PHYS 3446 – Lecture #3

Wednesday, Jan. 26, 2005
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1. Rutherford Scattering with Coulomb force
2. Scattering Cross Section
3. Differential Cross Section of Rutherford Scattering
4. Measurement of Cross Sections
5. A few measurements of differential cross sections

Announcements

- I have eleven subscribed the distribution list. You still have time for 3 extra credit points…
- Extra credit points for early completion of Homework:

Rutherford Scattering

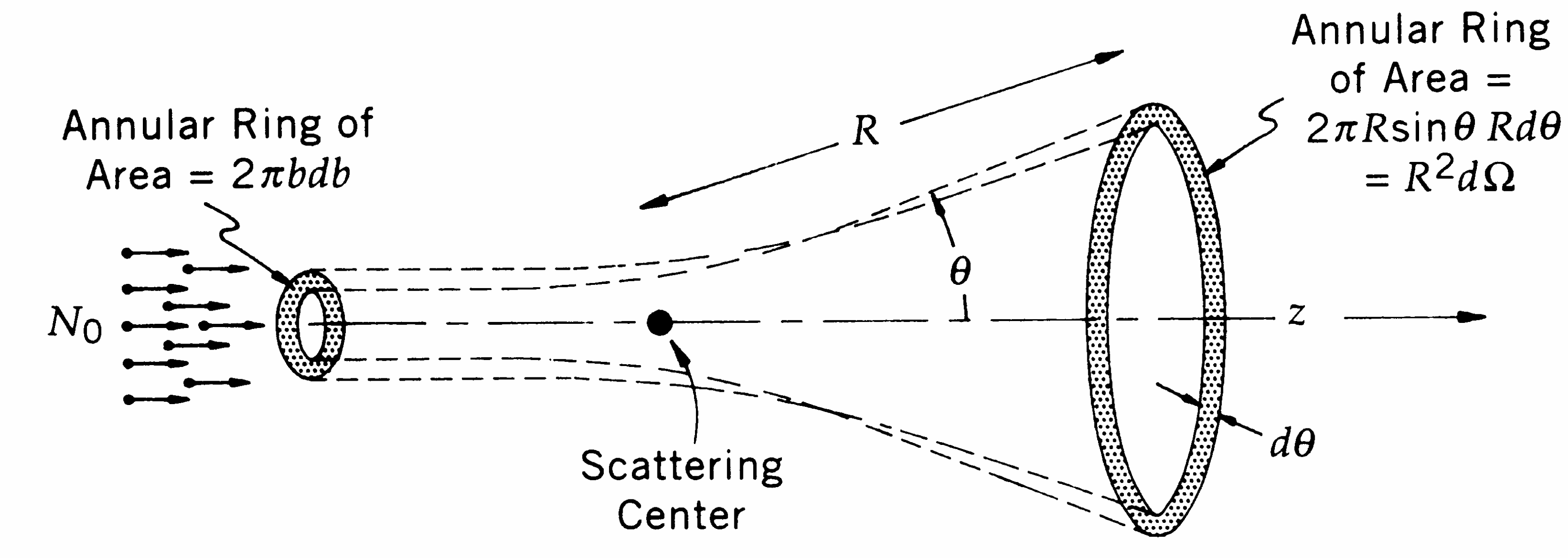
$$b = \frac{ZZ'e^2}{2E}\cot\frac{\theta}{2}$$

- From the solution for b, we can learn the following
  1. For fixed b, E and Z'
     - The scattering is larger for a larger value of Z.
       - Makes perfect sense since Coulomb potential is stronger with larger Z.
       - Results in larger deflection.
  2. For a fixed b, Z and Z'
     - The scattering angle is larger when E is smaller.
       - If particle has low energy, its velocity is smaller
       - Spends more time in the potential, suffering greater deflection
  3. For fixed Z, Z', and E
     - The scattering angle is larger for smaller impact parameter b
       - Makes perfect sense also, since as the incident particle is closer to the nucleus, it feels stronger Coulomb force.

