1. Nuclear Models
   • Shell Model Predictions
   • Collective Model
   • Super-deformed nuclei

2. Nuclear Radiation
   • Alpha Decay
   • Beta Decay
   • Gamma Decay
Announcements

- All of you have been given accounts at a DPCC computer
  - Please pick up your account sheet and bring it to Wednesday tutorial
- Tutorial Wednesday
  - Takes place in SH203
  - Gather in SH200 first and move to the next door
  - Your Mav-express cards will allow you access to SH203 for your projects after today
- Quiz results
  - Top score: 67
  - Average: 38.5
- First term exam
  - Date and time: 1:00 – 2:30pm, Monday, Feb. 21
  - Location: SH125
  - Covers: Appendix A (special relativity) + CH1 – CH4.4
Nuclear Models

• **Liquid Droplet Model:**
  - Ignore individual nucleon quantum properties
  - Assume spherical shape of nuclei
  - A core with saturated nuclear force + loosely bound surface nucleons
  - Describes BE of light nuclei reasonably well

• **Fermi Gas Model:**
  - Assumes nucleus as a gas of free protons and neutrons confined to the nuclear volume
  - Takes into account quantum effects w/ discrete nucleon energy levels
  - Accounts for strong spin pairing of nucleons

• **Shell Model**
  - Takes into account individual nucleon quantum properties
  - Needed to postulate a few potential shapes for nucleus
  - The model using spin-orbit potential seems reproduce all the desired magic numbers
Predictions of the Shell Model

• Spin-Parity of large number of odd-A nuclei predicted well
  – Nucleons obey Pauli exclusion principle \(\Rightarrow\) Fill up ground state energy levels in pairs
  – Ground state of all even-even nuclei have zero total angular momentum

• Single particle shell model cannot predict odd-odd nuclei spins
  – No prescription for how to combine the unpaired proton and neutron spins
Predictions of the Shell Model

• Magnetic Moment of neutron and proton are
  \[ \mu_p \approx 2.79 \mu_N \quad \mu_n \approx -1.91 \mu_N \]

• Intrinsic magnetic moment of unpaired nucleon to contribute to total magnetic moment of nuclei
  – Deuteron
    \[ \mu_D = \mu_p + \mu_n = 2.79 \mu_N - 1.91 \mu_N = 0.88 \mu_N \]
    • Measured value is \( \mu_D = 0.86 \mu_N \)
  – For Boron \(^{10}\text{B}^5\), the neutrons and protons have the same level structure: \((1S_{1/2})^2(1P_{3/2})^3\), leaving one of each unpaired and one proton in angular momentum \(l=1\) state
    \[ \mu_B = \mu_p + \mu_n + \mu_{\text{orbit}} = 2.79 \mu_N - 1.91 \mu_N + \mu_N = 1.88 \mu_N \]
    • Measured value is \( \mu_B = 1.80 \mu_N \)

• Does not work well with heavy nuclei
Collective Model

• For heavy nuclei, shell model predictions do not agree with experimental measurements
  – Especially in magnetic dipole moments
• Measured values of quadrupole moments for closed shells differ significantly with experiments
  – Some nuclei’s large quadrupole moments suggests significant nonspherical shapes
  – The assumption of rotational symmetry in shell model does not seem quite right
• These deficiencies are somewhat covered through the reconciliation of liquid drop model with Shell model
  – Bohr, Mottelson and Rainwater’s collective model, 1953
Collective Model

• Assumption
  – Nucleus consists of hard core of nucleons in filled shells
  – Outer valence nucleons behave like the surface molecules in a liquid drop
  – Non-sphericity of central core caused by the surface motion of the valence nucleon
• Thus, in collective model, the potential is a shell model with a spherically asymmetric potential
  – Aspherical nuclei can produce additional energy levels upon rotation while spherical ones cannot
• Important predictions of collective model:
  – Existence of rotational and vibrational energy levels in nuclei
  – Accommodate decrease of spacing between first excite state and the ground level for even-even nuclei as A increases, since moment of inertia increases with A
  – Spacing is largest for closed shell nuclei, since they tend to be spherical
Super-deformed Nuclei

