PHYS 1441 – Section 004
Lecture #23

Wednesday, Apr. 28, 2004

Dr. Jaehoon Yu

- Period and Sinusoidal Behavior of SHM
- Pendulum
- Damped Oscillation
- Forced Vibrations, Resonance
- Waves

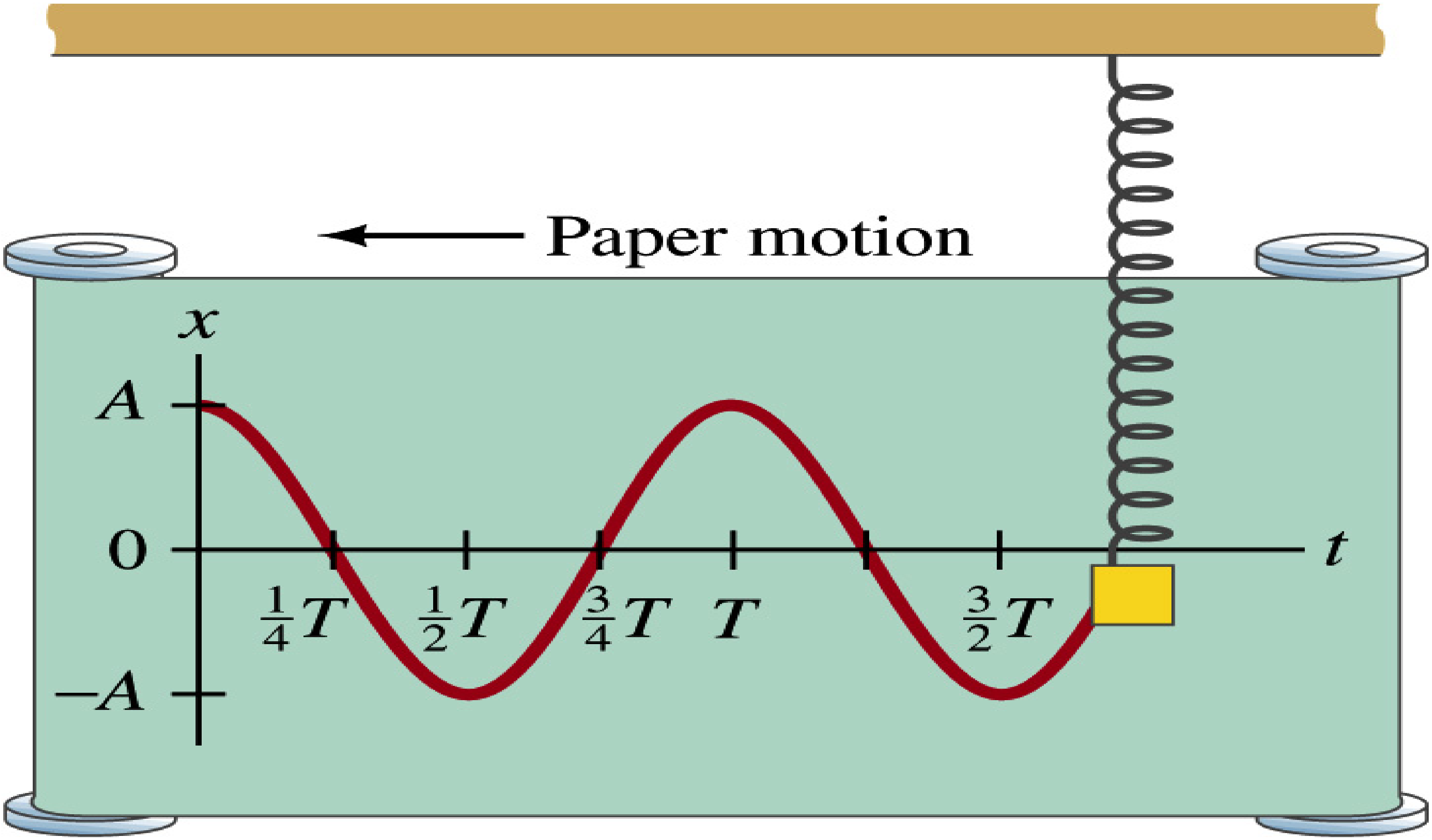
Today's homework is #13 and is due 1pm, next Wednesday !

