PHYS 3446 – Lecture #8

Monday, Feb. 14, 2005

Dr. Jae Yu

1. Nuclear Models
   - Shell Model Predictions
   - Collective Model
   - Super-deformed nuclei
2. Nuclear Radiation
   - Alpha Decay
   - Beta Decay
   - Gamma Decay

Announcements

- All of you have been given accounts at a DPCC computer
  - Please pick up your account sheet and bring it to Wednesday tutorial
- Tutorial Wednesday
  - Takes place in SH203
  - Gather in SH200 first and move to the next door
  - Your Mav-express cards will allow you access to SH203 for your projects after today
- Quiz results
  - Top score: 67
  - Average: 38.5
- First term exam
  - Date and time: 1:00 – 2:30pm, Monday, Feb. 21
  - Location: SH125
  - Covers: Appendix A (special relativity) + CH1 – CH4.4

Nuclear Models

- Liquid Droplet Model:
  - Ignore individual nucleon quantum properties
  - Assume spherical shape of nuclei
  - A core with saturated nuclear force + loosely bound surface nucleons
  - Describes BE of light nuclei reasonably well
- Fermi Gas Model:
  - Assumes nucleus as a gas of free protons and neutrons confined to the nuclear volume
  - Takes into account quantum effects w/ discrete nucleon energy levels
  - Accounts for strong spin pairing of nucleons
- Shell Model
  - Takes into account individual nucleon quantum properties
  - Needed to postulate a few potential shapes for nucleus
  - The model using spin-orbit potential seems reproduce all the desired magic numbers

Predictions of the Shell Model

- Spin-Parity of large number of odd-A nuclei predicted well
  - Nucleons obey Pauli exclusion principle → Fill up ground state energy levels in pairs
  - Ground state of all even-even nuclei have zero total angular momentum
- Single particle shell model cannot predict odd-odd nuclei spins
  - No prescription for how to combine the unpaired proton and neutron spins

Predictions of the Shell Model

- Magnetic Moment of neutron and proton are

$$\mu_p \approx 2.79\mu_N \qquad \mu_n \approx -1.91\mu_N$$

- Intrinsic magnetic moment of unpaired nucleon to contribute to total magnetic moment of nuclei
  - Deuteron

$$\mu_D = \mu_p + \mu_n = 2.79\,\mu_N - 1.91\mu_N = 0.88\,\mu_N$$

    - Measured value is $\mu_D = 0.86\,\mu_N$
  - For Boron ($^{10}B^5$) , the neutrons and protons have the same level structure: $(1S_{1/2})^2(1P_{3/2})^3$, leaving one of each unpaired and one proton in angular momentum l=1 state ➔ $\mu = \frac{e\hbar}{2m_N c} l = \mu_N$

$$\mu_B = \mu_p + \mu_n + \mu_{orbit} = 2.79\,\mu_N - 1.91\mu_N + \mu_N = 1.88\,\mu_N$$

    - Measured value is $\mu_B = 1.80\,\mu_N$
- Does not work well with heavy nuclei

Collective Model

- For heavy nuclei, shell model predictions do not agree with experimental measurements
  - Especially in magnetic dipole moments
- Measured values of quadrupole moments for closed shells differ significantly with experiments
  - Some nuclei's large quadrupole moments suggests significant nonspherical shapes
  - The assumption of rotational symmetry in shell model does not seem quite right
- These deficiencies are somewhat covered through the reconciliation of liquid drop model with Shell model
  - Bohr, Mottelson and Rainwater's collective model, 1953

Collective Model

- Assumption
  - Nucleus consists of hard core of nucleons in filled shells
  - Outer valence nucleons behave like the surface molecules in a liquid drop
  - Non-sphericity of central core caused by the surface motion of the valence nucleon
- Thus, in collective model, the potential is a shell model with a spherically asymmetric potential
  - Aspherical nuclei can produce additional energy levels upon rotation while spherical ones cannot
- Important predictions of collective model:
  - Existence of rotational and vibrational energy levels in nuclei
  - Accommodate decrease of spacing between first excite state and the ground level for even-even nuclei as A increases, since moment of inertia increases with A
  - Spacing is largest for closed shell nuclei, since they tend to be spherical

