PHYS 1441 – Section 004
Lecture #22

Monday, Apr. 26, 2004
Dr. Jaehoon Yu

- Simple Harmonic Motion
- Simple Block Spring System
- Energy of a Simple Harmonic Oscillator
- Pendulum

Quiz this Wednesday, Apr. 28!

Announcements

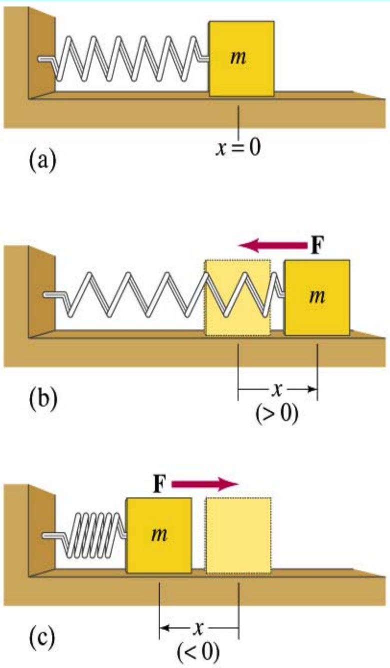
- Instructor Evaluation Today
- Next quiz on Wednesday, Apr. 28
  – Covers: ch. 10.4 – ch.11.2, including all that are covered in the lecture

Vibration or Oscillation

What are the things that vibrate/oscillate?

- Tuning fork
- A pendulum
- A car going over a bump
- Building and bridges
- The spider web with a prey

So what is a vibration or oscillation? A periodic motion that repeats over the same path.



A simplest case is a block attached at the end of a coil spring.

When a spring is stretched from its equilibrium position by a length x, the force acting on the mass is

$$F = -kx$$

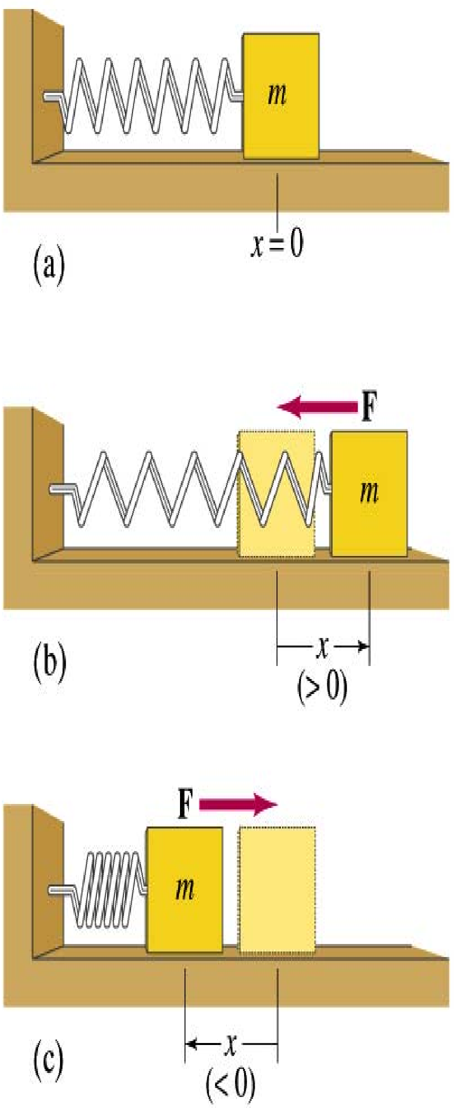
The sign is negative, because the force resists against the change of length, directed toward the equilibrium position.

Acceleration is proportional to displacement from the equilibrium

Acceleration is opposite direction to displacement

This system is doing a simple harmonic motion (SHM).

