PHYS 3446 – Lecture #7

Wednesday, Feb. 9, 2005

Dr. Jae Yu

1. Nuclear Models
    - Liquid Drop Model
    - Fermi-gas Model
    - Shell Model
    - Collective Model
    - Super-deformed nuclei

Announcements

- How many of you did send an account request to Patrick at (mcguigan@cse.uta.edu)?
  - Three of you still have to contact him for accounts.
  - Account information will be given to you next Monday in class.
  - There will be a linux and root tutorial session next Wednesday, Feb. 16, for your class projects.
  - You MUST make the request for the account by today.
- First term exam
  - Date and time: 1:00 – 2:30pm, Monday, Feb. 21
  - Location: SH125
  - Covers: Appendix A + from CH1 to CH4
- Jim, James and Casey need to fill out a form for safety office ➔ Margie has the form. Please do so ASAP.

Ranges in Yukawa Potential

- From the form of the Yukawa potential

$$V(r) \propto \frac{e^{-\frac{mc}{\hbar}r}}{r} = \frac{e^{-r/\bar{\lambda}}}{r}$$

- The range of the interaction is given by some characteristic value of r, Compton wavelength of the mediator with mass, m: $\bar{\lambda} = \frac{\hbar}{mc}$
- Thus once the mass of the mediator is known, range can be predicted or vise versa
- For nuclear force, range is about 1.2x10^{-13}cm, thus the mass of the mediator becomes:

$$mc^2 = \frac{\hbar c}{\bar{\lambda}} \approx \frac{197\,MeV - fm}{1.2\,fm} \approx 164\,MeV$$

- This is close to the mass of a well known π meson (pion)

$$m_{\pi^+} = m_{\pi^-} = 139.6\,MeV/c^2; \quad m_{\pi^0} = 135\,MeV/c^2$$

- Thus, it was thought that π are the mediators of the nuclear force

Nuclear Models

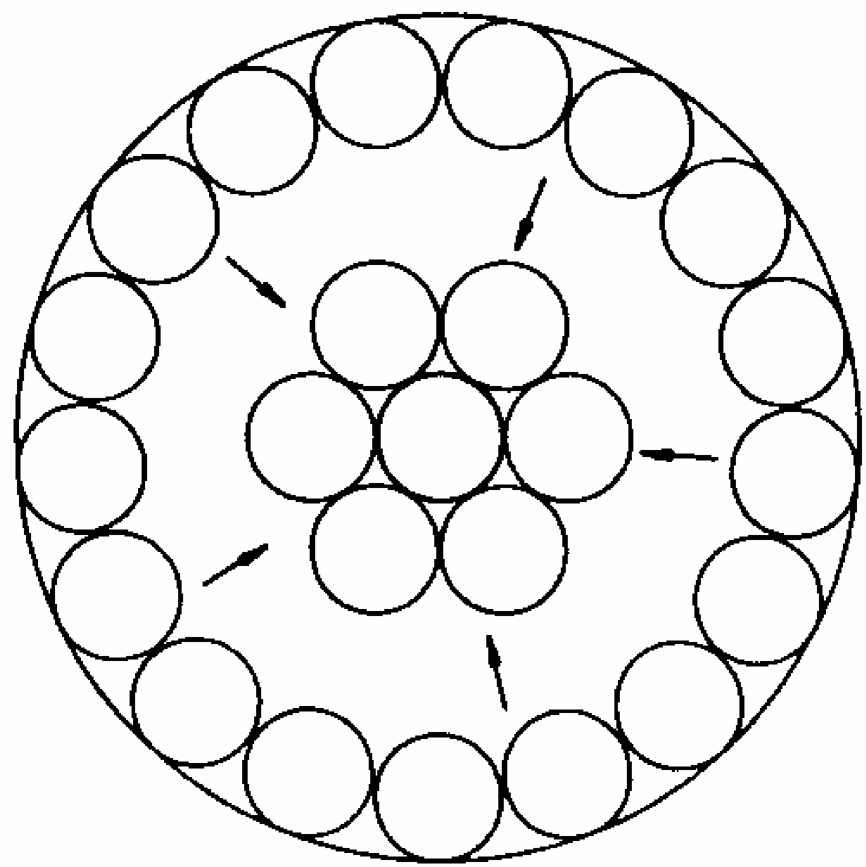
- Experiments demonstrated the dramatically different characteristics of nuclear forces to classical physics
- Quantification of nuclear forces and the structure of nucleus were not straightforward
  - Fundamentals of nuclear force were not well understood
- Several phenomenological models (not theories) that describe only limited cases of experimental findings
- Most the models assume central potential, just like Coulomb potential

