PHYS 3446 – Lecture #2

Monday, Jan. 24, 2005
Dr. Jae Yu

1. Introduction
2. History on Atomic Models
3. Rutherford Scattering
4. Rutherford Scattering with Coulomb force
5. Scattering Cross Section
6. Measurement of Cross Sections

Announcements

- World scientific acquired 30 copies of the book
- You might want to make sure that the book store knows you want a copy
- Until the book comes, the author generously allowed us to copy the relevant part of the book and use it.
- I have five subscribed the distribution list. You still have time for 5 extra credit points…
- Class assignments:
  - $W \rightarrow e\nu+X$: James, Carlos, Justin and Elisha
  - $W \rightarrow \mu\nu+X$: Jeremy, Jay, Jason and Jim
  - $Z \rightarrow ee+X$: Casey, David and Mathew
  - $Z \rightarrow \mu\mu+X$: John, Jacom and Sabine
- Your accounts will soon be created and a new machine for your login will be available at the end of the week.

Most importantly… **We should have fun in the class!**

Why do Physics?

Exp.
- To understand nature through experimental observations and measurements (Research)

Theory
- Establish limited number of fundamental laws, usually with mathematical expressions
- Predict the nature's course

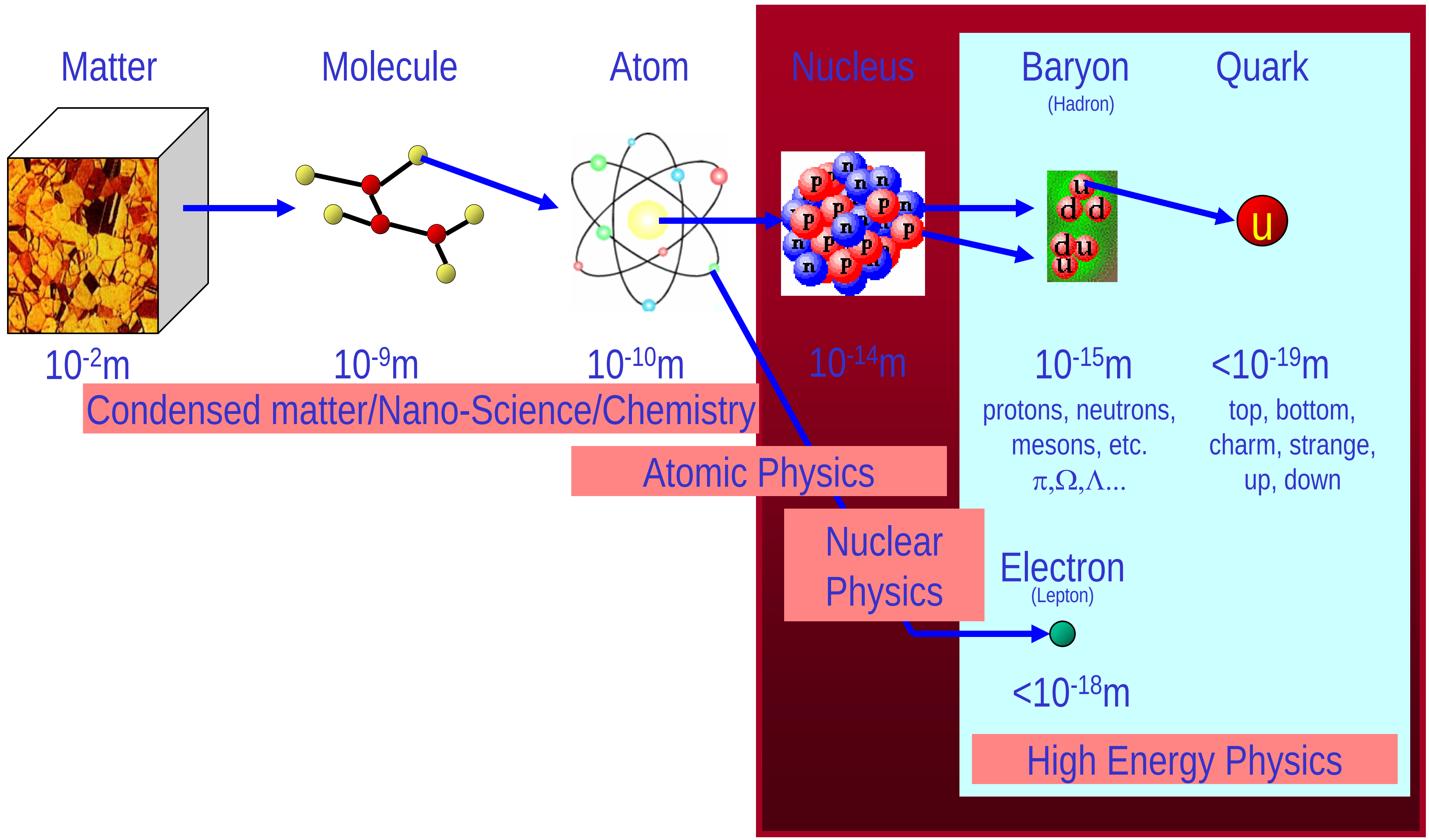
$\Rightarrow$Theory and Experiment work hand-in-hand