Scattering Cross Section

- Scattering of a particle in a potential is completely determined once impact parameter and energy of the particle are known.
- For fixed energy, deflection is defined by the impact parameter, b.
- What do we need to perform a scattering experiment?
  - Incident flux of beam particles with known E
  - Device that can measure number of scattered particles at various angle, θ.
  - These measurements reflect
    - Impact parameters of the particles
    - The effective size of the scattering center

Scattering Cross Section



- N_0: The number of particles incident on the target foil per unit area per unit time.
- Any incident particles entering with impact parameter b and b+db will scatter to the angle θ and θ-dθ.
- Will scatter into a solid angle dΩ.
- The number of scattered particles per unit time is $2\pi N_0 bdb$.

Scattering Cross Section

- For a central potential, spherical symmetry makes the scattering center as presenting an effective transverse x-sectional area of

$$\Delta\sigma = 2\pi b db$$

- More generalized cases, $\Delta\sigma$ depends on both θ and ϕ.

$$\Delta\sigma(\theta,\phi) = bdbd\phi = -\frac{d\sigma}{d\Omega}(\theta,\phi)d\Omega = -\frac{d\sigma}{d\Omega}(\theta,\phi)\sin\theta d\theta d\phi$$

- With an spherical symmetry, ϕ can be integrated out:

$$\Delta\sigma(\theta) = -\frac{d\sigma}{d\Omega}(\theta)2\pi\sin\theta d\theta = 2\pi b db$$

reorganize

$$\frac{d\sigma}{d\Omega}(\theta) = -\frac{b}{\sin\theta}\frac{db}{d\theta}$$

Differential Cross Section

What is the dimension of the differential cross section?

Area!!

Scattering Cross Section

- For a central potential, measuring the yield as a function of θ, or differential cross section, is equivalent to measuring the entire effect of the scattering
- So what is the physical meaning of the differential cross section?

$\Rightarrow$ Measurement of yield as a function of specific experimental variable

$\Rightarrow$ This is equivalent to measuring the probability of certain process in a specific kinematic phase space

- Cross sections are measured in the unit of barns:

$$1\ \text{barn} = 10^{-24}\,\text{cm}^2$$

Where does this come from?

Total Cross Section

- Total cross section is the integration of the differential cross section over the entire solid angle, Ω:

$$\sigma_{Total} = \int_0^{4\pi} \frac{d\sigma}{d\Omega}(\theta,\phi)\, d\Omega = 2\pi \int_0^{\pi} d\theta \sin\theta \frac{d\sigma}{d\Omega}(\theta)$$

- Total cross section represents the effective size of the scattering center at all possible impact parameter

Cross Section of Rutherford Scattering

- Impact parameter in Rutherford scattering is

$$b = \frac{ZZ'e^2}{2E}\cot\frac{\theta}{2}$$

- Thus,

$$\frac{db}{d\theta} = -\frac{1}{2}\frac{ZZ'e^2}{2E}\operatorname{cosec}^2\frac{\theta}{2}$$

- Differential cross section of Rutherford scattering

$$\frac{d\sigma}{d\Omega}(\theta) = -\frac{b}{\sin\theta}\frac{db}{d\theta} = \left(\frac{ZZ'e^2}{2E}\right)^2\operatorname{cosec}^4\frac{\theta}{2} = \left(\frac{ZZ'e^2}{2E}\right)^2\frac{1}{\sin^4\frac{\theta}{2}}$$

Cross Section of Rutherford Scattering

- Let's plug in the numbers
  - Z_{Au}=79
  - Z_{He}=2
  - For E=10keV
- Differential cross section of Rutherford scattering

$$\frac{d\sigma}{d\Omega}(\theta) = -\left(\frac{79\cdot 2\cdot\left(1.6\times 10^{-19}\right)^2}{2\cdot 10\times 10^3\times 1.6\times 10^{-19}}\right)^2\frac{1}{\sin^4\frac{\theta}{2}} = \frac{1.59\times 10^{-42}}{\sin^4\frac{\theta}{2}}cm^2$$

$$\frac{d\sigma}{d\Omega}(\theta) = \frac{1.59\times 10^{-18}}{\sin^4\frac{\theta}{2}}barns$$