Final Exam Monday, May. 10!

Announcements

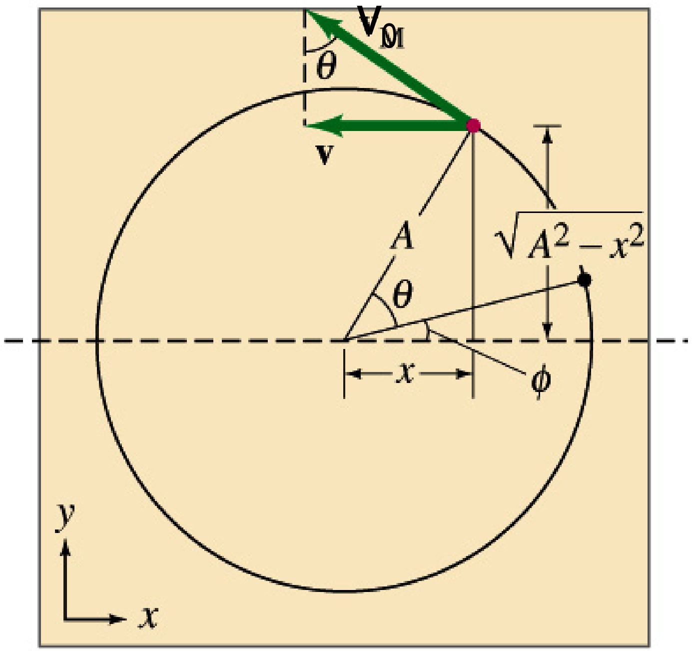
- Final exam Monday, May 10
  - Time: 11:00am – 12:30pm in SH101
  - Chapter 8 – whatever we cover next Monday
  - Mixture of multiple choices and numeric problems
  - Will give you exercise test problems next Monday
- Review next Wednesday, May 5.

Sinusoidal Behavior of SHM



The Period and Sinusoidal Nature of SHM

Consider an object moving on a circle with a constant angular speed ω



$$\sin\theta = \frac{v}{v_0} = \frac{\sqrt{A^2-x^2}}{A} = \sqrt{1-\left(\frac{x}{A}\right)^2} \Rightarrow v = v_0\sqrt{1-\left(\frac{x}{A}\right)^2}$$

Since it takes T to complete one full circular motion

$$v_0 = \frac{2\pi A}{T} = 2\pi A f \Rightarrow T = \frac{2\pi A}{v_0}$$

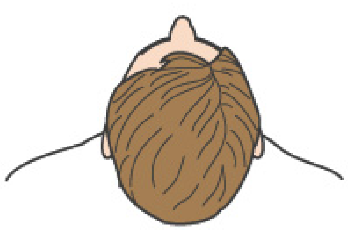
From an energy relationship in a spring SHM

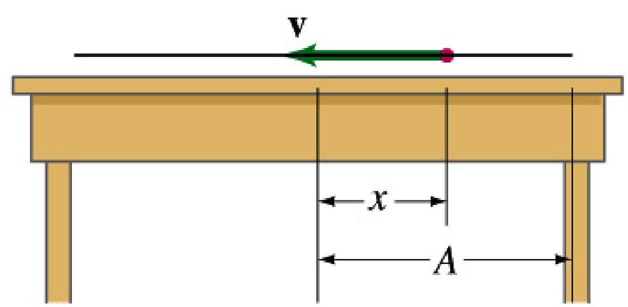
$$\frac{1}{2}mv_0^2 = \frac{1}{2}kA^2 \Rightarrow v_0 = \sqrt{\frac{k}{m}}A$$

Thus, T is

$$T = 2\pi\sqrt{\frac{m}{k}} \Rightarrow f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

Natural Frequency





If you look at it from the side, it looks as though it is doing a SHM

Example 11-5

Car springs. When a family of four people with a total mass of 200kg step into their 1200kg car, the car's springs compress 3.0cm. The spring constant of the spring is 6.5×10^4N/m. What is the frequency of the car after hitting the bump? Assume that the shock absorber is poor, so the car really oscillates up and down.