Vibration or Oscillation Properties



The maximum displacement from the equilibrium is

Amplitude

One cycle of the oscillation

The complete to-and-fro motion from an initial point

Period of the motion, T

The time it takes to complete one full cycle

Unit? s

Frequency of the motion, f

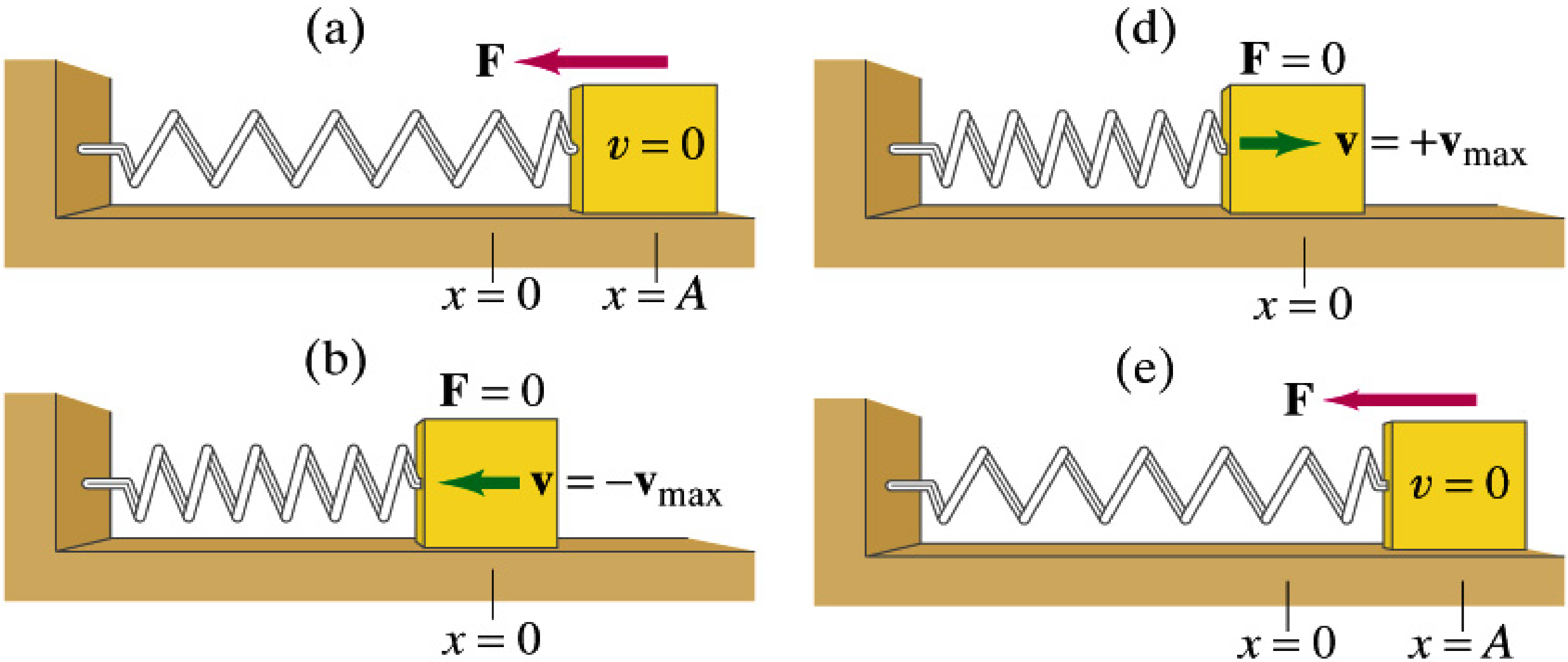
The number of complete cycles per second

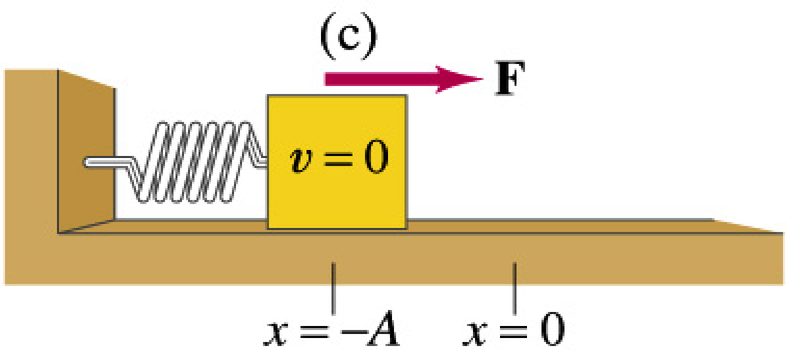
Unit? s^{-1}

Relationship between period and frequency?

$$f = \frac{1}{T} \quad \text{or} \quad T = \frac{1}{f}$$

Vibration or Oscillation Properties





Amplitude? A

- When is the force greatest?
- When is the velocity greatest?
- When is the acceleration greatest?
- When is the potential energy greatest?
- When is the kinetic energy greatest?

Example 11-1

Car springs. When a family of four people with a total mass of 200kg step into their 1200kg car, the car's springs compress 3.0cm. (a) What is the spring constant of the car's spring, assuming they act as a single spring? (b) How far will the car lower if loaded with 300kg?