$\Rightarrow$Theory works generally under restricted conditions

$\Rightarrow$Discrepancies between experimental measurements and theory are good for improvements

$\Rightarrow$Improves our everyday lives, though some laws can take a while till we see amongst us

Structure of Matter

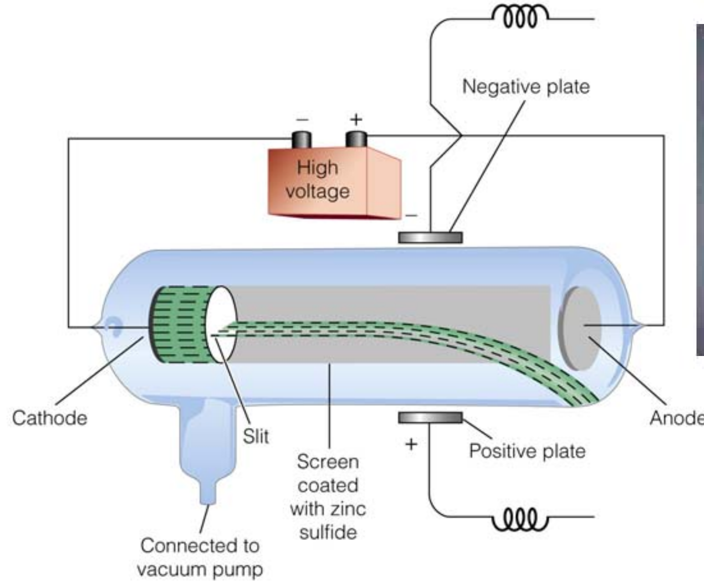


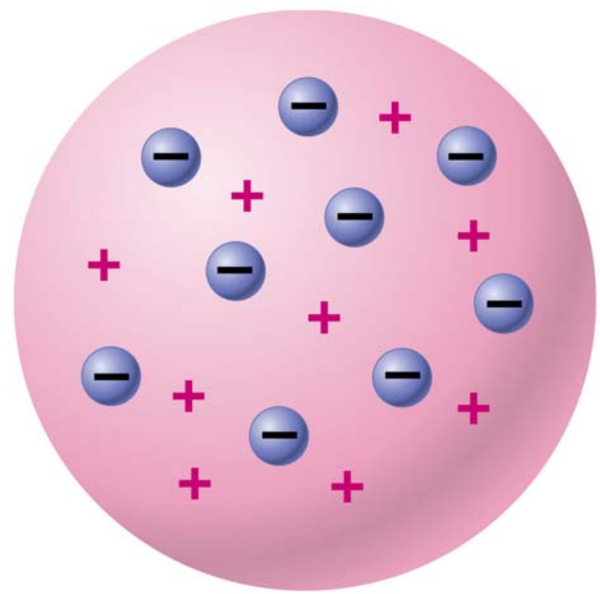
Theory for Microscopic Scale, Quantum Mechanics

- Since we deal with extremely small objects, it is difficult to explain the phenomena with classical mechanics and Electro-magnetism
- The study of atomic structure, thus, led us to quantum mechanics ➔ Extremely successful
  - Long range EM force is responsible for holding atom together
  - EM force is sufficiently weak so that the properties of atoms can be estimated reliably based on perturbative QM calculations
- However, when we step into nucleus regime, the simple Coulomb force does not work since the force in nucleus holds positively charged particles together
- The known forces in nature
  - Strong ~ 1
  - Electro-magnetic ~ 10^{-2}
  - Weak ~ 10^{-5}
  - Gravitational ~ 10^{-38}