Nuclear Models: Liquid Droplet Model

- An earliest phenomenological success in describing binding energy of a nucleus
- Nuclei are essentially spherical with the radii proportional to $A^{1/3}$.
  - Densities are independent of the number of nucleons
- Led to a model that envisions the nucleus as an incompressible liquid droplet
  - In this model, nucleons are equivalent to molecules
- Quantum properties of individual nucleons are ignored

Nuclear Models: Liquid Droplet Model

- Nucleus is imagined to consist of
  - A stable central core of nucleons where nuclear force is completely saturated
  - A surface layer of nucleons that are not bound tightly
    - This weaker binding at the surface decreases the effective binding energy per nucleon (B/A)
    - Provides an attraction of the surface nucleons towards the core as the surface tension to the liquid

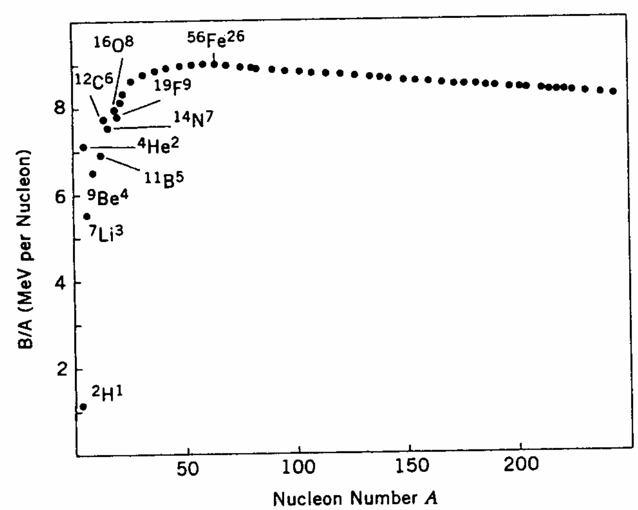


Liquid Droplet Model: Binding Energy

- If a constant BE per nucleon is attributed to the saturation of the nuclear force, a general form for the nuclear BE can be written as:

$$BE = -a_1 A + a_2 A^{2/3}$$

- What do you think each term does?
  - First term: volume energy for uniform saturated binding. Why?
  - Second term corrects for weaker surface tension
- This can explain the low BE/nucleon behavior of low A nuclei. How?
  - For low A nuclei, the proportion of the second term is larger.
  - Reflects relatively large surface nucleons than the core.



Liquid Droplet Model: Binding Energy

- Small decrease of BE for heavy nuclei can be understood as due to Coulomb repulsion
  - The electrostatic energies of protons have destabilizing effect
- Reflecting this effect, the empirical formula takes the correction

$$BE = -a_1 A + a_2 A^{2/3} + a_3 Z^2 A^{-1/3}$$

- All terms of this formula have classical origin.
- This formula does not take into account the fact that
  - The lighter nuclei with the equal number of protons and neutrons are stable or have a stronger binding
  - Natural abundance of even-even nuclei or paucity of odd-odd nuclei
- These could mainly arise from quantum effect of spins.

