PHYS 3446 – Lecture #5

Wednesday, Feb. 2, 2005
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1. Nuclear Phenomenology
2. Properties of Nuclei
   - Labeling
   - Masses
   - Sizes
   - Nuclear Spin and Dipole Moment
   - Stability and Instability of Nuclei
3. Nature of the Nuclear Force

Announcements

- I have only one more to go on the distribution list.
  - A test message will be sent out this afternoon
- I asked you to derive a few equations for you to
  - Understand the physics behind such calculations
  - To follow through the complete calculations yourselves once in your life
- You must keep up with the homework
  - HW constitutes 15% of your grade!!!

Nuclear Phenomenology

- Rutherford scattering experiment clearly demonstrated the existence of a positively charged central core in an atom
- The formula deviated for high energy α particles (E>25MeV), especially for low Z nuclei.
- 1920's James Chadwick noticed serious discrepancies between Coulomb scattering expectation and the elastic scattering of α particle on He.
- None of the known effects, including quantum effect, described the discrepancy.
- Clear indication of something more than Coulomb force involved in the interactions.
- Before Chadwick's discovery of neutron in 1932, people thought nucleus contain protons and electrons. ➔ We now know that there are protons and neutrons (nucleons) in nuclei.

Properties of Nuclei: Labeling

$$^{A}X^{Z}$$

- The nucleus of an atom X can be labeled uniquely by its:
  - Electrical Charge or atomic number Z (number of protons).
  - Total number of nucleons A ($=N_p+N_n$)
- Isotopes: Nuclei with the same Z but different A
  - Same number of protons but different number of neutrons
  - Have similar chemical properties
- Isobars: Nuclei with same A but different Z
  - Same number of nucleons but different number of protons
- Isomers or resonances of the ground state: Excited nucleus to a higher energy level
- Mirror nuclei: Nuclei with the same A but with switched N_p and N_n

Nuclear Properties: Masses of Nuclei

- A nucleus of ${}^{A}X^{Z}$ has N_p=Z and N_n=A-Z
- Naively one would expect

$$M(A,Z) = Zm_p + (A-Z)m_n$$

- Where m_p~938.27MeV/c^2 and m_n=939.56MeV/c^2
- However measured mass turns out to be

$$M(A,Z) < Zm_p + (A-Z)m_n$$

- This is one of the explanations for nucleus not falling apart into its nucleon constituents

Nuclear Properties: Binding Energy

- The mass deficit

$$\Delta M(A,Z) = M(A,Z) - Zm_p - (A-Z)m_n$$

- Is always negative and is proportional to the nuclear binding energy
- How are the BE and mass deficit related?

$$B.E = \Delta M(A,Z)c^2$$

- What is the physical meaning of BE?
  – A minimum energy required to release all nucleons from a nucleus