Evolution of Atomic Models

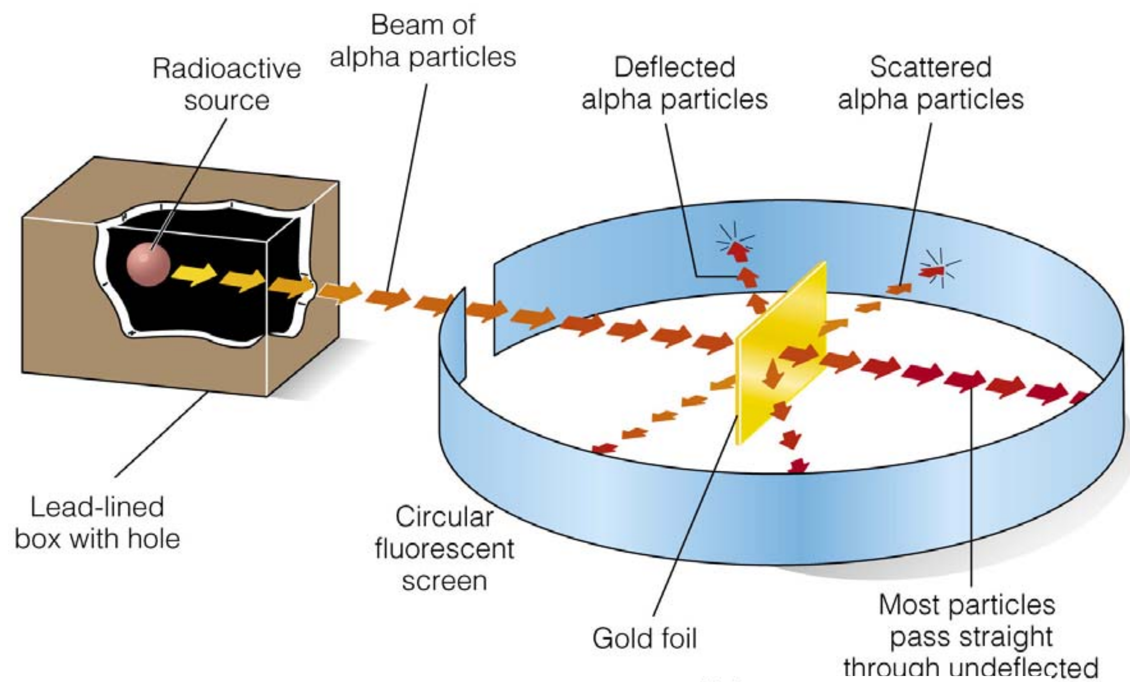
- 1897: J.J. Thompson Discovered electrons
- 1904: J.J. Thompson Proposed a "plum pudding" model of atoms ➔ Negatively charged electrons embedded in a uniformly distributed positive charge