(a) What is the force on the spring? $F = mg = 200 \cdot 9.8 = 1960N$

From Hooke's law $F = -kx = -k \cdot 0.03 = -mg = 1960N$

$$k = \frac{F_g}{x} = \frac{mg}{x} = \frac{1960}{0.03} = 6.5 \times 10^4 N/m$$

(b) Now that we know the spring constant, we can solve for x in the force equation

$$F = -kx = -mg = -300 \cdot 9.8$$

$$x = \frac{mg}{k} = \frac{300 \cdot 9.8}{6.5 \times 10^4} = 4.5 \times 10^{-2} m$$

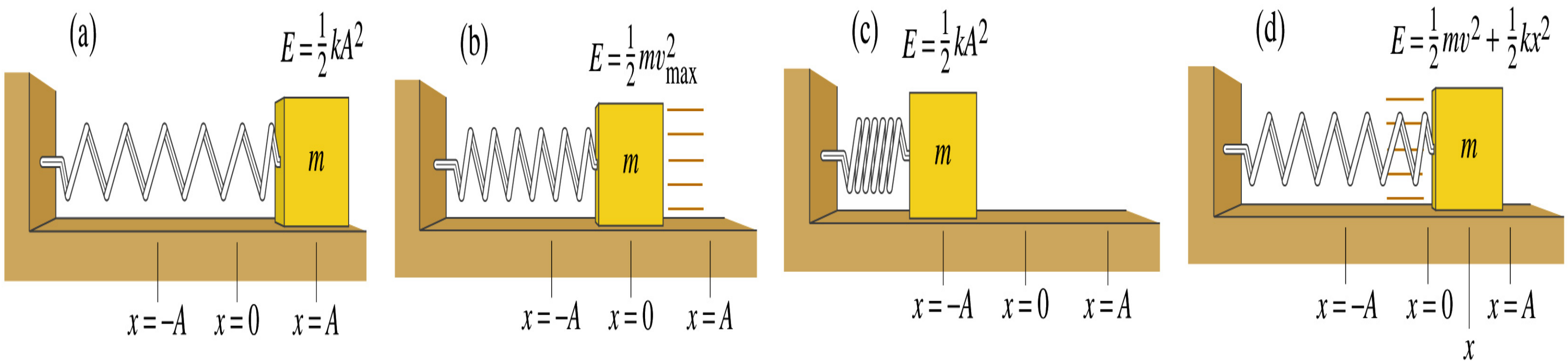
Energy of the Simple Harmonic Oscillator

How do you think the mechanical energy of the harmonic oscillator look without friction?

Kinetic energy of a harmonic oscillator is $$KE = \frac{1}{2}mv^2$$

The elastic potential energy stored in the spring $$PE = \frac{1}{2}kx^2$$

Therefore the total mechanical energy of the harmonic oscillator is $$E = KE + PE = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$



Total mechanical energy of a simple harmonic oscillator is proportional to the square of the amplitude.

