

# PHYS 1441 – Section 004

## Lecture #20

*Wednesday, Apr. 14, 2004*

*Dr. Jaehoon Yu*

- Variation of Pressure vs Depth
- Pascal's Principle and Hydraulics
- Absolute and Relative Pressure
- Buoyant Force and Archimedes' Principle
- Flow Rate and Equation of Continuity
- Bernoulli's Equation

Today's homework is #11, due at noon, next Wednesday, Apr. 21.



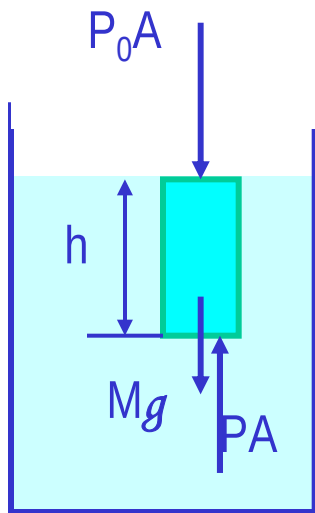
# Announcements

- Mid-term grade one-on-one discussion
  - In my office, RM 242-A, SH
  - Today is the last day for this...
    - Today: All remaining students who haven't met me in person yet
- Wrapping up the semester
  - There will be two more homework.
  - One more quiz on the week of Apr. 26
  - Some more opportunities for strong participation for extra credit



# Variation of Pressure and Depth

Water pressure increases as a function of depth, and the air pressure decreases as a function of altitude. Why?



It seems that the pressure has a lot to do with the total mass of the fluid above the object that puts weight on the object.

Let's imagine a liquid contained in a cylinder with height  $h$  and cross sectional area  $A$  immersed in a fluid of density  $\rho$  at rest, as shown in the figure, and the system is in its equilibrium.

If the liquid in the cylinder is the same substance as the fluid, the mass of the liquid in the cylinder is  $M = \rho V = \rho Ah$

Since the system is in its equilibrium

$$PA - P_0A - Mg = PA - P_0A - \rho Ahg = 0$$

Therefore, we obtain  $P = P_0 + \rho gh$

Atmospheric pressure  $P_0$  is

$$1.00 \text{ atm} = 1.013 \times 10^5 \text{ Pa}$$

The pressure at the depth  $h$  below the surface of a fluid open to the atmosphere is greater than atmospheric pressure by  $\rho gh$ .

What else can you learn from this?

# Pascal's Principle and Hydraulics

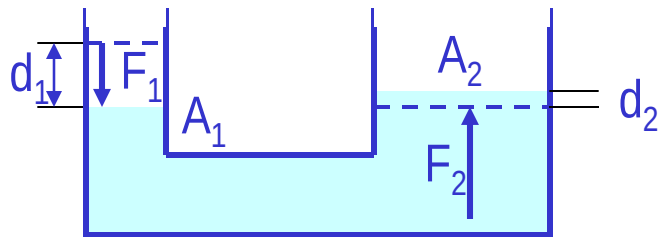
A change in the pressure applied to a fluid is transmitted undiminished to every point of the fluid and to the walls of the container.

$$P = P_0 + \rho gh$$

What happens if  $P_0$  is changed?

The resultant pressure  $P$  at any given depth  $h$  increases as much as the change in  $P_0$ .

This is the principle behind hydraulic pressure. How?



Since the pressure change caused by the force  $F_1$  applied on to the area  $A_1$  is transmitted to the  $F_2$  on an area  $A_2$ .

$$P = \frac{F_1}{A_1} = \frac{F_2}{A_2}$$

Therefore, the resultant force  $F_2$  is

$$F_2 = \frac{A_2}{A_1} F_1$$

In other words, the force gets multiplied by the ratio of the areas  $A_2/A_1$  and is transmitted to the force  $F_2$  on the surface.

This seems to violate some kind of conservation law, doesn't it?

No, the actual displaced volume of the fluid is the same. And the work done by the forces are still the same.

$$F_2 = \frac{d_1}{d_2} F_1$$



# Example for Pascal's Principle

In a car lift used in a service station, compressed air exerts a force on a small piston that has a circular cross section and a radius of 5.00cm. This pressure is transmitted by a liquid to a piston that has a radius of 15.0cm. What force must the compressed air exert to lift a car weighing 13,300N? What air pressure produces this force?

Using the Pascal's principle, one can deduce the relationship between the forces, the force exerted by the compressed air is

$$F_1 = \frac{A_2}{A_1} F_2 = \frac{\pi (0.15)^2}{\pi (0.05)^2} \times 1.33 \times 10^4 = 1.48 \times 10^3 \text{ N}$$

Therefore the necessary pressure of the compressed air is

$$P = \frac{F_1}{A_1} = \frac{1.48 \times 10^3}{\pi (0.05)^2} = 1.88 \times 10^5 \text{ Pa}$$



# Example for Pascal's Principle

Estimate the force exerted on your eardrum due to the water above when you are swimming at the bottom of the pool with a depth 5.0 m.