Liquid Droplet Model: Binding Energy

- Additional corrections to compensate the deficiency, give corrections to the empirical formula

$$BE = -a_1 A + a_2 A^{2/3} + a_3 Z^2 A^{-1/3} + a_4 \frac{(N-Z)^2}{A} \pm a_5 A^{-3/4}$$

  - The parameters are assumed to be positive
  - The forth term reflects N=Z stability
  - The last term
    - Positive sign is chosen for odd-odd nuclei, reflecting instability
    - Negative sign is chosen for even-even nuclei
    - For odd-A nuclei, a_5 is chosen to be 0.

Liquid Droplet Model: Binding Energy

- The parameters are determined by fitting experimentally observed BE for a wide range of nuclei:

$$a_1 \approx 15.6MeV \quad a_2 \approx 16.8MeV \quad a_3 \approx 0.72MeV$$
$$a_4 \approx 23.3MeV \qquad a_5 \approx 34MeV;$$

- Now we can write an empirical formula for masses of nuclei

$$M(A,Z) = (A-Z)m_n + Zm_p + \frac{BE}{c^2} = (A-Z)m_n + Zm_p$$
$$-\frac{a_1}{c^2}A + \frac{a_2}{c^2}A^{2/3} + \frac{a_3}{c^2}Z^2A^{-1/3} + \frac{a_4}{c^2}\frac{(N-Z)^2}{A} \pm \frac{a_5}{c^2}A^{-3/4}$$

- This is Bethe-Weizsacker semi-empirical mass formula
  – Used to predict stability and masses of unknown nuclei of arbitrary A and Z

Nuclear Models: Fermi Gas Model

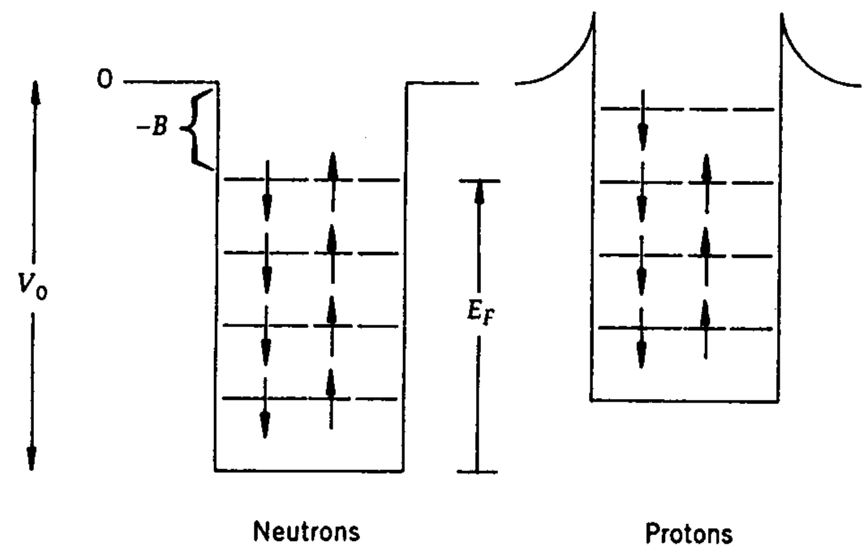
- Early attempt to incorporate quantum effects
- Assumes nucleus as a gas of free protons and neutrons confined to the nuclear volume
  - The nucleons occupy quantized (discrete) energy levels
  - Nucleons are moving inside a spherically symmetric well with the range determined by the radius of the nucleus
  - Depth of the well is adjusted to obtain correct binding energy
- Protons carry electric charge → Senses slightly different potential than neutrons

Nuclear Models: Fermi Gas Model

- Nucleons are Fermions (spin ½ particles) ➔ Obey Pauli exclusion principle
  - Any given energy level can be occupied by at most two identical nucleons – opposite spin projections
- For a greater stability, the energy levels fill up from the bottom
- Fermi level: Highest, fully occupied energy level (E_F)
- Binding energies are given
  - No Fermions above E_F: BE of the last nucleon= E_F
  - The level occupied by Fermion reflects the BE of the last nucleon