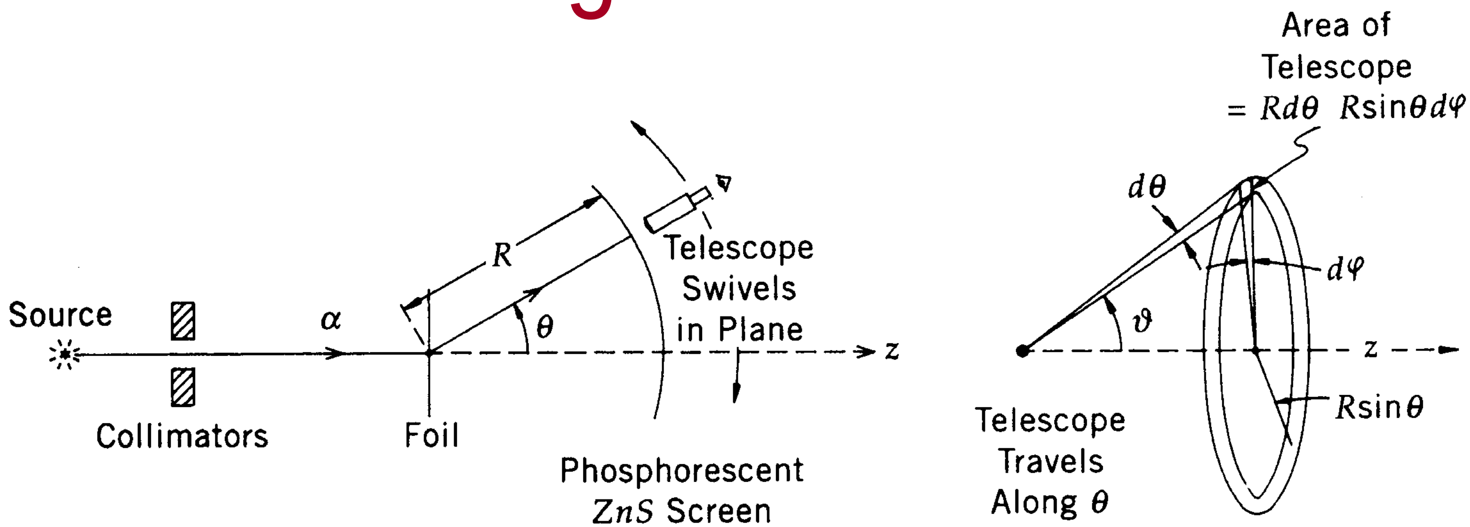
Total X-Section of Rutherford Scattering

- To obtain total cross section of Rutherford scattering, one integrates over all θ:

$$\sigma_{Total} = 2\pi \int_0^{\pi} \frac{d\sigma}{d\Omega}(\theta) d\theta = 8\pi \left(\frac{ZZ'e^2}{2E} \right)^2 \int_0^1 d\left(\sin\frac{\theta}{2} \right) \frac{1}{\sin^3\frac{\theta}{2}}$$

- What is the result of this integration?
  - Infinity!!
- Does this make sense?
  - Yes
- Why?
  - Since the Coulomb force's range is infinity.
- Is this physically meaningful?
  - No
- What would be the sensible thing to do?
  - Integrate to a cut off angle since after certain distance the force is too weak to impact the scattering. ($\theta=\theta_0>0$)

Measuring Cross Sections



- For Rutherford scattering experiment
  - Used a collimated beam of α particles emitted from Radon
  - A thin Au foil target
  - A scintillating glass screen with ZnS phosphor deposit
  - Telescope to view limited area of solid angle
  - Telescope only need to move along θ not ϕ. Why?

Measuring Cross Section

- With the flux of N_0 per unit area per second
- Any a particles in b and b+db will be scattered to θ and θ-dθ
- The telescope aperture limits the measurable area to

$$A_{Tele} = Rd\theta \cdot R\sin\theta d\phi = R^2 d\Omega$$

- How could they have increased the rate of measurement?
  - By constructing an annular telescope
  - By how much would it increase? $\boxed{2\pi/d\phi}$

Measuring Cross Section

- Fraction of incident particles approaching the target in the small area $\Delta\sigma$=bdϕdb at impact parameter b is –dn/N_0.
  - These scatters into $R^2 d\Omega$, the aperture of the telescope
- This ratio is the same as the sum of all $\Delta\sigma$ over all the N nuclear centers throughout the foil divided by the total area (S) of the foil.
  - Probability for incident particle to enter within the N areas of annular ring and subsequently scatters into the telescope
- So this ratio can be expressed as

$$-\frac{dn}{N_0} = \frac{Nbd\phi d\theta}{S} = \frac{N}{S}\Delta\sigma(\theta,\phi)$$

