the net torque must be 0 about any arbitrary axis.

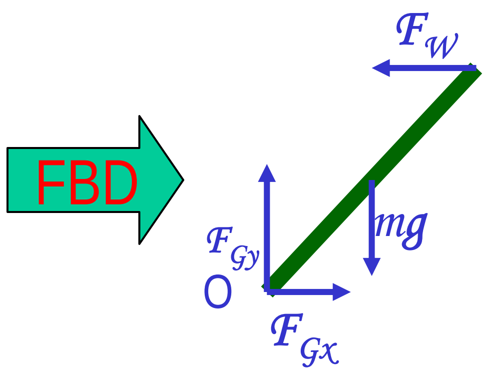
Why is this true?

Because the object is ***not moving***, no matter what the rotational axis is, there should not be a motion. It is simply a matter of mathematical calculation.

Example 9 – 9

A 5.0 m long ladder leans against a wall at a point 4.0m above the ground. The ladder is uniform and has mass 12.0kg. Assuming the wall is frictionless (but ground is not), determine the forces exerted on the ladder by the ground and the wall.





First the translational equilibrium, using components

$$\sum F_x = F_{Gx} - F_W = 0$$

$$\sum F_y = -mg + F_{Gy} = 0$$

Thus, the y component of the force by the ground is

$$F_{Gy} = mg = 12.0 \times 9.8N = 118N$$

The length x_0 is, from Pythagorian theorem

$$x_0 = \sqrt{5.0^2 - 4.0^2} = 3.0m$$

Example 9 – 9 cont'd

From the rotational equilibrium $\sum \tau_O = -mg\, x_0/2 + F_W 4.0 = 0$

Thus the force exerted on the ladder by the wall is

$$F_W = \frac{mg\, x_0/2}{4.0} = \frac{118 \cdot 1.5}{4.0} = 44N$$

Tx component of the force by the ground is

$$\sum F_x = F_{Gx} - F_W = 0$$

Solve for F_{Gx}

$$F_{Gx} = F_W = 44N$$

Thus the force exerted on the ladder by the ground is

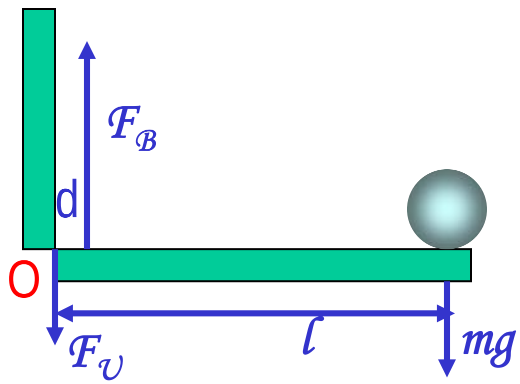
$$F_G = \sqrt{F_{Gx}^2 + F_{Gy}^2} = \sqrt{44^2 + 118^2} \approx 130N$$

The angle between the ladder and the wall is

$$\theta = \tan^{-1}\left(\frac{F_{Gy}}{F_{Gx}}\right) = \tan^{-1}\left(\frac{118}{44}\right) = 70^\circ$$

Example for Mechanical Equilibrium

A person holds a 50.0N sphere in his hand. The forearm is horizontal. The biceps muscle is attached 3.00 cm from the joint, and the sphere is 35.0cm from the joint. Find the upward force exerted by the biceps on the forearm and the downward force exerted by the upper arm on the forearm and acting at the joint. Neglect the weight of forearm.



Since the system is in equilibrium, from the translational equilibrium condition

$$\sum F_x = 0$$

$$\sum F_y = F_B - F_U - mg = 0$$

From the rotational equilibrium condition

$$\sum \tau = F_U \cdot 0 + F_B \cdot d - mg \cdot l = 0$$

Thus, the force exerted by the biceps muscle is

$$F_B \cdot d = mg \cdot l$$

$$F_B = \frac{mg \cdot l}{d} = \frac{50.0 \times 35.0}{3.00} = 583N$$

Force exerted by the upper arm is

$$F_U = F_B - mg = 583 - 50.0 = 533\,N$$

How do we solve equilibrium problems?

1. Identify all the forces and their directions and locations
2. Draw a free-body diagram with forces indicated on it
3. Write down vector force equation for each x and y component with proper signs
4. Select a rotational axis for torque calculations ➔ Selecting the axis such that the torque of one of the unknown forces become 0.
5. Write down torque equation with proper signs
6. Solve the equations for unknown quantities

Elastic Properties of Solids

We have been assuming that the objects do not change their shapes when external forces are exerting on it.   It this realistic?