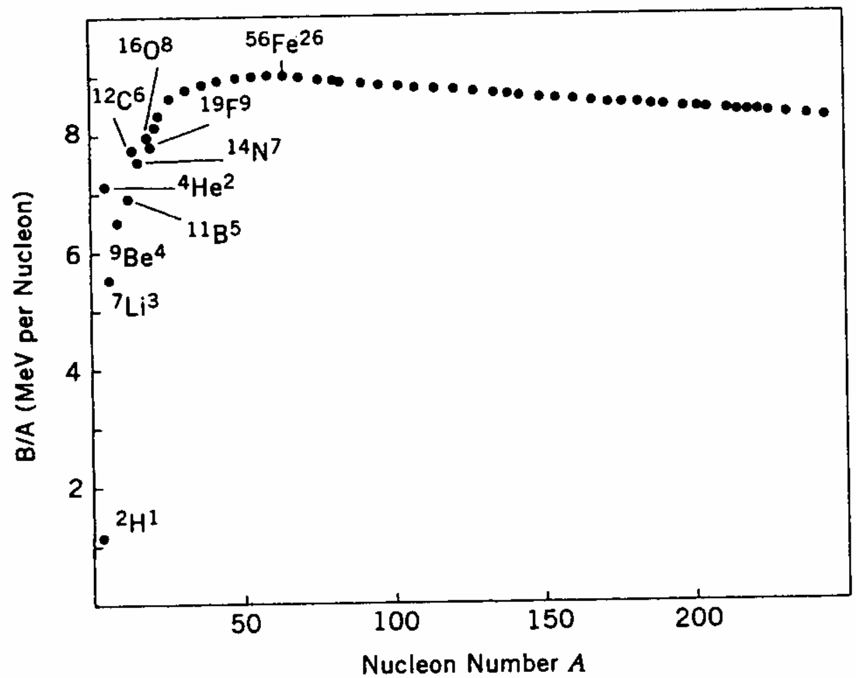
Nuclear Properties: Binding Energy

- BE per nucleon is

$$\frac{B}{A}=\frac{-BE}{A}$$

$$=\frac{-\Delta M(A,Z)c^2}{A}$$

$$=\frac{\left(Zm_p+(A-Z)m_n-M(A,Z)\right)c^2}{A}$$



- Rapidly increase with A till A~60 at which point BE~9MeV.
- A>60, the B.E gradually decrease ➔ For most the large A nucleus, BE~8MeV.

Nuclear Properties: Binding Energy

- de Broglie's wavelength: $$\lambdabar = \frac{\hbar}{p}$$
  - Where $\hbar$ is the Planck's constant
  - And $\lambdabar$ is the reduced wave length
- Assuming 8MeV was given to a nucleon (m~940MeV), the wavelength is

$$\lambdabar = \frac{\hbar}{p} = \frac{\hbar}{\sqrt{2mT}} = \frac{\hbar c}{\sqrt{2mc^2 T}} \approx \frac{197 Mev - fm}{\sqrt{2 \cdot 940 \cdot 8}} \approx 1.6\, fm$$

- Makes sense for nucleons to be inside a nucleus since the size is small.
- If it were electron with 8MeV, the wavelength is ~10fm, a whole lot larger than a nucleus.

Nuclear Properties: Sizes

- Sizes of subatomic particles are not as crisp clear as normal matter
  - Must be treated quantum mechanically via probability distributions or expectation values
  - Atoms: The average coordinate of the outermost electron and calculable
  - Nucleus: Not calculable and must be relied on experimental measurements
- For Rutherford scattering of low E projectile $r_0^{\min} = \frac{ZZ'e^2}{E}$
  - DCA provides an upper bound on the size of a nucleus
  - These result in $R_{Au}<3.2\times10^{-12}$cm or $R_{Ag}<2\times10^{-12}$cm

Nuclear Properties: Sizes

- Scatter very high E projectiles for head-on collisions
  - As E increases DCA becomes 0.
  - High E particles can probe deeper into nucleus

$$r_0^{\min} = \frac{ZZ'e^2}{E}$$

- Use electrons to probe the charge distribution (form factor) in a nucleus
  - What are the advantages of using electrons?
    - Electrons are fundamental particles ➔ No structure of their own
    - Electrons primarily interact through electromagnetic force
    - Electrons do not get affected by the nuclear force
  - The radius of charge distribution can be regarded as an effective size of the nucleus

Nuclear Properties: Sizes

- At relativistic energies the magnetic moment of electron also contributes to the scattering
  - Neville Mott formulated Rutherford scattering in QM and included the spin effects
  - R. Hofstadter, et al., discovered effect of spin, nature of nuclear (& proton) form factor in late 1950s
- Mott scattering x-sec (scattering of a point particle) is related to Rutherford x-sec: $\left(\frac{d\sigma}{d\Omega}\right)_{Mott} = 4\cos^2\frac{\theta}{2}\left(\frac{d\sigma}{d\Omega}\right)_{Rutherford}$
- Deviation from the distribution expected for point-scattering provides a measure of size (structure)