Energy of the Simple Harmonic Oscillator cont'd

Maximum KE is when PE=0

$$KE_{max} = \frac{1}{2}mv_{max}^2 = \frac{1}{2}kA^2$$

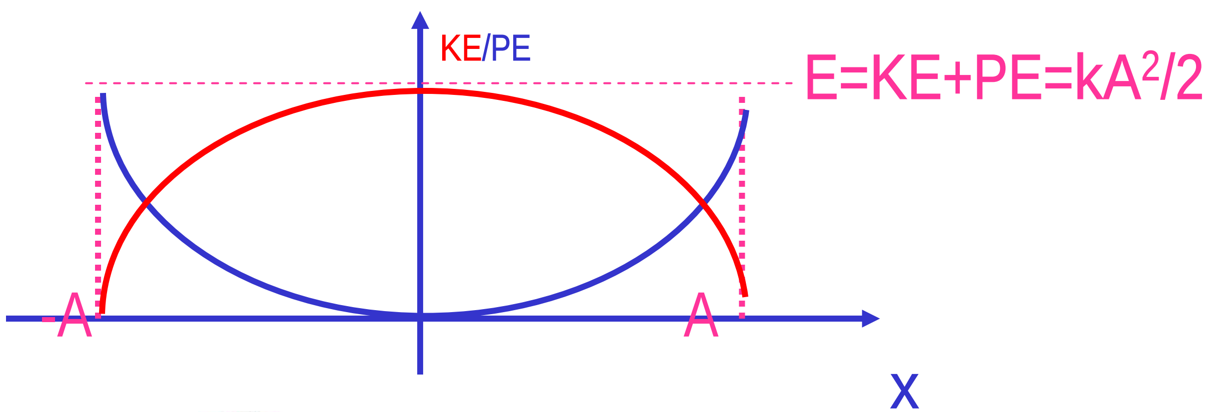
Maximum speed

$$v_{max} = \sqrt{\frac{k}{m}}A$$

The speed at any given point of the oscillation

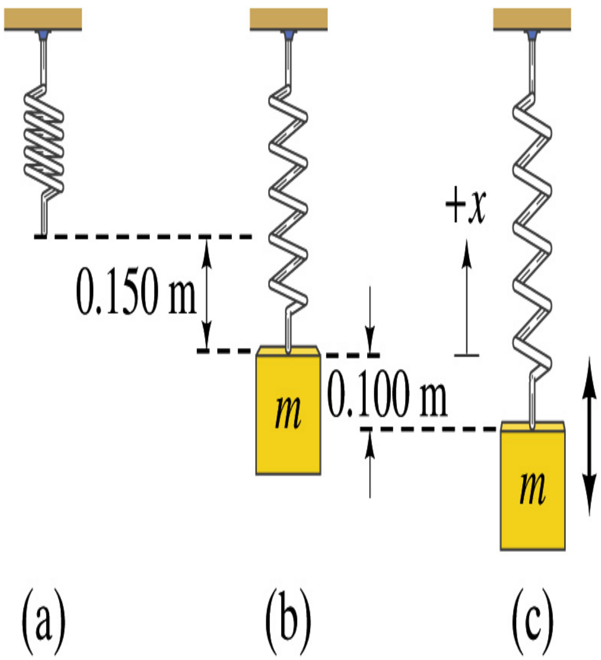
$$E = KE + PE = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$$

$$v = +\sqrt{k/m\left(A^2 - x^2\right)} = \pm v_{max}\sqrt{1-\left(\frac{x}{A}\right)^2}$$



Example 11-3

Spring calculations. A spring stretches 0.150m when a 0.300-kg mass is hung from it. The spring is then stretched an additional 0.100m from this equilibrium position and released.



(a) Determine the spring constant.

From Hooke's law

$$F = -kx = -mg = -0.300 \cdot 9.8N$$

$$k = \frac{mg}{x} = \frac{0.300 \cdot 9.8}{0.150} = 19.6N/m$$

(b) Determine the amplitude of the oscillation.

Since the spring was stretched 0.100m from equilibrium, and is given no initial speed, the amplitude is the same as the additional stretch.

$$A = 0.100m$$

Example cont'd

(c) Determine the maximum velocity v_{max}.

$$v_{max} = \sqrt{\frac{k}{m}}A = \sqrt{\frac{19.6}{0.300}}0.100 = 0.808m/s$$

(d) Determine the magnitude of velocity, v, when the mass is 0.050m from equilibrium.

$$v = \left| v_{max}\sqrt{1-\left(\frac{x}{A}\right)^2} \right| = \left| 0.808\sqrt{1-\left(\frac{0.050}{0.100}\right)^2} \right| = 0.700m/s$$

(d) Determine the magnitude of the maximum acceleration of the mass.

Maximum acceleration is at the point where the mass is stopped to return.

$$F = ma = kA$$

Solve for a

$$a = \frac{kA}{m} = \frac{19.6 \cdot 0.100}{0.300} = 6.53m/s^2$$

Example for Energy of Simple Harmonic Oscillator

A 0.500kg cube connected to a light spring for which the force constant is 20.0 N/m oscillates on a horizontal, frictionless track. a) Calculate the total energy of the system and the maximum speed of the cube if the amplitude of the motion is 3.00 cm.

From the problem statement, A and k are

$$k = 20.0 N/m$$
$$A = 3.00cm = 0.03m$$

The total energy of the cube is

$$E = KE + PE = \frac{1}{2}kA^2 = \frac{1}{2}(20.0)\times(0.03)^2 = 9.00\times10^{-3} J$$

Maximum speed occurs when kinetic energy is the same as the total energy

$$KE_{\max} = \frac{1}{2}mv_{\max}^2 = E = \frac{1}{2}kA^2$$

$$v_{\max} = A\sqrt{\frac{k}{m}} = 0.03\sqrt{\frac{20.0}{0.500}} = 0.190m/s$$

Example for Energy of Simple Harmonic Oscillator

b) What is the velocity of the cube when the displacement is 2.00 cm.

velocity at any given displacement is

$$v = \sqrt{k/m\left(A^2 - x^2\right)}$$

$$= \sqrt{20.0 \cdot \left(0.03^2 - 0.02^2\right)/0.500} = 0.141 m/s$$

c) Compute the kinetic and potential energies of the system when the displacement is 2.00 cm.

Kinetic energy, KE

$$KE = \frac{1}{2}mv^2 \quad = \frac{1}{2}0.500 \times (0.141)^2 = 4.97 \times 10^{-3}\,J$$

Potential energy, PE

$$PE = \frac{1}{2}kx^2 \quad = \frac{1}{2}20.0 \times (0.02)^2 = 4.00 \times 10^{-3}\,J$$