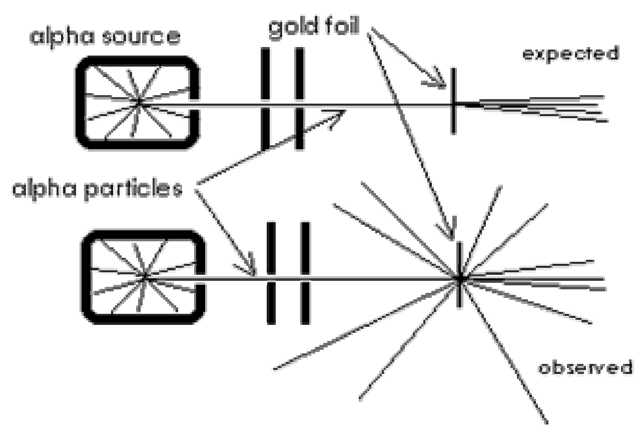


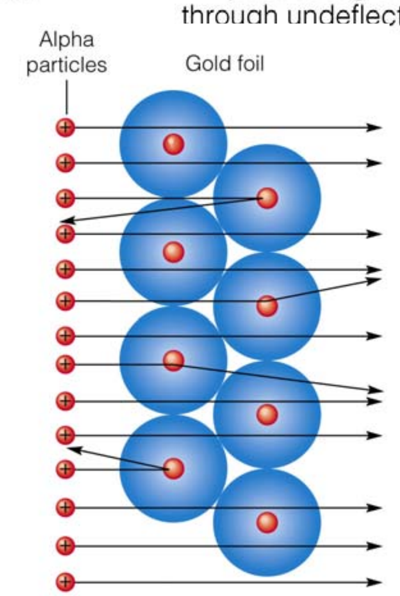


History of Atomic Models

- 1911: Geiger and Marsden with Rutherford perform a scattering experiment with alpha particles shot on a thin gold foil