Super-deformed Nuclei

- Nuclei tend to have relatively small intrinsic spins
- Particularly stable nuclei predicted for A between 150 and 190 with spheroidal character
  - Semi-major axis about a factor of 2 larger than semi-minor
- Heavy ion collisions in late 1980s produced super-deformed nuclei with angular momentum of $\sim 60\hbar$
- The energy level spacings of these observed through photon radiation seem to be essentially fixed
- Different nuclei seem to have identical emissions as they spin down
- Problem with collective model and understanding of strong pairing of nucleon binding energy
- Understanding nuclear structure still in progress

Nuclear Radiation: Alpha Decay

- Represents the disintegration of a parent nucleus to a daughter through an emission of a He nucleus
- Reaction equation is

$$^{A}X^{Z} \rightarrow {}^{A-4}Y^{Z-2} + {}^{4}He^{2}$$

- α-decay is a spontaneous fission of the parent nucleus into two daughters of highly asymmetric masses
- Assuming parent at rest, from the energy conservation

$$M_P c^2 = M_D c^2 + T_D + M_\alpha c^2 + T_\alpha$$

- Can be re-written as

$$T_D + T_\alpha = \left(M_P - M_D - M_\alpha\right)c^2 = \Delta M c^2$$

Nuclear Radiation: Alpha Decay

- Since electron masses cancel, we could use atomic mass expression

$$T_D + T_\alpha = \left(M(A,Z) - M(A-4, Z-2) - M(4,2)\right)c^2 \equiv Q$$

- This is the definition of the disintegration energy or Q-value
  - Difference of rest masses of the initial and final states
  - Q value is equal to the sum of the final state kinetic energies
- For non-relativistic particles, KE are

$$T_D = \frac{1}{2}M_D v_D^2 \qquad T_\alpha = \frac{1}{2}M_\alpha v_\alpha^2$$

Nuclear Radiation: Alpha Decay

- Since the parent is at rest, from the momentum conservation

$$M_D v_D = M_\alpha v_\alpha \qquad v_D = \frac{M_\alpha}{M_D} v_\alpha$$

- If $M_D \gg M_\alpha$, $v_D \ll v_\alpha$, then $T_D \ll T_\alpha$
- We can write the relationship of KE and Q-value as

$$T_D + T_\alpha = \frac{1}{2} M_D v_D^2 + \frac{1}{2} M_\alpha v_\alpha^2 = \frac{1}{2} M_D \left(\frac{M_\alpha}{M_D} v_\alpha \right)^2 + \frac{1}{2} M_\alpha v_\alpha^2$$

$$T_D + T_\alpha = T_\alpha \frac{M_\alpha + M_D}{M_D} \qquad \boxed{T_\alpha = \frac{M_D}{M_\alpha + M_D} Q}$$

- T_α is unique for the given nuclei
- Direct consequence of 2-body decay of a rest parent

Nuclear Radiation: Alpha Decay

- KE of the emitted α must be positive
- Thus for an α-decay to occur, it must be an exorthermic process $\Delta M \geq 0, \ Q \geq 0$
- For massive nuclei, the daughter's KE is

$$T_D = Q - T_\alpha = \frac{M_\alpha}{M_\alpha + M_D} Q = \frac{M_\alpha}{M_D} T_\alpha \ll T_\alpha$$