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{1400}{6.5\times 10^4}} = 0.92s$$

$$f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{6.5\times 10^4}{1400}} = 1.09Hz$$

Example 11-6

Spider Web. A small insect of mass 0.30 g is caught in a spider web of negligible mass. The web vibrates predominantly with a frequency of 15Hz. (a) Estimate the value of the spring constant k for the web.

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = 15Hz$$

Solve for k

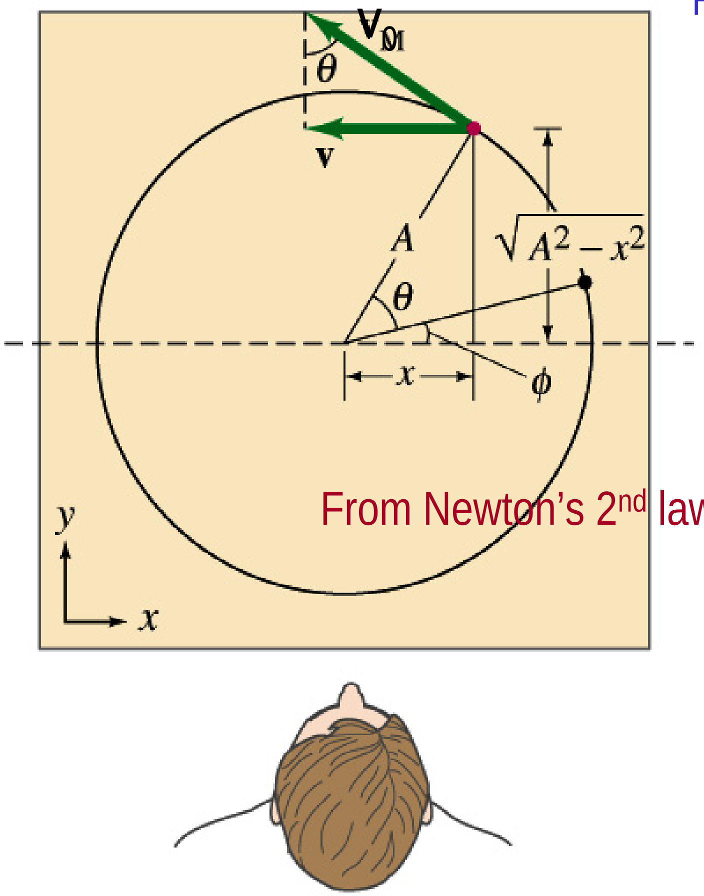
$$k = 4\pi^2 mf^2 = 4\pi^2 \cdot 3\times10^{-4} \cdot (15)^2 = 2.7 N/m$$

(b) At what frequency would you expect the web to vibrate if an insect of mass 0.10g were trapped?

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{2.7}{1\times10^{-4}}} = 26Hz$$

The SHM Equation of Motion

The object is moving on a circle with a constant angular speed ω



(a)

From Newton's 2nd law

How is x, its position at any given time expressed with the known quantities?

$$x = A\cos\theta \quad \text{since} \quad \theta = \varpi t \quad \text{and} \quad \varpi = 2\pi f$$

$$x = A\cos\varpi t = A\cos 2\pi ft$$

How about its velocity v at any given time?

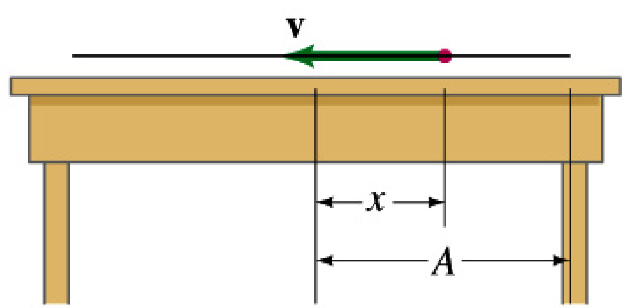
$$v_0 = \sqrt{\frac{k}{m}}A$$

$$v = -v_0 \sin\theta = -v_0 \sin\left(\varpi t\right) = -v_0 \sin\left(2\pi ft\right)$$

How about its acceleration a at any given time?