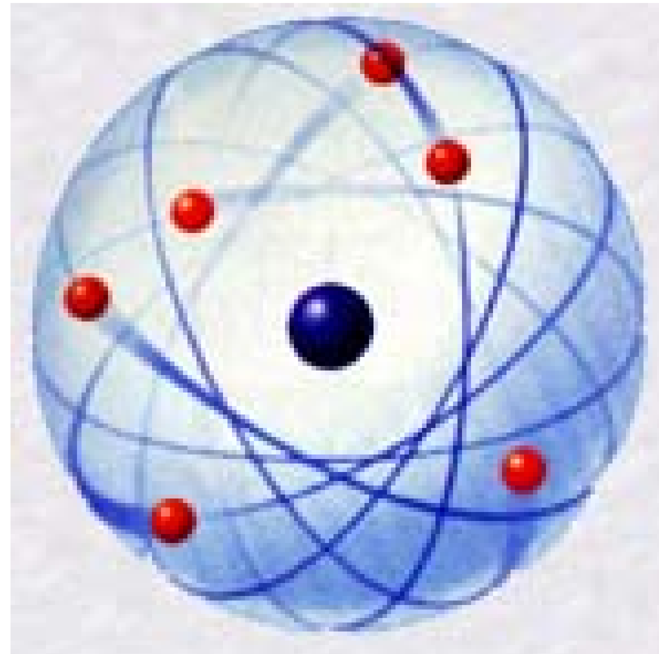


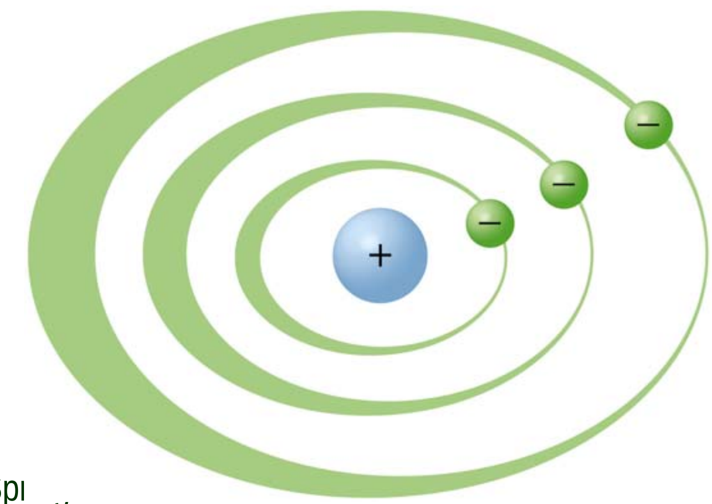




History of Atomic Models

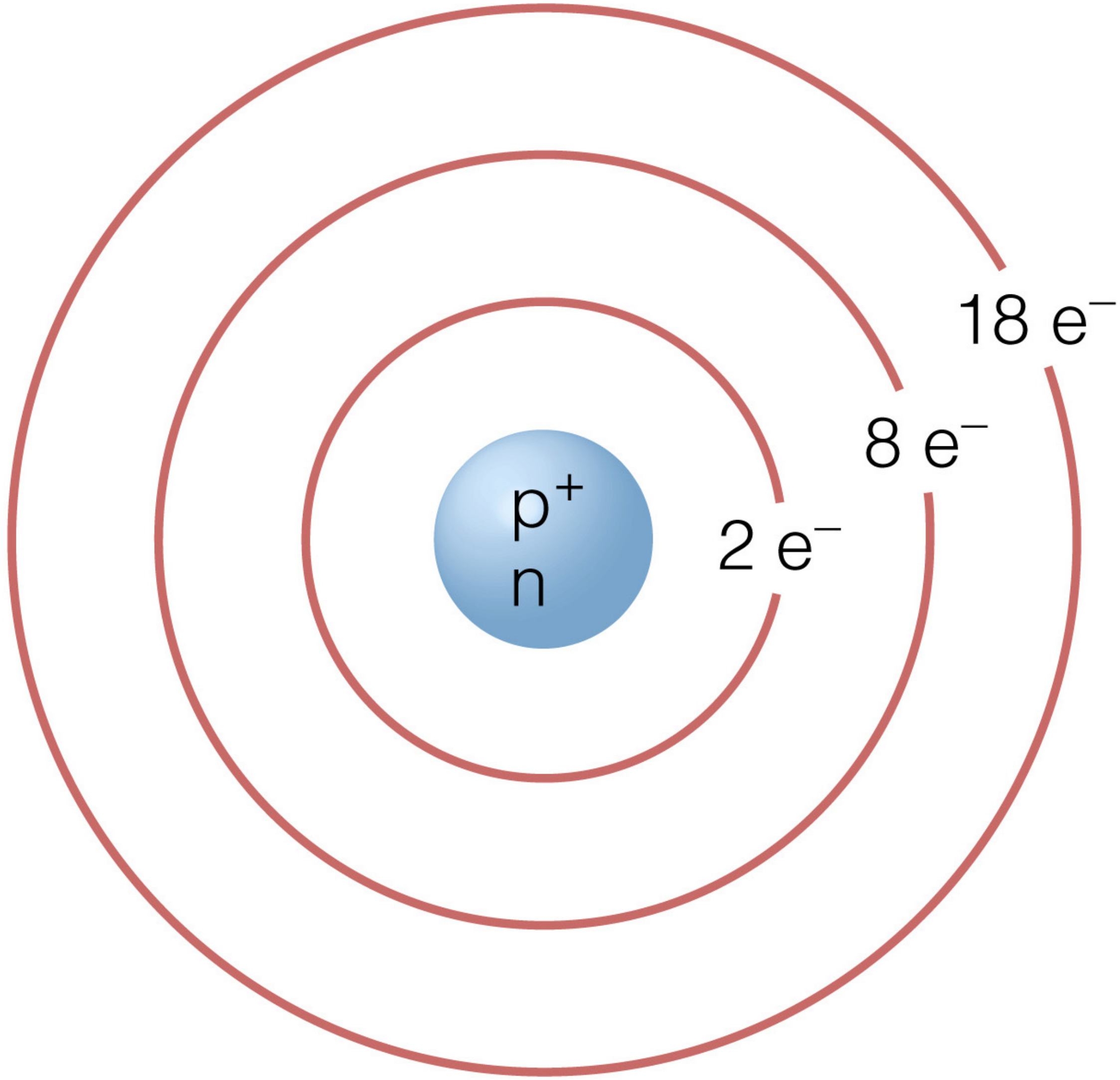
- 1912: Rutherford proposes atomic model with a positively charged core surrounded by electrons
  - Rutherford postulated a heavy nucleus with electrons circulating it like planets around the sun
  - Deficiency of instability





