PHYS 3446 – Lecture #6

Monday, Feb. 7, 2005

Dr. Jae Yu

1. Nature of the Nuclear Force
   - Short Range Nature of the Nuclear Force
   - Shape of the Nuclear Potential
   - Yukawa Potential
   - Range of Yukawa Potential
2. Nuclear Models
   - Liquid Drop Model
   - Fermi-gas Model
   - Shell Model

Announcements

- How many of you did send an account request to Patrick at (mcquigan@cse.uta.edu)?
  - I was told that only 7 of you have contacted him for accounts.
  - There will be a linux and root tutorial session next Wednesday, Feb. 16, for your class projects.
  - You must make the request for the account by this Wednesday.
- First term exam
  - Date and time: 1:00 – 2:30pm, Monday, Feb. 21
  - Location: SH125
  - Covers: Appendix A + from CH1 to what we cover next Monday, Feb. 14

Nature of the Nuclear Force

- Scattering experiments helped to
  - Determine the properties of nuclei
  - More global information on the characteristics of the nuclear force
- From what we have learned, it is clear that there is no classical analog to nuclear force
  - Gravitational force is too weak to provide the binding
  - Can’t have an electromagnetic origin
    - Deuteron nucleus has one neutron and one proton
    - Coulomb force destabilizes the nucleus

Short-range Nature of the Nuclear Force

- Atomic structure is well explained by the electromagnetic interaction
  - Thus the range of nucleus cannot be much greater than the radius of the nucleus
  - Nuclear force range ~ $10^{-13} – 10^{-12}$cm
- Binding energy is constant per each nucleon, essentially independent of the size of the nucleus
  - If the nuclear force is long-ranged, like the Coulomb force
  - For A nucleons, there would be ½ A(A-1) pair-wise interactions
  - Thus, the BE which reflects all possible interactions among the nucleons would grow as a function of A

$$B \propto A(A-1) \quad \xrightarrow{\text{For large A}} \quad \frac{B}{A} \propto A$$

Short-range Nature of the Nuclear Force

- If the nuclear force is long-ranged and is independent of the presence of other nucleons, BE per nucleon will increase linearly with A
  - This is because long-range forces do not saturate
  - Since any single particle can interact with as many other particle as are available
  - ⇒Binding becomes tighter as the number of interacting objects increases
  - The size of the interacting region stays fairly constant
  - Atoms with large number of electrons have the sizes compatible to those with smaller number of electrons
- Long-rangeness of nuclear force is disputed by the experimental measurement that the BE/nucleon stays constant
  - Nuclear force must saturate
  - Any given nucleon can only interact with a finite number of nucleons in its vicinity

Short-range Nature of the Nuclear Force

- What does adding more nucleons to a nucleus do?
  - Only increases the size of the nucleus
    - Recall that $R \sim A^{1/3}$
  - The size of a nucleus grows slowly with A and keep the nuclear density constant