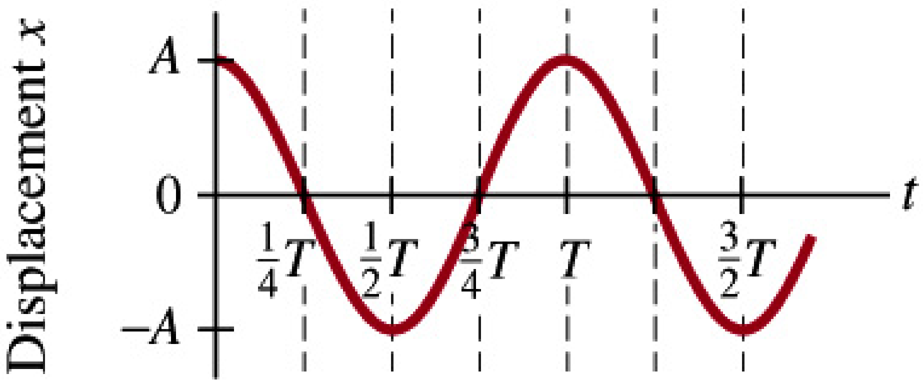
$$a = \frac{F}{m} = \frac{-kx}{m} = -\left(\frac{kA}{m}\right)\cos\left(2\pi ft\right) = -a_0 \cos\left(2\pi ft\right)$$

$$a_0 = \frac{kA}{m}$$

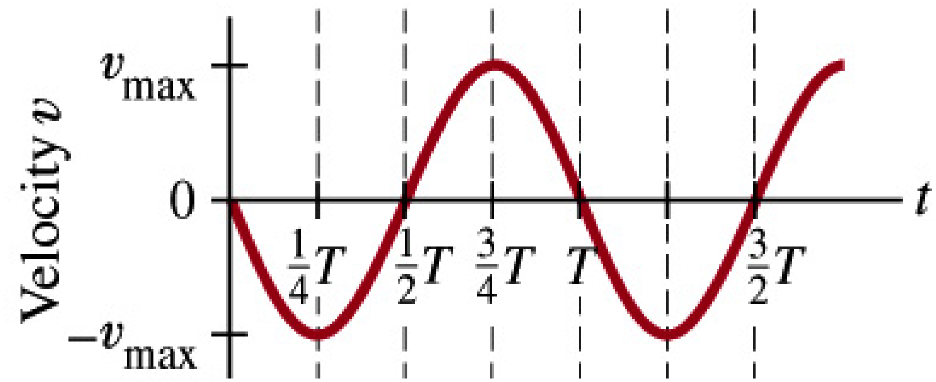


(b)