We first need to find out the pressure difference that is being exerted on the eardrum. Then estimate the area of the eardrum to find out the force exerted on the eardrum.

Since the outward pressure in the middle of the eardrum is the same as normal air pressure

$$P - P_0 = \rho_w gh = 1000 \times 9.8 \times 5.0 = 4.9 \times 10^4 \text{ Pa}$$

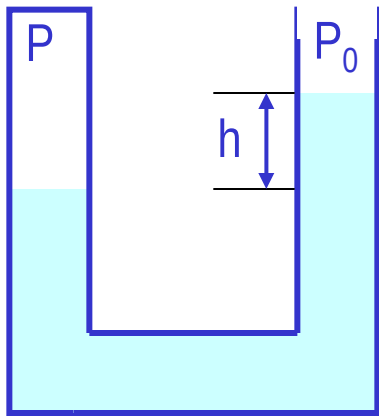
Estimating the surface area of the eardrum at  $1.0\text{cm}^2 = 1.0 \times 10^{-4} \text{ m}^2$ , we obtain

$$F = (P - P_0)A \approx 4.9 \times 10^4 \times 1.0 \times 10^{-4} \approx 4.9 \text{ N}$$



# Absolute and Relative Pressure

How can one measure pressure?



One can measure pressure using an open-tube manometer, where one end is connected to the system with unknown pressure  $P$  and the other open to air with pressure  $P_0$ .

The measured pressure of the system is  $P = P_0 + \rho gh$

This is called the absolute pressure, because it is the actual value of the system's pressure.

In many cases we measure pressure difference with respect to atmospheric pressure due to changes in  $P_0$  depending on the environment. This is called gauge or relative pressure.

$$P_G = P - P_0 = \rho gh$$

The common barometer which consists of a mercury column with one end closed at vacuum and the other open to the atmosphere was invented by Evangelista Torricelli.

Since the closed end is at vacuum, it does not exert any force. 1 atm is

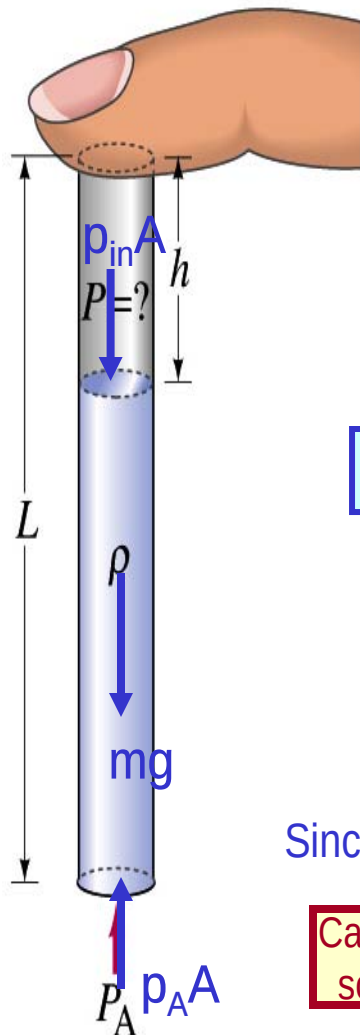
$$\begin{aligned} P_0 &= \rho gh = (13.595 \times 10^3 \text{ kg} / \text{m}^3)(9.80665 \text{ m} / \text{s}^2)(0.7600 \text{ m}) \\ &= 1.013 \times 10^5 \text{ Pa} = 1 \text{ atm} \end{aligned}$$

If one measures the tire pressure with a gauge at 220kPa the actual pressure is 101kPa+220kPa=303kPa.



# Finger Holds Water in Straw

You insert a straw of length  $L$  into a tall glass of your favorite beverage. You place your finger over the top of the straw so that no air can get in or out, and then lift the straw from the liquid. You find that the straw strains the liquid such that the distance from the bottom of your finger to the top of the liquid is  $h$ . Does the air in the space between your finger and the top of the liquid have a pressure  $P$  that is (a) greater than, (b) equal to, or (c) less than, the atmospheric pressure  $P_A$  outside the straw? **Less**



What are the forces in this problem?

Gravitational force on the mass of the liquid  $F_g = mg = \rho A(L-h)g$

Force exerted on the top surface of the liquid by inside air pressure  $F_{in} = p_{in}A$

Force exerted on the bottom surface of the liquid by outside air  $F_{out} = -p_A A$

Since it is at equilibrium  $F_{out} + F_g + F_{in} = 0 \Rightarrow p_A A + \rho g(L-h)A + p_{in}A = 0$

Cancel A and solve for  $p_{in}$

$$p_{in} = p_A - \rho g(L-h)$$

So  $p_{in}$  is less than  $P_A$  by  $\rho gh$ .