$\Rightarrow$Another supporting evidence of short-range nature of nuclear force

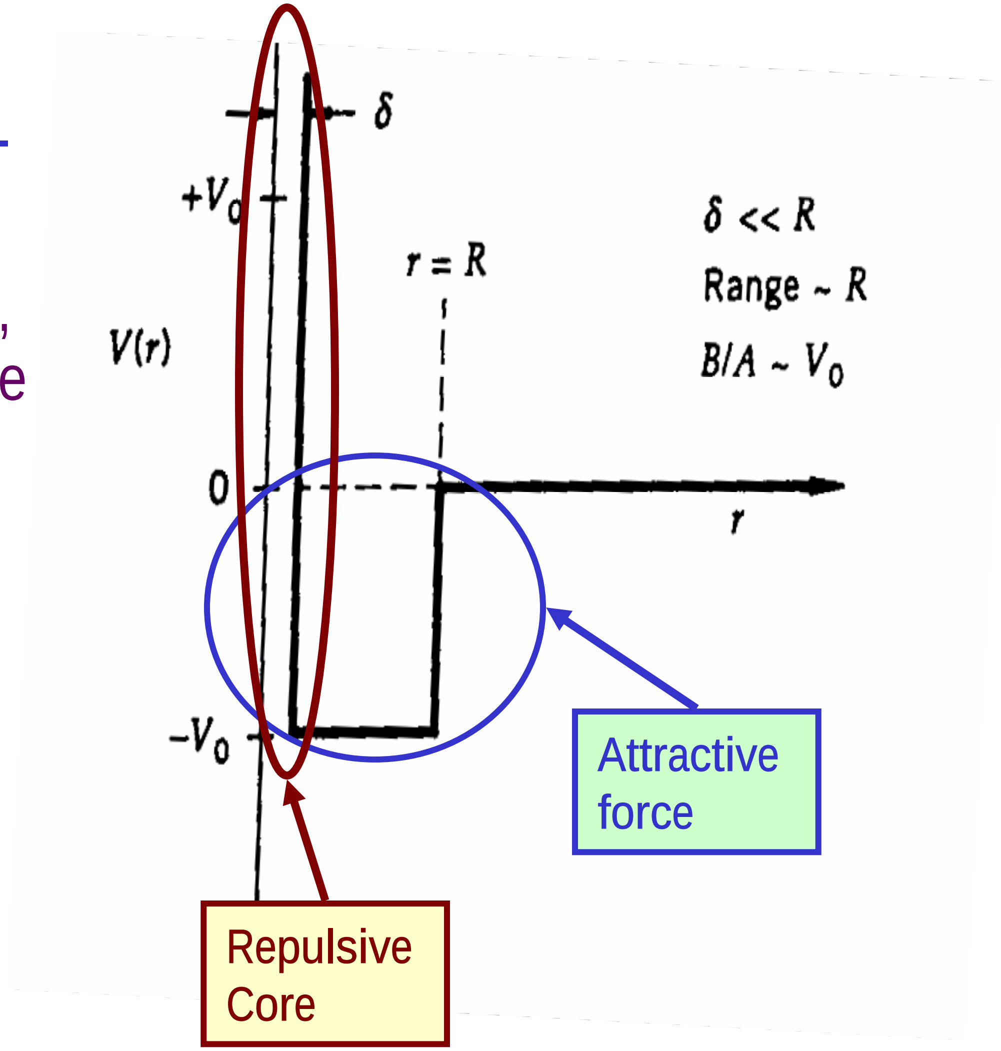
Shape of the Nuclear Potential

- Nuclear force keeps the nucleons within the nucleus.
  – What does this tell you about the nature of the nuclear force?

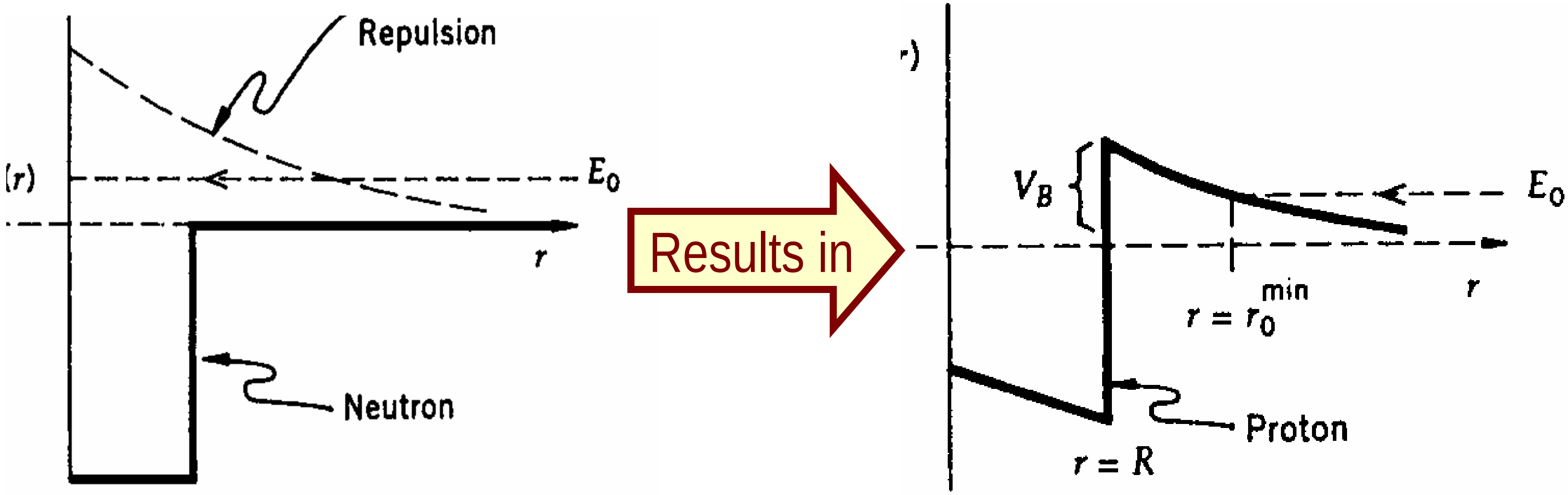
  $\Rightarrow$It must be attractive!!
- However, scattering Experiments with high energy revealed a repulsive core!!
  – Below a certain length scale, the nuclear force changes from attractive to repulsive.
  – What does this tell you?
    - Nucleons have a substructure….
- This feature is good, why?
  – If the nuclear force were attractive at all distances, the nucleus would collapse in on itself.