No. In reality, the objects get deformed as external forces act on it, though the internal forces resist the deformation as it takes place.

Deformation of solids can be understood in terms of Stress and Strain

Stress: A quantity proportional to the force causing deformation.
Strain: Measure of degree of deformation

It is empirically known that for small stresses, strain is proportional to stress

The constants of proportionality are called Elastic Modulus

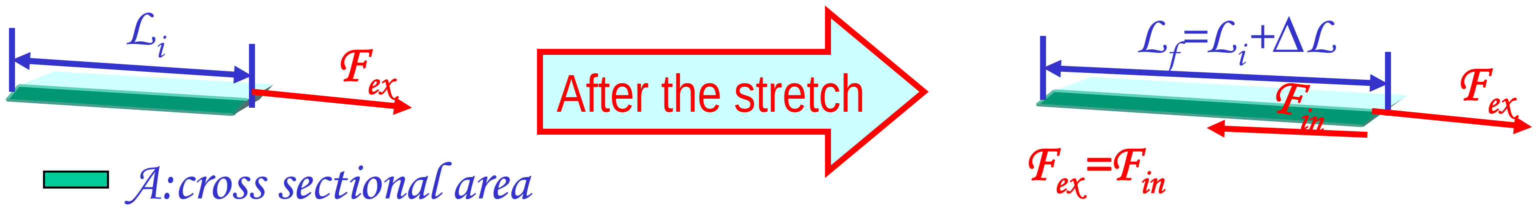
$$\text{Elastic Modulus} \equiv \frac{\text{stress}}{\text{strain}}$$

Three types of Elastic Modulus

1. **Young's modulus**: Measure of the elasticity in length
2. **Shear modulus**: Measure of the elasticity in plane
3. **Bulk modulus**: Measure of the elasticity in volume

Young's Modulus

Let's consider a long bar with cross sectional area A and initial length $\mathcal{L}_i$.



Tensile stress $$\text{Tensile Stress} \equiv \frac{F_{ex}}{A}$$

Tensile strain $$\text{Tensile Strain} \equiv \frac{\Delta L}{L_i}$$

Young's Modulus is defined as $$Y \equiv \frac{\text{Tensile Stress}}{\text{Tensile Strain}} = \frac{F_{ex}/A}{\Delta L/L_i}$$

Used to characterize a rod or wire stressed under tension or compression

What is the unit of Young's Modulus?

Force per unit area

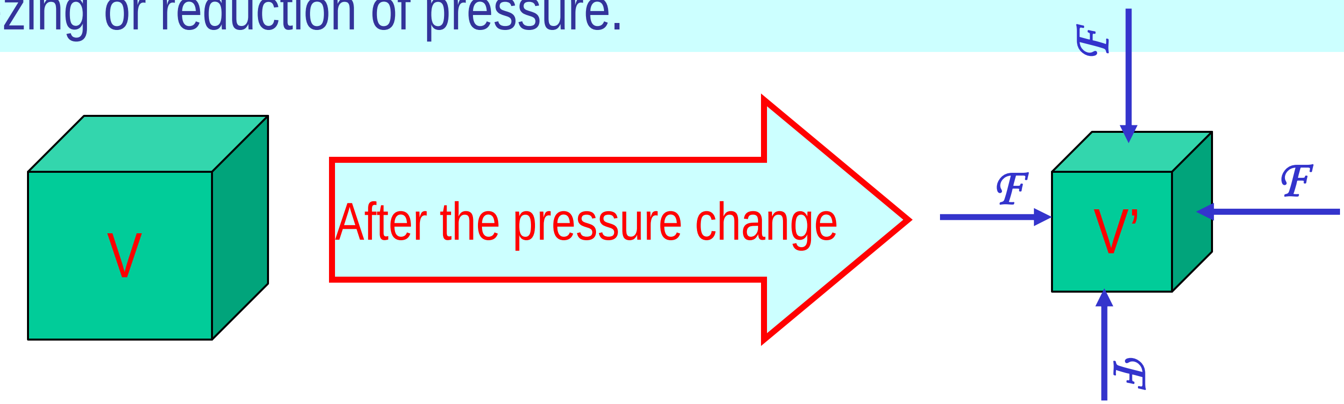
Experimental Observations

1. For fixed external force, the change in length is proportional to the original length
2. The necessary force to produce a given strain is proportional to the cross sectional area

Elastic limit: Maximum stress that can be applied to the substance before it becomes permanently deformed

Bulk Modulus

Bulk Modulus characterizes the response of a substance to uniform squeezing or reduction of pressure.