# Buoyant Forces and Archimedes' Principle

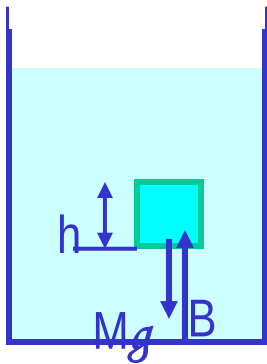
Why is it so hard to put a beach ball under water while a piece of small steel sinks in the water?

The water exerts force on an object immersed in the water.  
This force is called Buoyant force.

How does the  
Buoyant force work?

The magnitude of the buoyant force always equals the weight of the fluid in the volume displaced by the submerged object.

This is called, Archimedes' principle. What does this mean?



Let's consider a cube whose height is  $h$  and is filled with fluid and at its equilibrium. Then the weight  $Mg$  is balanced by the buoyant force  $B$ .

$$B = F_g = Mg$$

And the pressure at the bottom of the cube is larger than the top by  $\rho gh$ .

Therefore,  $\Delta P = B / A = \rho gh$

$$B = \Delta P A = \rho gh A = \rho V g$$

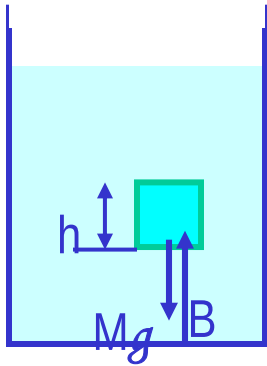
$$B = F_g = \rho V g = Mg$$

Where  $Mg$  is the weight of the fluid.

# More Archimedes' Principle

Let's consider buoyant forces in two special cases.

Case 1: Totally submerged object Let's consider an object of mass  $M$ , with density  $\rho_0$ , is immersed in the fluid with density  $\rho_f$ .



The magnitude of the buoyant force is  $B = \rho_f Vg$

The weight of the object is  $F_g = Mg = \rho_0 Vg$

Therefore total force of the system is  $F = B - F_g = (\rho_f - \rho_0) Vg$

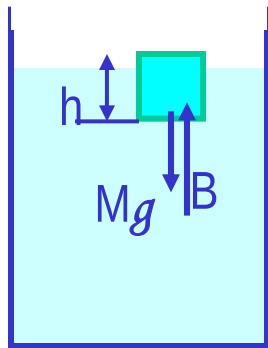
What does this tell you?

The total force applies to different directions, depending on the difference of the density between the object and the fluid.

1. If the density of the object is smaller than the density of the fluid, the buoyant force will push the object up to the surface.
2. If the density of the object is larger than the fluid's, the object will sink to the bottom of the fluid.

# More Archimedes' Principle

## Case 2: Floating object



Let's consider an object of mass  $M$ , with density  $\rho_0$ , is in static equilibrium floating on the surface of the fluid with density  $\rho_f$ , and the volume submerged in the fluid is  $V_f$

The magnitude of the buoyant force is  $B = \rho_f V_f g$

The weight of the object is  $F_g = Mg = \rho_0 V_0 g$

Therefore total force of the system is  $F = B - F_g = \rho_f V_f g - \rho_0 V_0 g = 0$

Since the system is in static equilibrium  $\rho_f V_f g = \rho_0 V_0 g$

$$\frac{\rho_0}{\rho_f} = \frac{V_f}{V_0}$$

What does this tell you?

Since the object is floating its density is always smaller than that of the fluid.

The ratio of the densities between the fluid and the object determines the submerged volume under the surface.

# Example for Bernoulli's Equation

Water circulates throughout a house in a hot-water heating system. If the water is pumped at a speed of 0.5m/s through a 4.0cm diameter pipe in the basement under a pressure of 3.0atm, what will be the flow speed and pressure in a 2.6cm diameter pipe on the second 5.0m above? Assume the pipes do not divide into branches.

Using the equation of continuity, flow speed on the second floor is

$$v_2 = \frac{A_1 v_1}{A_2} = \frac{\pi r_1^2 v_1}{\pi r_2^2} = 0.5 \times \left( \frac{0.020}{0.013} \right)^2 = 1.2 \text{ m / s}$$

Using Bernoulli's equation, the pressure in the pipe on the second floor is

$$\begin{aligned} P_2 &= P_1 + \frac{1}{2} \rho (v_1^2 - v_2^2) + \rho g (y_1 - y_2) \\ &= 3.0 \times 10^5 + \frac{1}{2} 1 \times 10^3 (0.5^2 - 1.2^2) + 1 \times 10^3 \times 9.8 \times (-5) \\ &= 2.5 \times 10^5 \text{ N / m}^2 \end{aligned}$$