- Since $M_\alpha / M_D \approx 4/(A-4)$, we obtain

$$T_\alpha \approx \frac{A-4}{4} Q \qquad T_D \approx \frac{4}{A} Q$$

Nuclear Radiation: Alpha Decay

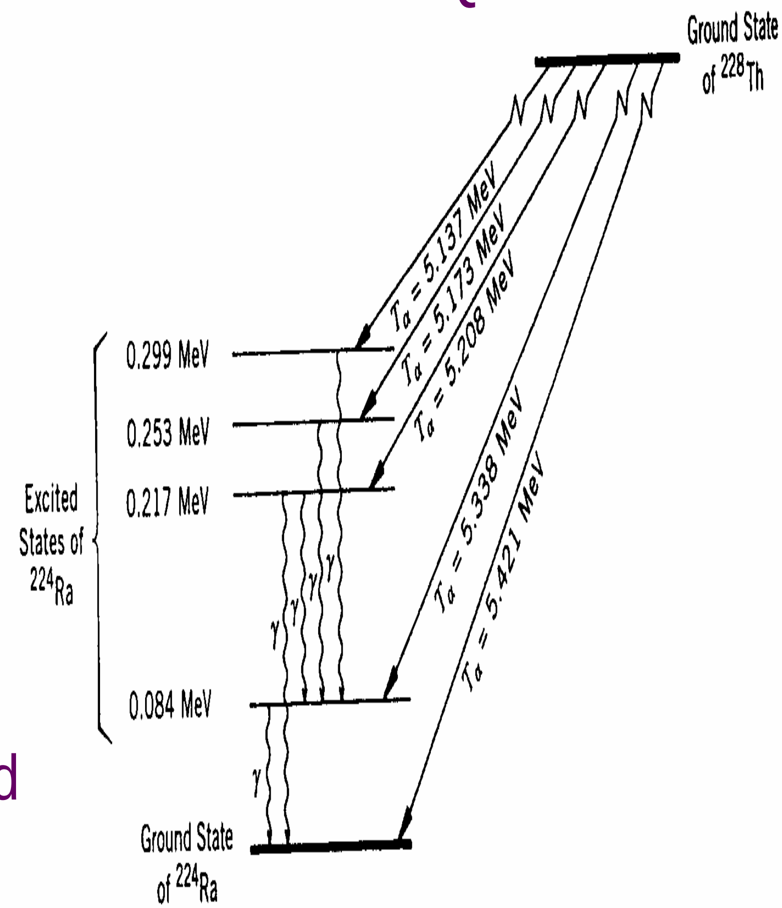
- Most energetic α-particles produced alone
  - Parent nucleus decays to the ground state of a daughter and produces an α-particle whose KE is the entire Q value
- Less energetic ones accompany photons – mostly delayed…
  - Indicates quantum energy levels
  - Parent decays to an excited state of the daughter after emitting an α

$${}^{A}X^{Z} \rightarrow {}^{A-4}Y^{*Z-2} + {}^{4}He^{2}$$

  - Daughter then subsequently de-excite by emitting a photon

$${}^{A-4}Y^{*Z-2} \rightarrow {}^{A-4}Y^{Z-2} + \gamma$$

  - Difference in the two Q values correspond to photon energy



Nuclear Radiation: α-Decay Example

- ^{240}Pu94 decay reaction is

$$^{240}Pu^{94} \rightarrow ^{236}U^{92} + ^{4}He^{2}$$

- α particles observed with 5.17MeV and 5.12 MeV
- Since $Q = \frac{A}{A-4} T_\alpha$
- We obtain the two Q-values

$$Q_1 \approx \frac{240}{236} 5.17MeV = 5.26MeV \qquad Q_2 \approx \frac{240}{236} 5.12MeV = 5.21MeV$$

- Which yields photon energy of $E_\gamma = \Delta Q = Q_1 - Q_2 = 0.05MeV$
- Consistent with experimental measurement, 45KeV
- Indicates the energy level spacing of order 100KeV for nuclei
  - Compares to order 1eV spacing in atomic levels