Measuring Cross Section

- For a foil thickness t, density ρ, atomic weight A:

$$N = \frac{\rho t S}{A} A_0$$

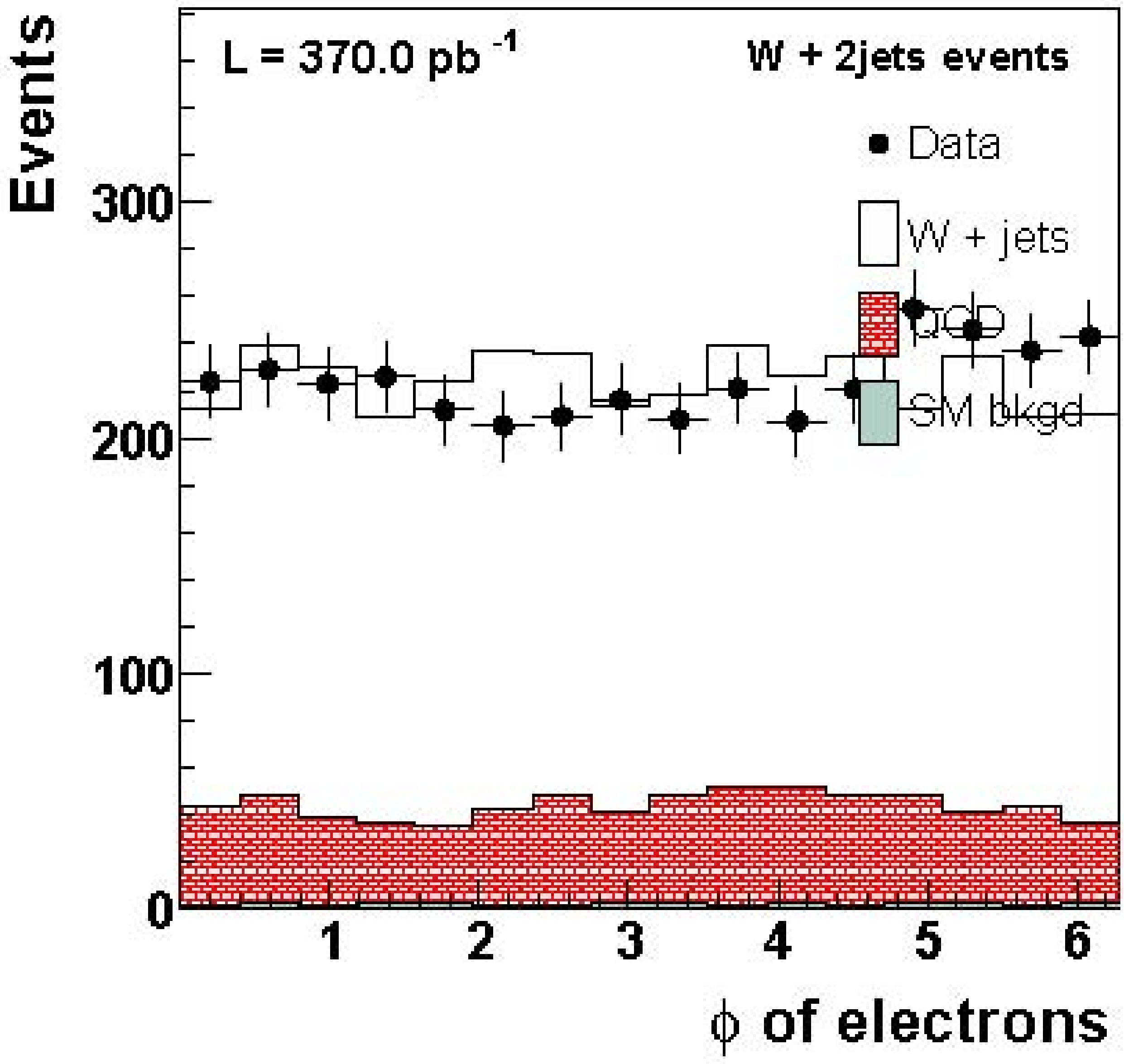
A_0: Avogadro's number

- The number of α scattered into the detector angle (θ,ϕ) is

$$dn = \frac{N_0 \rho t A_0}{A} \frac{d\sigma(\theta,\phi)}{d\Omega} d\Omega = N_0 \frac{N}{S} \frac{d\sigma(\theta,\phi)}{d\Omega} d\Omega$$