- Nuclei tend to have relatively small intrinsic spins
- Particularly stable nuclei predicted for A between 150 and 190 with spheroidal character
  - Semi-major axis about a factor of 2 larger than semi-minor
- Heavy ion collisions in late 1980s produced super-deformed nuclei with angular momentum of $\sim 60\hbar$
- The energy level spacings of these observed through photon radiation seem to be essentially fixed
- Different nuclei seem to have identical emissions as they spin down
- Problem with collective model and understanding of strong pairing of nucleon binding energy
- Understanding nuclear structure still in progress
Nuclear Radiation: Alpha Decay

• Represents the disintegration of a parent nucleus to a daughter through an emission of a He nucleus

• Reaction equation is

\[ ^A_XZ \rightarrow ^{A-4}_YZ^{-2} + ^4He^2 \]

\( \alpha \)-decay is a spontaneous fission of the parent nucleus into two daughters of highly asymmetric masses

• Assuming parent at rest, from the energy conservation

\[ M_Pc^2 = M_Dc^2 + T_D + M_\alpha c^2 + T_\alpha \]

• Can be re-written as

\[ T_D + T_\alpha = (M_P - M_D - M_\alpha )c^2 = \Delta Mc^2 \]
Nuclear Radiation: Alpha Decay

- Since electron masses cancel, we could use atomic mass expression

\[ T_D + T_\alpha = \left( M(A, Z) - M(A-4, Z-2) - M(4, 2) \right) c^2 \equiv Q \]

- This is the definition of the disintegration energy or Q-value
  - Difference of rest masses of the initial and final states
  - Q value is equal to the sum of the final state kinetic energies

- For non-relativistic particles, KE are

\[ T_D = \frac{1}{2} M_D v_D^2 \quad T_\alpha = \frac{1}{2} M_\alpha v_\alpha^2 \]
Nuclear Radiation: Alpha Decay

- Since the parent is at rest, from the momentum conservation
  \[ M_D v_D = M_\alpha v_\alpha \quad v_D = \frac{M_\alpha}{M_D} v_\alpha \]

- If \( M_D \gg M_\alpha, \ v_D \ll v_\alpha \), then \( T_D \ll T_\alpha \)

- We can write the relationship of KE and Q-value as
  \[ T_D + T_\alpha = \frac{1}{2} M_D v_D^2 + \frac{1}{2} M_\alpha v_\alpha^2 = \frac{1}{2} M_D \left( \frac{M_\alpha}{M_D} v_\alpha \right)^2 + \frac{1}{2} M_\alpha v_\alpha^2 \]

\[ T_D + T_\alpha = T_\alpha \frac{M_\alpha + M_D}{M_D} \]

\[ T_\alpha = \frac{M_D}{M_\alpha + M_D} Q \]

- \( T_\alpha \) is unique for the given nuclei

- Direct consequence of 2-body decay of a rest parent
Nuclear Radiation: Alpha Decay

- KE of the emitted $\alpha$ must be positive
- Thus for an $\alpha$-decay to occur, it must be an exothermic process $\Delta M \geq 0$, $Q \geq 0$
- For massive nuclei, the daughter’s KE is

$$T_D = Q - T_\alpha = \frac{M_\alpha}{M_\alpha + M_D} Q = \frac{M_\alpha}{M_D} T_\alpha \ll T_\alpha$$

- Since $M_\alpha / M_D \approx 4 / (A-4)$, we obtain

$$T_\alpha \approx \frac{A-4}{4} Q \quad T_D \approx \frac{4}{A} Q$$
Nuclear Radiation: Alpha Decay

- **Most energetic \( \alpha \)-particles produced alone**
  - Parent nucleus decays to the ground state of a daughter and produces an \( \alpha \)-particle whose KE is the entire Q value

- **Less energetic ones accompany photons – mostly delayed...**
  - Indicates quantum energy levels
  - Parent decays to an excited state of the daughter after emitting an \( \alpha \)
    \[
    A^XZ \rightarrow A^{4-4}Y^{*Z-2} + \frac{4}{2}He^2
    \]
  - Daughter then subsequently de-excite by emitting a photon
    \[
    A^{4-4}Y^{*Z-2} \rightarrow A^{4-4}Y^{Z-2} + \gamma
    \]
  - Difference in the two Q values correspond to photon energy
Nuclear Radiation: $\alpha$-Decay Example