Nuclear Radiation: β-Decays

- Three kinds of β-decays
  - Electron emission
    - Nucleus with large N_n
    - Proton number increases by one

$$^{A}X^{Z} \rightarrow {}^{A}Y^{Z+1} + e^{-}$$

  - Positron emission
    - Nucleus with many protons
    - Proton number decreases by one

$$^{A}X^{Z} \rightarrow {}^{A}Y^{Z-1} + e^{+}$$

  - Electron capture
    - Nucleus with many protons
    - Absorbs a K-shell atomic electron
    - Proton number decreases by one
    - Causes cascade x-ray emission from the transition of remaining atomic electrons

$$^{A}X^{Z} + e^{-} \rightarrow {}^{A}Y^{Z-1}$$

- For b-decay: ΔA=0 and |ΔZ|=1

Nuclear Radiation: β-Decays

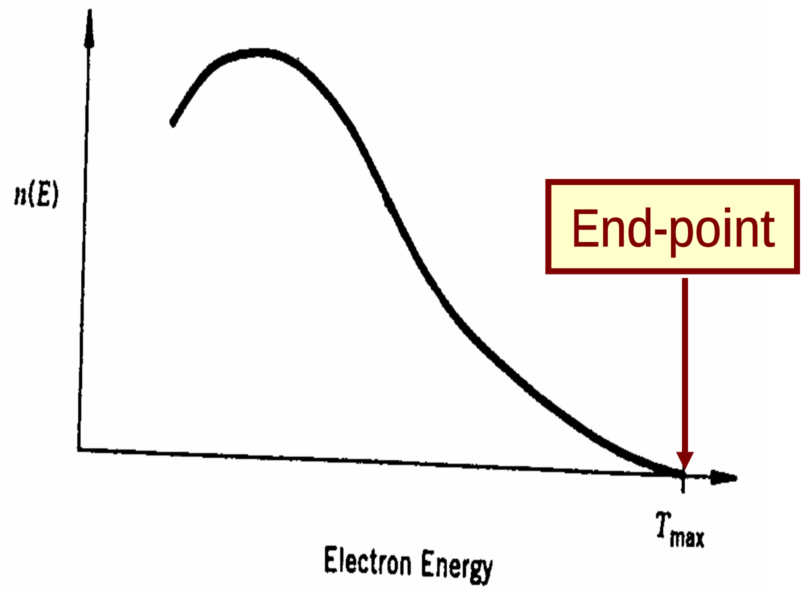
- Initially assumed to be 2-body decay
- From the conservation of energy

$$E_X = E_Y + E_{e^-} = E_Y + T_e + m_e c^2$$

- Since the lighter electron carries most the energy

$$T_e = \left(E_X - E_Y - m_e c^2\right) = \left(m_X - m_Y - m_e\right)c^2 - T_Y = Q - T_Y \approx Q$$

- Will result in a unique values as in α-decay.
- In reality, electrons emitted with continuous E spectrum with an end-point given by the formula above
- Energy conservation is violated!!!!



Nuclear Radiation: β-Decays

- Angular momentum is also in trouble
- In β-decays total number of nucleons is conserved
- Electrons are fermions with spin $\hbar/2$
- Independent of any changes of an integer orbital angular momentum, the total angular momentum cannot be conserved
- Angular momentum conservation is violated!!!

Nuclear Radiation: β-Decays

- Pauli proposed an additional particle emitted in β-decays
  - No one saw this particle in experiment
    - Difficult to detect
  - Charge is conserved in b-decay
    - Electrically neutral
  - Maximum energy of electrons is the Q values
    - Massless
  - Must conserve the angular momentum
    - Must be a fermion with spin $\hbar/2$
- This particle is called neutrino (by Feynman) and expressed as ν

Nuclear Radiation: Neutrinos

- Have anti-neutrinos $\bar{\nu}$, just like other particles
- Neutrinos and anti-neutrinos are distinguished by magnetic moment
  - Helicity is used to distinguish them $H \propto \vec{p} \cdot \vec{s}$
    - Left-handed (spin and momentum opposite direction) anti-electron-neutrinos are produced in β-decays
    - Right-handed electron-neutrinos are produced in positron emission
  - e^- is a particle and e^+ is an anti-particle
  - ν_e is a particle and $\bar{\nu}_e$ is an anti-particle

Assignments

1. End of the chapter problems: 3.2
2. Derive the following equations:
   - Eq. 4.8 starting from conservation of energy
   - Eq. 4.11 both the formula

- Due for these homework problems is next Wednesday, Feb. 23.