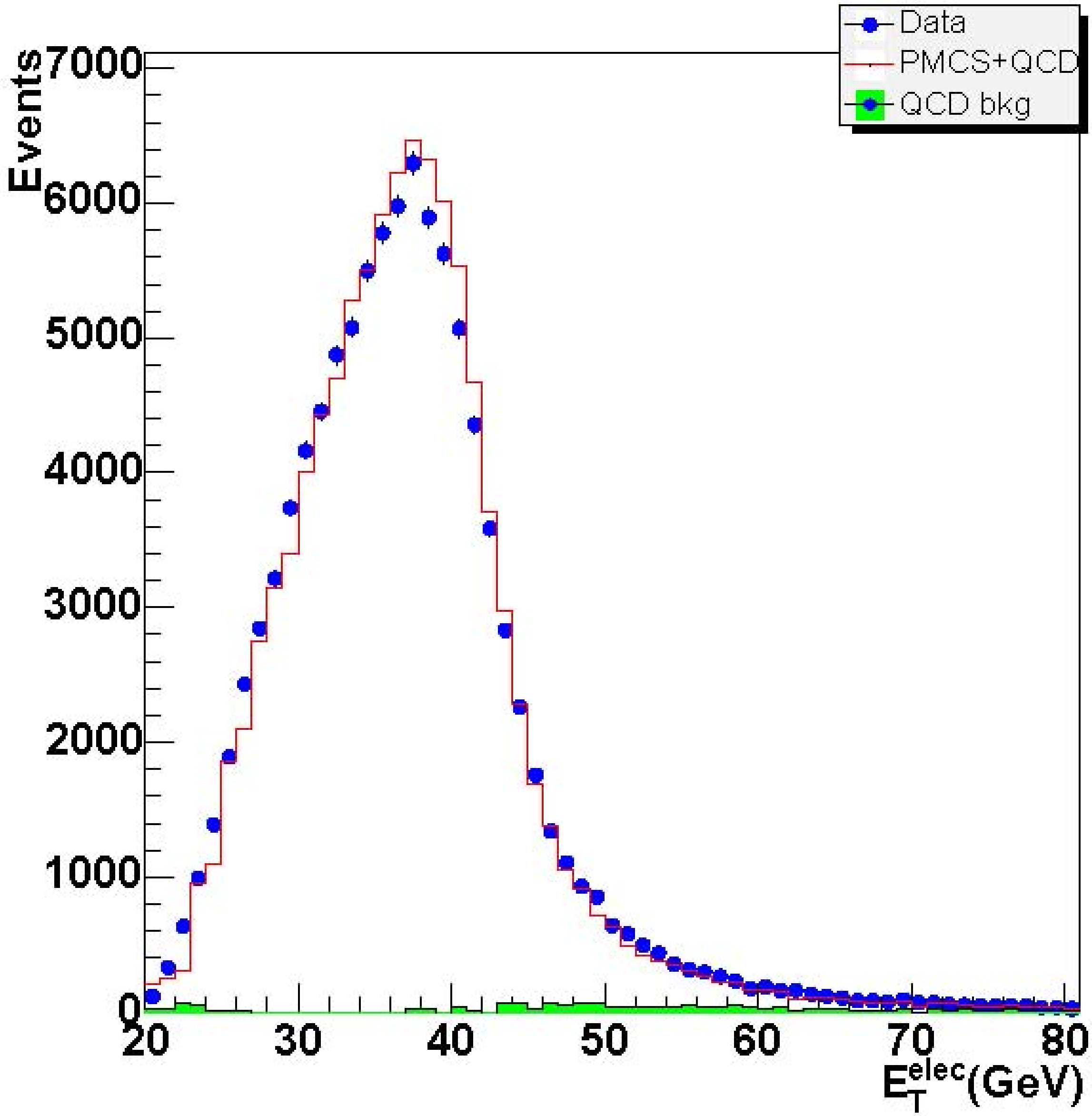
- This is a general expression for any scattering process, independent of the existence of theory
- This gives an observed counts per second