Shape of the Nuclear Potential

- We can turn these behaviors into a square-well potential
  - For low energy particles, the repulsive core can be ignored, why?
- This model is too simplistic, since there are too many abrupt changes in potential.
  - There would be additional effects by the Coulomb force



Nuclear Potential w/ Coulomb Corrections



- Classically an incident proton with total energy E_0 cannot be closer than $r=r_0$. Why?
  - For $R<r<r_0$, $V(r) > E_0$ and KE<0 ➔ Physically impossible
- What about a neutron?
  - Could penetrate into the nuclear center.
- Low energy scattering experiment did not provide the exact shape of the potential but the range and height of the potential
- The square-well shape provides a good phenomenological description of the nuclear force.

Nuclear Potential

- Description of nuclear potential using a square well shape suggests the basis of quantum theory with discrete energy levels and corresponding bound state as in atoms
  - Presence of such nuclear quantum states have been confirmed through
    - Scattering experiments
    - Studies of the energies emitted in nuclear radiation
- Studies of mirror nuclei and the scattering of protons and neutrons demonstrate
  - Once Coulomb effects have been corrected, the forces between two neutrons, two protons and a proton and a neutron are the same → Nuclear force is charge independent!!
  - Inferred as charge independence of nuclear force.

Nuclear Potential

- Strong nuclear force is independent of the electric charge carried by nucleons
  - Concept of strong isotopic-spin symmetry.
  - Under this symmetry, proton and neutron are the two different iso-spin state of the same particle, nucleon
  - If Coulomb effect can be turned off, protons and neutrons would be indistinguishable in their nuclear interactions
  - This is analogues to the indistinguishability of spin up and down states in the absence of a magnetic field!!
- Iso-spin symmetry!!!

Range of Nuclear Force

- EM force can be understood as a result of a photon exchange
  - Photon propagation is described by the Maxwell's equation
  - Photons propagate at the speed of light.
  - What does this tell you about the mass of the photon?
    - Massless
- Coulomb potential is expressed as $$V(r) \propto \frac{1}{r}$$
- What does this tell you about the range of the Coulomb force?
  - Long range. Why?

Yukawa Potential

- For massive particle exchanges, the potential takes the form

$$V(r) \propto \frac{e^{-\frac{mc}{\hbar}r}}{r}$$

  – What is the mass, m, in this expression?
    - Mass of the particle exchanged in the interaction, or the force mediator
- This form of potential is called Yukawa Potential
  – Formulated by Hideki Yukawa in 1934
- In the limit m →0, Yukawa potential turns into Coulomb potential

Ranges in Yukawa Potential

- From the form of the Yukawa potential

$$V(r) \propto \frac{e^{-\frac{mc}{\hbar}r}}{r} = \frac{e^{-r/\lambda\!\!\!^{-}}}{r}$$

- The range of the interaction is given by some characteristic value of r, Compton wavelength of the mediator with mass, m:

$$\lambda\!\!\!^{-} = \frac{\hbar}{mc}$$

- Thus once the mass of the mediator is known, range can be predicted or vise versa
- For nuclear force, range is about 1.2x10^{-13}cm, thus the mass of the mediator becomes:

$$mc^2 = \frac{\hbar c}{\lambda\!\!\!^{-}} \approx \frac{197\,MeV - fm}{1.2\,fm} \approx 164\,MeV$$

- This is close to the mass of a well known π meson (pion)

$$m_{\pi^+} = m_{\pi^-} = 139.6\,MeV/c^2; \quad m_{\pi^0} = 135\,MeV/c^2$$

- Thus, it was thought that π are the mediators of the nuclear force

Assignments

1. End of the chapter problems:
   - 2.2, 2.5, 2.9.
2. Draw Yukawa potential for particles with the following masses as a function of the radial distance, r, in the range of $10^{-14} – 10^{-20}$ m in a semi-logarithmic scale.
   - 130 MeV/c^2
   - 80 GeV/c^2
   - 115 GeV/c^2

- Due for these homework problems is next Monday, Feb. 16.