Nuclear Properties: Sizes

- Another way is to use the strong nuclear force using sufficiently energetic strongly interacting particles (π mesons or protons, etc)
  - What is the advantage of using these particles?
    - If energy is high, Coulomb interaction can be neglected
    - These particles readily interact with nuclei, getting "absorbed" into the nucleus
  - These interactions can be treated the same way as the light absorptions resulting in diffraction, similar to that of light passing through gratings or slits
- The size of a nucleus can be inferred from the diffraction pattern
- From all these phenomenological investigation provided the simple formula for the radius of the nucleus to its number of nucleons or atomic number, A:

$$R = r_0 A^{1/3} \approx 1.2 \times 10^{-13} A^{1/3} cm = 1.2 A^{1/3} \mathrm{fm}$$

How would you interpret this formula?

Nuclear Properties: Spins

- Both protons and neutrons are fermions with spins ½.
- Nucleons inside a nucleus can have orbital angular momentum
- In QM orbital angular momenta are integers
- Thus the total angular momenta of the nucleus are
  - Integers: if even number of nucleons in the nucleus
  - Half integers: if odd number of nucleons in the nucleus
- Interesting facts are
  - All nucleus with even number of p and n are spin 0.
  - Large nuclei have very small spins in their ground state
- Hypothesis: Nucleon spins in the nucleus are very strongly paired to minimize their overall effect

Nuclear Properties: Magnetic Dipole Moments

- Every charged particle has a magnetic dipole moment associated with its spin

$$\vec{\mu} = g\frac{e}{2mc}\vec{S}$$

- e, m and S are the charge, mass and the intrinsic spin of the charged particle
- Constant g is called Lande factor with its value:
  - $g = 2$: for a point like particle, such as the electron
  - $g \neq 2$: Particle possesses an anomalous magnetic moment, an indication of having a substructure

Nuclear Properties: Magnetic Dipole Moments

- For electrons, $\mu_e \sim \mu_B$, where μ_B is Bohr Magneton

$$\mu_B = \frac{e\hbar}{2m_e c} = 5.79 \times 10^{-11} \text{ MeV/T}$$

- For nucleons, magnetic dipole moment is measured in nuclear magneton, defined using proton mass

$$\mu_N = \frac{e\hbar}{2m_p c}$$

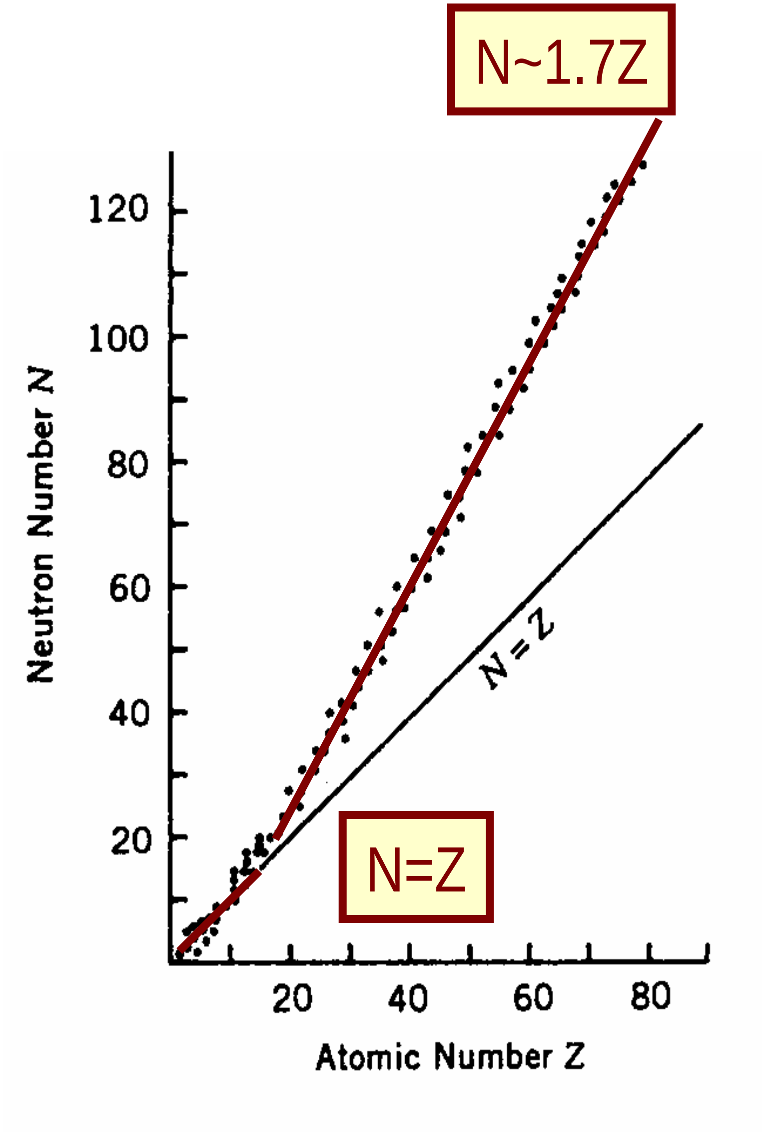
- Magnetic moment of proton and neutron are:

$$\mu_p \approx 2.79\mu_N \qquad \mu_n \approx -1.91\mu_N$$

- What important information do you get from these?
  - The Lande factors of the nucleons deviate significantly from 2.
    - Strong indication of substructure
  - An electrically neutral neutron has a significant magnetic moment
    - Must have extended charge distributions
- Measurements show that mangetic moment of nuclei lie $-3\mu_N \sim 10\mu_N$
  - Indication of strong pairing
  - Electrons cannot reside in nucleus

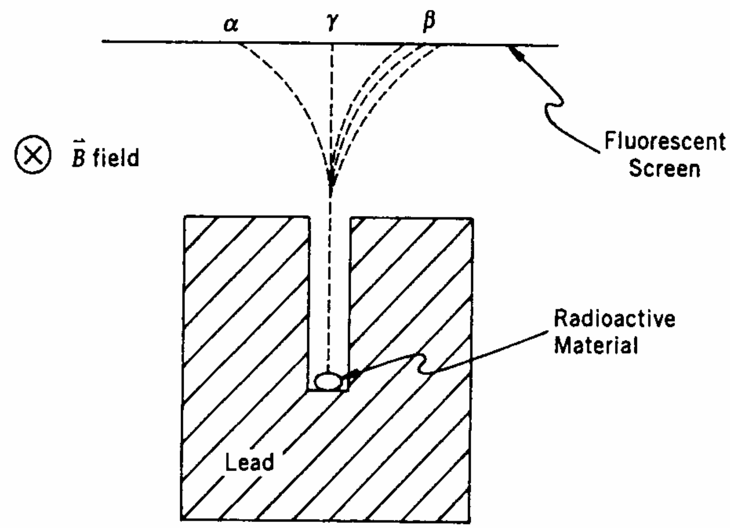
Nuclear Properties: Stability

- The number of protons and neutrons inside the stable nuclei are
  - A<40: Equal (N=Z)
  - A>40: N~1.7Z
  - Neutrons out number protons
  - Most are even-p + even–n
    - See table 2.1
  - Support strong pairing



Nuclear Properties: Instability

- H. Becquerel discovered natural radioactivity in 1896 via an accident
- Nuclear radio activity involves emission of three radiations: α, β, and γ
- These can be characterized using the device on the right
  - α: Nucleus of He
  - β: electrons
  - γ: photons



- What do you see from above?
  - α and β are charged particles while γ is neutral.
  - α is mono-energetic
  - β has broad spectrum
- What else do you see?

Assignments

1. Compute the mass density of a nucleus.
   - Pick two nucleus for this. I would like you guys to do different ones.
2. Compute the de Broglie wavelengths for
   - Protons in Fermilab's Tevatron Collider
   - Protons in CERN's Large Hadron Collider (LHC)
   - 500 GeV electrons in a Linear Collider
3. Compute the actual value of the nuclear magneton

- Due for these homework problems is next Wednesday, Feb. 9.