Nuclear Models: Fermi Gas Model

- Experimental observations demonstrates BE is charge independent
- If well depth is the same, BE for the last nucleon would be charge dependent for heavy nuclei (Why?)
- E_F must be the same for protons and neutrons. How do we make this happen?
  - Protons for heavy nuclei moves in to shallower potential wells
- What happens if this weren't the case?
  - Nucleus is unstable.
  - All neutrons at higher energy levels would undergo a β-decay and transition to lower proton levels



Fermi Gas Model: E_F vs n_F

- Fermi momentum: $p_F = \sqrt{2mE_F}$
- Volume for momentum space up to Fermi level $V_{p_F} = \frac{4\pi}{3} p_F^3$
- Total volume for the states (kinematic phase space)
  – Proportional to the total number of quantum states in the system

$$V_{TOT} = V \cdot V_{p_F} = \frac{4\pi}{3} r_0^3 A \cdot \frac{4\pi}{3} p_F^3 = \left(\frac{4\pi}{3}\right)^2 A \left(r_0 p_F\right)^3$$

- Using Heisenberg's uncertainty principle: $\Delta x \Delta p \geq \hbar/2$
- The minimum volume associated with a physical system becomes $V_{state} = \left(2\pi\hbar\right)^3$
- n_F that can fill up to E_F is

$$n_F = 2\frac{V_{TOT}}{\left(2\pi\hbar\right)^3} = \frac{2}{\left(2\pi\hbar\right)^3}\left(\frac{4\pi}{3}\right)^2 A\left(r_0 p_F\right)^3 = \frac{4}{9\pi} A\left(\frac{r_0 p_F}{\hbar}\right)^3$$

Fermi Gas Model: E_F vs n_F

- Let's consider a nucleus with N=Z=A/2 and assume that all states up to fermi level are filled

$$N = Z = \frac{A}{2} = \frac{4}{9\pi} A \left(\frac{r_0 p_F}{\hbar} \right)^3 \quad \text{or} \quad p_F = \frac{\hbar}{r_0} \left(\frac{9\pi}{8} \right)^{1/3}$$

- What do you see about p_F above?
  - Fermi momentum is constant, independent of the number of nucleons

$$E_F = \frac{p_F^2}{2m} = \frac{1}{2m} \left(\frac{\hbar}{r_0} \right)^2 \left(\frac{9\pi}{8} \right)^{2/3} \approx \frac{2.32}{2mc^2} \left(\frac{\hbar c}{r_0} \right)^2 \approx \frac{2.32}{2 \cdot 940} \left(\frac{197 MeV - fm}{1.2\, fm} \right) \approx 33 MeV$$

- Using the average BE of -8MeV, the depth of potential well (V_0) is ~40MeV
  - Consistent with other findings
- This model is a natural way of accounting for a_4 term in Bethe-Weizsacker mass formula

Nuclear Models: Shell Model

- Exploit the success of atomic model
  - Uses orbital structure of nucleons
  - Electron energy levels are quantized
  - Limited number of electrons in each level based on available spin and angular momentum configurations
    - For n^{th} energy level, ℓ angular momentum (ℓ<n), one expects a total of $2(2\ell+1)$ possible degenerate states for electrons
- Quantum numbers of individual nucleons are taken into account to affect fine structure of spectra
- Magic numbers in nuclei just like inert atoms
  - Atoms: Z=2, 10, 18, 36, 54
  - Nuclei: N=2, 8, 20, 28, 50, 82, and 126 and Z=2, 8, 20, 28, 50, and 82
  - Magic Nuclei: Nuclei with either N or Z a magic number → Stable
  - Doubly magic nuclei: Nuclei with both N and Z magic numbers → Particularly stable
- Explains well the stability of nucleus