Some Example Cross Section Measurements



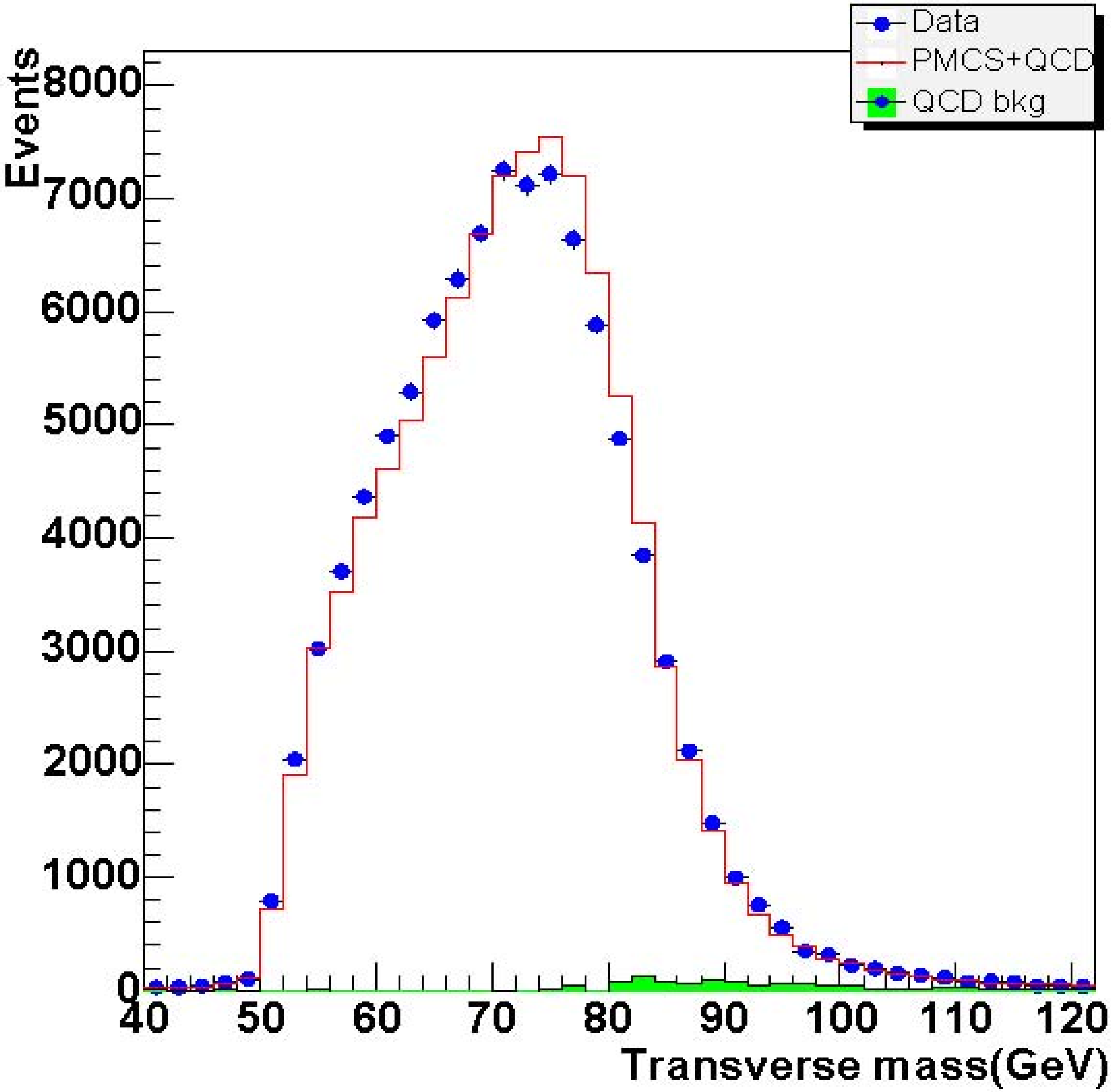
- Azimuthal angle distribution of electrons in W+2jets events