Volume stress =pressure

$$\text{Pressure} \equiv \frac{\text{Normal Force}}{\text{Surface Area the force applies}} = \frac{F}{A}$$

If the pressure on an object changes by $\Delta P = \Delta F/A$, the object will undergo a volume change ΔV.

Bulk Modulus is defined as

$$B \equiv \frac{\text{Volume Stress}}{\text{Volume Strain}} = -\frac{\Delta F / A}{\Delta V / V_i} = -\frac{\Delta P}{\Delta V / V_i}$$

Because the change of volume is reverse to change of pressure.

Compressibility is the reciprocal of Bulk Modulus

Example for Solid's Elastic Property

A solid brass sphere is initially under normal atmospheric pressure of $1.0\text{x}10^5 N/m^2$. The sphere is lowered into the ocean to a depth at which the pressures is $2.0\text{x}10^7 N/m^2$. The volume of the sphere in air is $0.5m^3$. By how much its volume change once the sphere is submerged?

Since bulk modulus is $$\mathrm{B} = -\frac{\Delta P}{\Delta V / V_i}$$

The amount of volume change is $$\Delta V = -\frac{\Delta P V_i}{\mathrm{B}}$$

From table 12.1, bulk modulus of brass is $6.1\text{x}10^{10}\ N/m^2$

The pressure change ΔP is $$\Delta P = P_f - P_i = 2.0\times10^7 - 1.0\times10^5 \approx 2.0\times10^7$$

Therefore the resulting volume change ΔV is $$\Delta V = V_f - V_i = -\frac{2.0\times10^7\times0.5}{6.1\times10^{10}} = -1.6\times10^{-4}m^3$$

The volume has decreased.

Density and Specific Gravity

Density, ρ (rho) , of an object is defined as mass per unit volume

$$\rho \equiv \frac{M}{V}$$

Unit? kg/m^3

Dimension? $[ML^{-3}]$

Specific Gravity of a substance is defined as the ratio of the density of the substance to that of water at 4.0 °C (ρ_{H2O}=1.00g/cm^3).

$$SG \equiv \frac{\rho_{\text{substance}}}{\rho_{H_2O}}$$

Unit? None

Dimension? None

What do you think would happen of a substance in the water dependent on SG?

$SG > 1$ Sink in the water

$SG < 1$ Float on the surface

Fluid and Pressure

What are the three states of matter?

Solid, Liquid, and Gas

How do you distinguish them?

By the time it takes for a particular substance to change its shape in reaction to external forces.

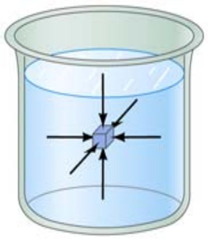
What is a fluid?

A collection of molecules that are randomly arranged and loosely bound by forces between them or by the external container.

We will first learn about mechanics of fluid at rest, *fluid statics*.

In what way do you think fluid exerts stress on the object submerged in it?

Fluid cannot exert shearing or tensile stress. Thus, the only force the fluid exerts on an object immersed in it is the forces perpendicular to the surfaces of the object.



This force by the fluid on an object usually is expressed in the form of the force on a unit area at the given depth, the pressure, defined as

$$P \equiv \frac{F}{A}$$

Expression of pressure for an infinitesimal area dA by the force dF is

$$P = \frac{dF}{dA}$$

Note that pressure is a scalar quantity because it's the magnitude of the force on a surface area A.

What is the unit and dimension of pressure?

Unit: N/m^2

Dim.: $[M][L^{-1}][T^{-2}]$

Special SI unit for pressure is Pascal

$$1Pa \equiv 1N/m^2$$

Example for Pressure

The mattress of a water bed is 2.00m long by 2.00m wide and 30.0cm deep. a) Find the weight of the water in the mattress.

The volume density of water at the normal condition (0^oC and 1 atm) is $1000kg/m^3$. So the total mass of the water in the mattress is

$$m = \rho_W V_M = 1000 \times 2.00 \times 2.00 \times 0.300 = 1.20 \times 10^3 kg$$

Therefore the weight of the water in the mattress is

$$W = mg = 1.20 \times 10^3 \times 9.8 = 1.18 \times 10^4 N$$

b) Find the pressure exerted by the water on the floor when the bed rests in its normal position, assuming the entire lower surface of the mattress makes contact with the floor.

Since the surface area of the mattress is $4.00\ m^2$, the pressure exerted on the floor is

$$P = \frac{F}{A} = \frac{mg}{A} = \frac{1.18 \times 10^4}{4.00} = 2.95 \times 10^3$$