Shell Model: Various Potential Shapes

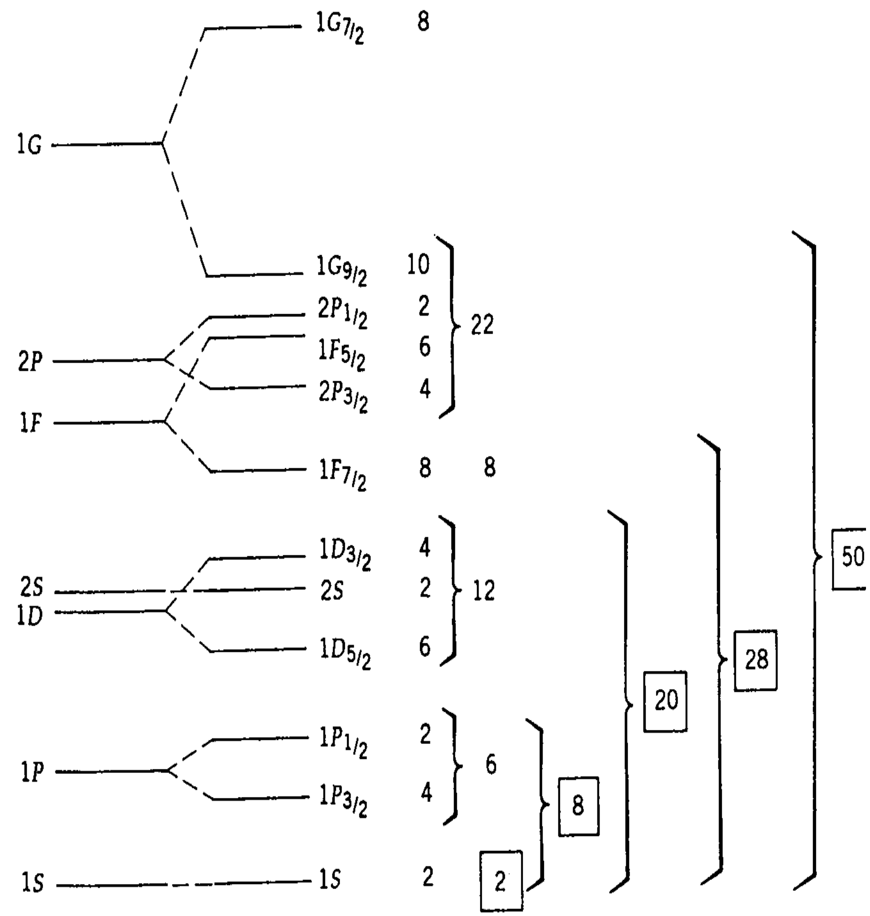
- To solve equation of motion in quantum mechanics, Schrodinger equation, one must know the shape of the potential
  - Details of nuclear potential not well known
- A few models of potential tried out
  - Infinite square well: Each shell can contain up to $2(2\ell+1)$ nucleons
    - Can predict 2, 8, 18, 32 and 50 but no other magic numbers
  - Three dimensional harmonic oscillator: $V(r) = \frac{1}{2} m\varpi^2 r^2$
    - Can predict 2, 8, 20 and 40 ➔ Not all magic numbers are predicted

Shell Model: Spin-Orbit Potential

- Central potential could not reproduce all magic numbers
- In 1940, Mayer and Jesen proposed a central potential + strong spin-orbit interaction w/

$$V_{TOT} = V(r) - f(r)\vec{L}\cdot\vec{S}$$

  - $f(r)$ is an arbitrary function of radial coordinates and chosen to fit the data
- The spin-orbit interaction with the properly chosen f(r), a finite square well can split
- Reproduces all the desired magic numbers



Spectroscopic notation: n L j

Orbit number

Orbital angular momentum

Projection of total momentum

Assignments

1. End of the chapter problems: 3.2

- Due for these homework problems is next Wednesday, Feb. 18.