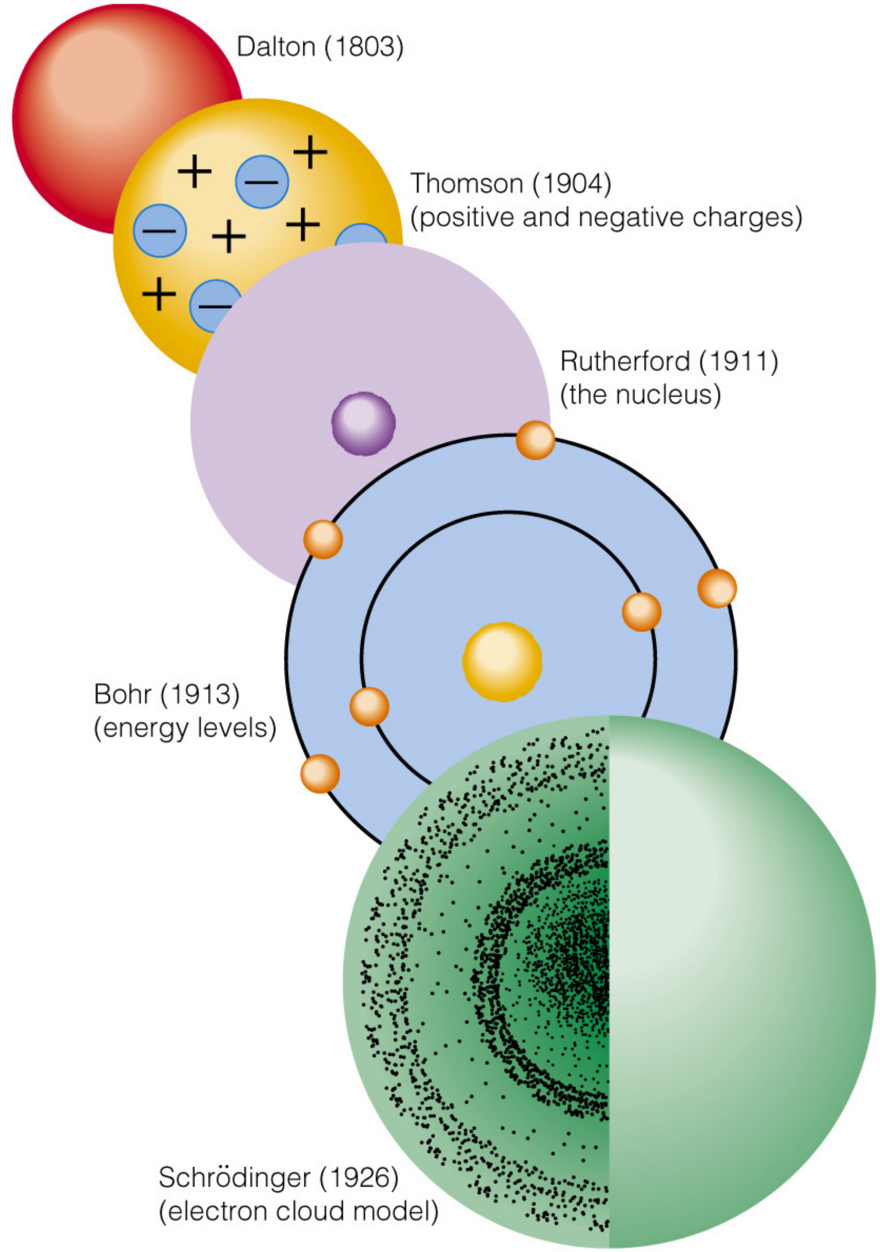
History of Atomic Models

- 1913: Neils Bohr proposed a quantified electron orbit
  - Electrons can only transition to pre-defined orbits



History of Atomic Models

- 1926: Schrodinger proposed an electron cloud model based on quantum mechanics



Rutherford Scattering

- A fixed target experiment with alpha particle as projectile shot on thin gold foil
  - Alpha particle's energy is low ➔ Speed is well below 0.1c (non-relativistic)
- An elastic scattering of the particles
- What are the conserved quantities in an elastic scattering?
  - Momentum
  - Kinetic Energy

Elastic Scattering

- From momentum conservation

$$\vec{v}_0 = \frac{m_\alpha \vec{v}_\alpha + m_t \vec{v}_t}{m_\alpha} = \vec{v}_\alpha + \frac{m_t}{m_\alpha}\vec{v}_t$$

- From kinetic energy conservation

$$v_0^2 = v_\alpha^2 + \frac{m_t}{m_\alpha} v_t^2$$

- From these two, we obtain

$$v_t^2\left(1 - \frac{m_t}{m_\alpha}\right) = 2\vec{v}_\alpha \cdot \vec{v}_t$$

The Analysis

$$v_t^2\left(1-\frac{m_t}{m_\alpha}\right)=2\vec{v}_\alpha\cdot\vec{v}_t$$

- If $m_t << m_\alpha$,
  - left-hand side become positive
  - v_α and v_t must be the same direction
  - Using the actual masses

$$m_e \approx 0.5 MeV/c^2$$

$$m_\alpha \approx 4\times 10^3 MeV/c^2$$

  - If $m_t = m_e$, then $m_t/m_\alpha \sim 10^{-4}$. ➔ $v_\alpha \approx v_0$
  - Thus, $p_e/p_{\alpha 0} < 10^{-4}$.
  - Change of momentum of alpha particle is negligible