Example Cross Section: W($\rightarrow$eν) +X



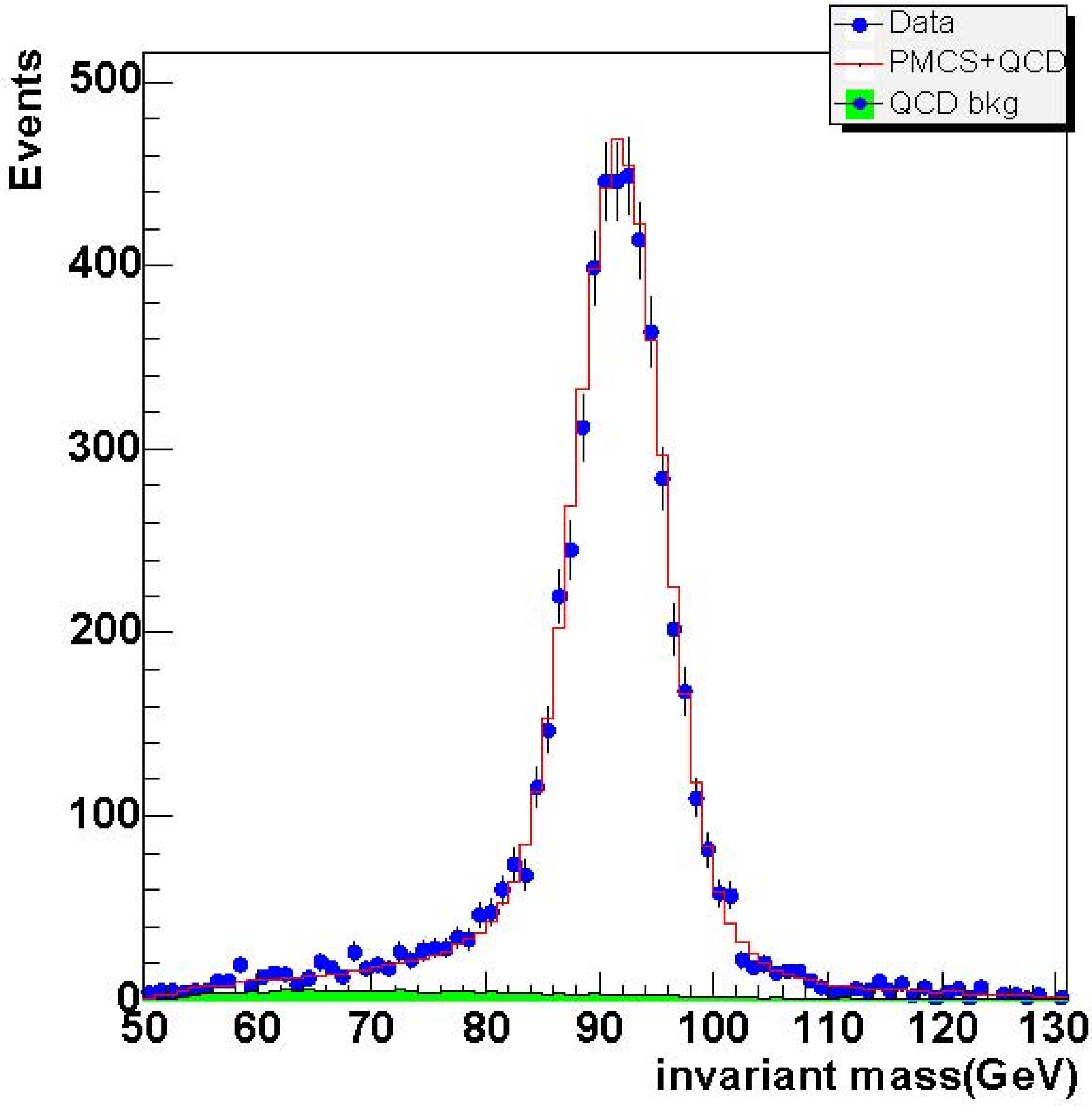
- Transverse momentum distribution of electrons in W+X events