- $^{240}\text{Pu}^{94}$ decay reaction is

  \[ ^{240}\text{Pu}^{94} \rightarrow ^{236}\text{U}^{92} + ^{4}\text{He}^{2} \]

- $\alpha$ particles observed with 5.17MeV and 5.12 MeV

- Since $Q = \frac{A}{A-4} T_\alpha$

- We obtain the two Q-values

  \[ Q_1 \approx \frac{240}{236} 5.17\text{MeV} = 5.26\text{MeV} \quad Q_2 \approx \frac{240}{236} 5.12\text{MeV} = 5.21\text{MeV} \]

- Which yields photon energy of

  \[ E_\gamma = \Delta Q = Q_1 - Q_2 = 0.05\text{MeV} \]

- Consistent with experimental measurement, 45KeV

- Indicates the energy level spacing of order 100KeV for nuclei

  - Compares to order 1eV spacing in atomic levels
Nuclear Radiation: $\beta$-Decays

- **Three kinds of $\beta$-decays**
  - **Electron emission**
    - Nucleus with large $N_n$
    - Proton number increases by one
  
  \[ A^Z X^Z \rightarrow A^Z Y^{Z+1} + e^- \]

  - **Positron emission**
    - Nucleus with many protons
    - Proton number decreases by one
  
  \[ A^Z X^Z \rightarrow A^Z Y^{Z-1} + e^+ \]

  - **Electron capture**
    - Nucleus with many protons
    - Absorbs a K-shell atomic electron
    - Proton number decreases by one
    - Causes cascade x-ray emission from the transition of remaining atomic electrons
  
  \[ A^Z X^Z + e^- \rightarrow A^Z Y^{Z-1} \]

- **For $\beta$-decay**: $\Delta A=0$ and $|\Delta Z|=1$
Nuclear Radiation: $\beta$-Decays

• Initially assumed to be 2-body decay

• From the conservation of energy

$$E_X = E_Y + E_e^- = E_Y + T_e + m_e c^2$$

• Since the lighter electron carries most the energy

$$T_e = (E_X - E_Y - m_e c^2) = (m_X - m_Y - m_e) c^2 - T_Y = Q - T_Y \approx Q$$

• Will result in a unique values as in $\alpha$-decay.

• In reality, electrons emitted with continuous E spectrum with an end-point given by the formula above

• Energy conservation is violated!!!!
Nuclear Radiation: $\beta$-Decays

- Angular momentum is also in trouble
- In $\beta$-decays total number of nucleons is conserved
- Electrons are fermions with spin $\hbar/2$
- Independent of any changes of an integer orbital angular momentum, the total angular momentum cannot be conserved
- Angular momentum conservation is violated!!!
Nuclear Radiation: $\beta$-Decays

- Pauli proposed an additional particle emitted in $\beta$-decays
  - No one saw this particle in experiment
    - Difficult to detect
  - Charge is conserved in $b$-decay
    - Electrically neutral
  - Maximum energy of electrons is the Q values
    - Massless
  - Must conserve the angular momentum
    - Must be a fermion with spin $\hbar/2$
- This particle is called neutrino (by Feynman) and expressed as $\nu$
Nuclear Radiation: Neutrinos

• Have anti-neutrinos $\bar{\nu}$, just like other particles

• Neutrinos and anti-neutrinos are distinguished by magnetic moment
  – Helicity is used to distinguish them $H \propto \vec{p} \cdot \vec{s}$
    • Left-handed (spin and momentum opposite direction) anti-electron-neutrinos are produced in $\beta$-decays
    • Right-handed electron-neutrinos are produced in positron emission

  – $e^-$ is a particle and $e^+$ is an anti-particle
  – $\nu_e$ is a particle and $\bar{\nu}_e$ is an anti-particle
Assignments

1. End of the chapter problems: 3.2
2. Derive the following equations:
   • Eq. 4.8 starting from conservation of energy
   • Eq. 4.11 both the formula
   • Due for these homework problems is next Wednesday, Feb. 23.