The Analysis

$$v_t^2\left(1-\frac{m_t}{m_\alpha}\right)=2\vec{v}_\alpha\cdot\vec{v}_t$$

- If $m_t \gg m_\alpha$,
  - left-hand side become negative
  - v_α and v_t is opposite direction
  - Using the actual masses

    $$m_t \approx m_{Au} \approx 2\times10^5\, MeV/c^2$$

    $$m_\alpha \approx 4\times10^3\, MeV/c^2$$
  - If $m_t=m_e$, then m_t/m_α~50. ➔ $v_\alpha \approx v_0$
  - Thus, $p_e/p_{\alpha0}$~$2p_{\alpha0}$.
  - Change of momentum of alpha particle is large.

Rutherford Scattering with EM Force 1

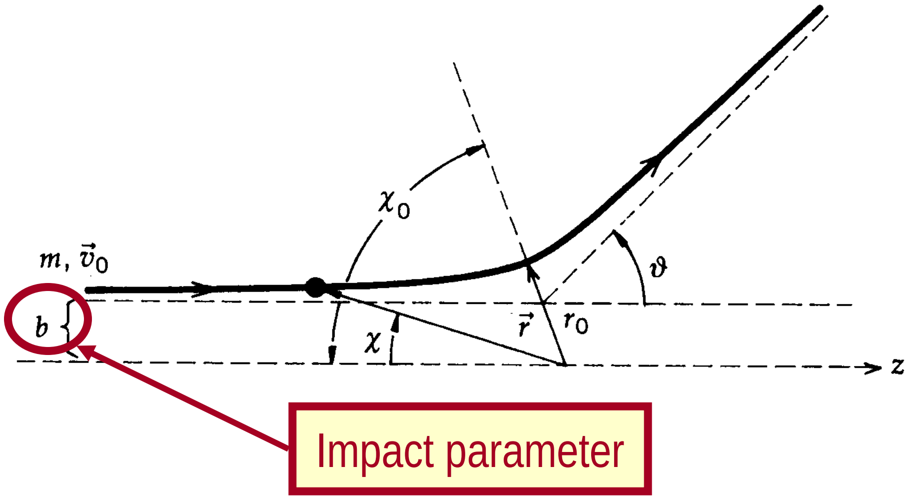
- We did not take into account electro-magnetic force between the alpha particle and the atom
- Coulomb force is a central force and thus a conservative force
- Coulomb potential between particles with Ze and Z'e electrical charge separated by distance r is

$$V(r) = \frac{ZZ'e^2}{r}$$

- Since the total energy is conserved,

$$E = \frac{1}{2}mv_0^2 = \text{constant} > 0 \Rightarrow \mathrm{v}_0 = \sqrt{\frac{2E}{m}}$$

Rutherford Scattering with EM Force 2



- The distance vector **r** is always the same direction as the force throughout the entire motion, net torque (**r**x**F**) is 0.
- Since there is no net torque, the angular momentum (**l**=**r**x**p**) is conserved. ➔ The magnitude of the angular momentum is *l=mvb*.

- From energy relation, we obtain

$$l = m\sqrt{2E/m}\,b = b\sqrt{2mE} \Rightarrow b^2 = l^2/2mE$$

- From the definition of angular momentum, we obtain an equation of motion $d\chi/dt = l/mr^2$
- From energy conservation, we obtain another equation of motion

$$\frac{dr}{dt} = \pm\sqrt{\frac{2}{m}\left(E - \boxed{V(r) - \frac{l^2}{2mr^2}}\right)}$$