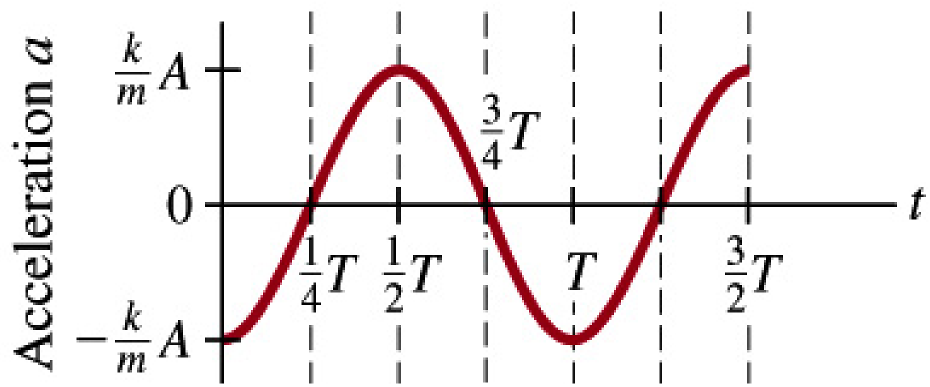
Sinusoidal Behavior of SHM



$$x = A\cos\left(2\pi ft\right)$$

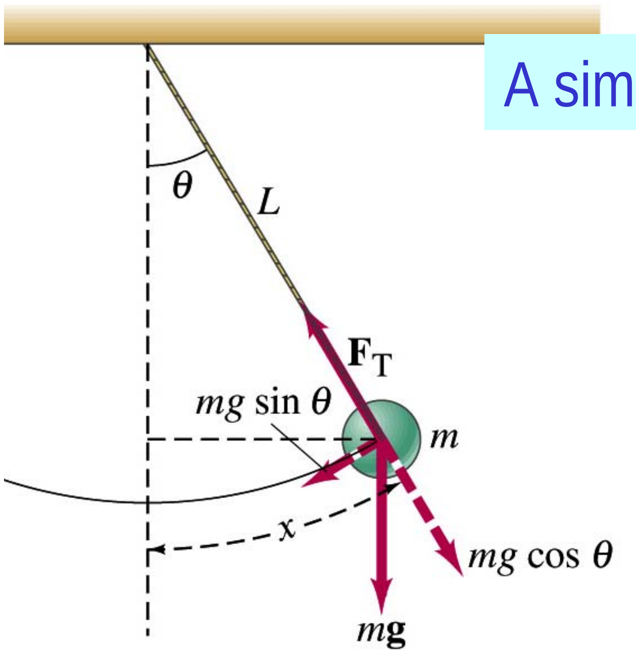


$$v = -v_0 \sin\left(2\pi ft\right)$$



$$a = -a_0 \cos\left(2\pi ft\right)$$

The Simple Pendulum



A simple pendulum also performs periodic motion.

The net force exerted on the bob is

$$\sum F_r = T - mg\cos\theta_A = 0$$

$$\sum F_t = -mg\sin\theta_A \approx -mg\theta$$

Since the arc length, x, is $x = L\theta$

$$F = \sum F_t = -\frac{mg}{L}x$$

Satisfies conditions for simple harmonic motion!

It's almost like Hooke's law with. $k = \frac{mg}{L}$

The period for this motion is $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{mL}{mg}} = 2\pi\sqrt{\frac{L}{g}}$

The period only depends on the length of the string and the gravitational acceleration

Example 11-8

Grandfather clock. (a) Estimate the length of the pendulum in a grandfather clock that ticks once per second.

Since the period of a simple pendulum motion is

$$T = 2\pi\sqrt{\frac{L}{g}}$$

The length of the pendulum in terms of T is

$$L = \frac{T^2 g}{4\pi^2}$$

Thus the length of the pendulum when T=1s is

$$L = \frac{T^2 g}{4\pi^2} = \frac{1\times 9.8}{4\pi^2} = 0.25m$$

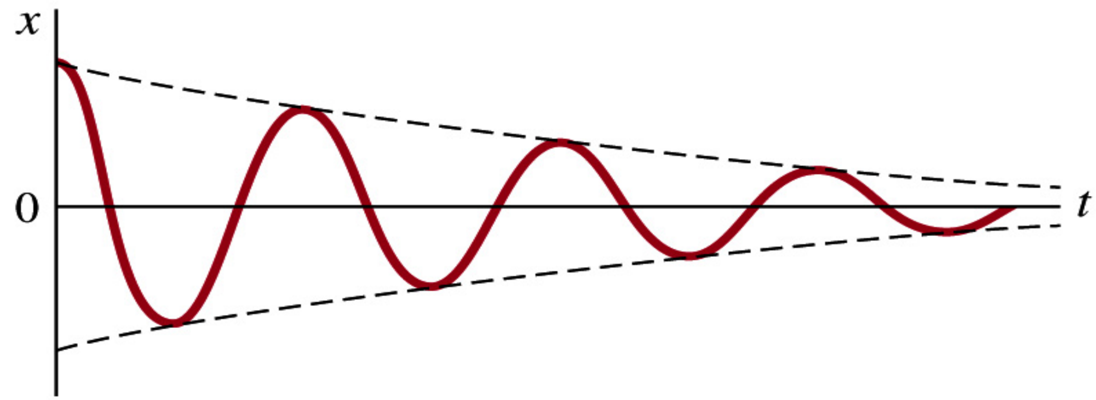
(b) What would be the period of the clock with a 1m long pendulum?