Example Cross Section: W($\rightarrow e\nu$) +X



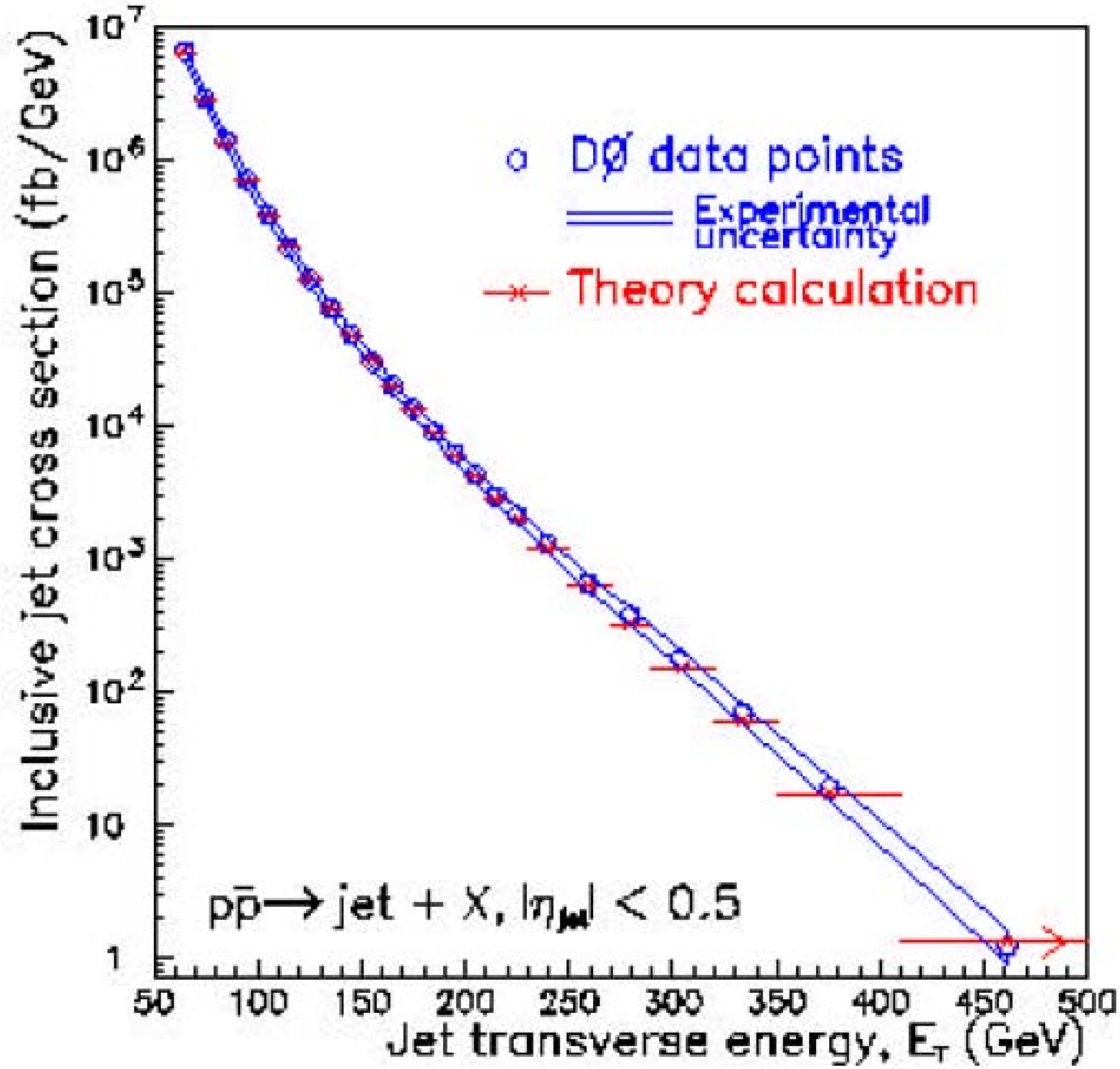
- Transverse mass distribution of electrons in W+X events

Example Cross Section: W(→ee) +X



- Invariant mass distribution of electrons in Z+X events

Example Cross Section: Jet +X



- Inclusive jet production cross section as a function of transverse energy

Assignments

1. Draw the plot of differential cross section of the Rutherford scattering as a function of the scattering angle θ with some sensible lower limit of the angle
   - Write down your opinion on the sensibility of the plot and the cross section
2. Reading assignment: Appendix A.

- These assignments are due next Wednesday, Feb. 2.