Centrifugal barrier

Effective potential

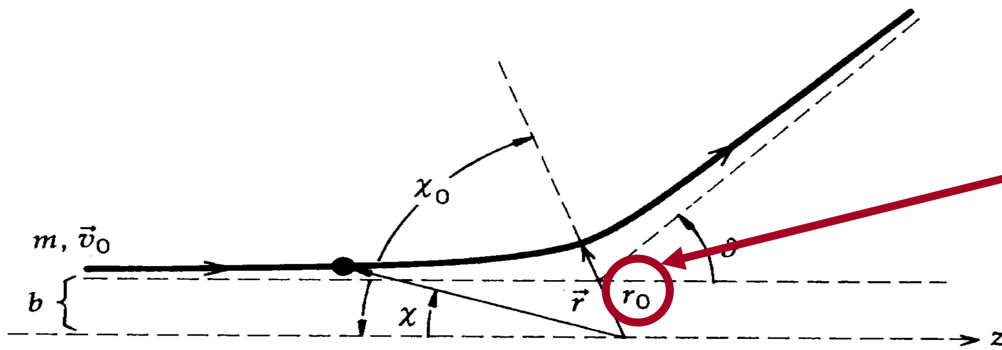
Rutherford Scattering with EM Force 3

- Rearranging the terms, we obtain

$$\frac{dr}{dt} = -\frac{l}{mrb}\sqrt{r^2\left(1-\frac{V(r)}{E}\right)-b^2}$$

- and $$d\chi = -\frac{b\,dr}{r\left[r^2\left(1-\frac{V(r)}{E}\right)-b^2\right]^{1/2}}$$

- Integrating this from r_0 to infinity gives the angular distribution of the outgoing alpha particle



Distance of closest approach

Rutherford Scattering with EM Force 4

- What happens at the DCA?
  - Kinetic energy reduces to 0. $$\left.\frac{dr}{dt}\right|_{r=r_0} = 0$$
  - The incident alpha could turn around and accelerate
  - We can obtain $$r_0^2\left(1-\frac{V(r_0)}{E}\right)-b^2=0$$
  - This allows us to determine DCA for a given potential and χ_0.
- Define scattering angle q as the changes in the asymptotic angles of the trajectory, we obtain

$$\theta=\pi-2\chi_0=\pi-2b\int_{r_0}^{\infty}\frac{dr}{r\left[r^2\left(1-\frac{V(r)}{E}\right)-b^2\right]^{1/2}}$$

Rutherford Scattering with EM Force 5

- For a Coulomb potential

$$V(r) = \frac{ZZ'e^2}{r}$$

- DCA can be obtained for a given impact parameter b,

$$r_0 = \frac{ZZ'e^2/E}{2}\left(1 + \sqrt{1 + 4b^2E^2/\left(ZZ'e^2\right)^2}\right)$$

- And the angular distribution becomes

$$\theta = \pi - 2b\int_{r_0}^{\infty} \frac{dr}{r\left[r^2\left(1 - \frac{ZZ'e^2}{rE}\right) - b^2\right]^{1/2}}$$

Rutherford Scattering with EM Force 6

- Replace the variable 1/r=x, and performing the integration, we obtain

$$\theta = \pi + 2b\cos^{-1}\left(\frac{1}{\sqrt{1+4b^2E^2/\left(ZZ'e^2\right)^2}}\right)$$

- This can be rewritten

$$\frac{1}{\sqrt{1+4b^2E^2/\left(ZZ'e^2\right)^2}} = \cos\left(\frac{\theta-\pi}{2}\right)$$

- Solving this for b, we obtain

$$b = \frac{ZZ'e^2}{2E}\cot\frac{\theta}{2}$$

Rutherford Scattering with EM Force 7

$$b = \frac{ZZ'e^2}{2E}\cot\frac{\theta}{2}$$

- From the solution for b, we can learn the following
  1. For fixed b. E and Z'
     - The scattering is larger for a larger value of Z.
       - Makes perfect sense since Coulomb potential is stronger with larger Z.
       - Results in larger deflection.
  2. For a fixed b, Z and Z'
     - The scattering angle is larger when E is smaller.
       - If particle has low energy, its velocity is smaller
       - Spends more time in the potential, suffering greater deflection
  3. For fixed Z, Z', and E
     - The scattering angle is larger for smaller impact parameter b
       - Makes perfect sense also, since as the incident particle is closer to the nucleus, it feels stronger Coulomb force.

Assignments

1. Compute the masses of electron and alpha particles in MeV/c^2.
2. Compute the gravitational and the Coulomb forces between two protons separated by 10^{-10}m and compare their strengths
3. Drive the following equations in your book:
   - Eq. # 1.3, 1.16, 1.19, 1.25, 1.32

- These assignments are due next Monday, Jan. 31.