$$T = 2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{1.0}{9.8}} = 2.0s$$

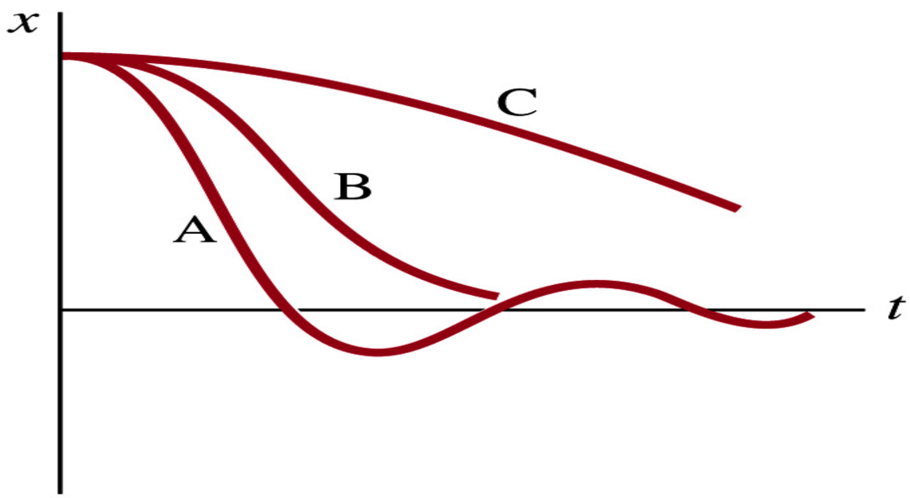
Damped Oscillation

More realistic oscillation where an oscillating object loses its mechanical energy in time by a retarding force such as friction or air resistance.

How do you think the motion would look?



Amplitude gets smaller as time goes on since its energy is spent.



Types of damping

A: Overdamped

B: Critically damped

C: Underdamped

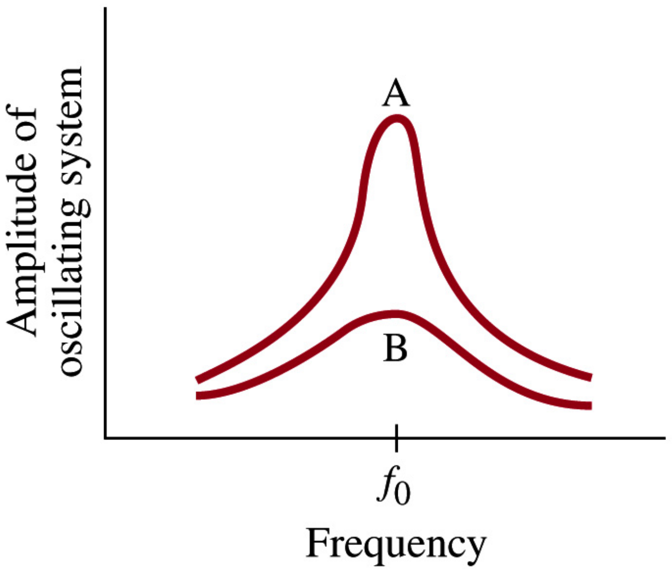
Forced Oscillation; Resonance

When a vibrating system is set into motion, it oscillates with its natural frequency f_0.

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

However a system may have an external force applied to it that has its own particular frequency (f), causing forced vibration.

For a forced vibration, the amplitude of vibration is found to be dependent on the different between f and f_0. and is maximum when $f=f_0$.



A: light damping

B: Heavy damping

The amplitude can be large when $f=f_0$, as long as damping is small.

This is called resonance. The natural frequency f_0 is also called resonant frequency.