PHYS 3446 – Lecture #9

Wednesday, Feb. 23, 2005
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1. Nuclear Radiation
   - Beta Decay
   - Gamma Decay
2. Application of Nuclear Physics

Announcements

- Please be sure to start doing the analysis using the computer skills you have learned
  - Generate some plots, especially those Venkat asked you to since they provide good benchmark
- Term exam results
  - Average: 63.2/100 including extra credit
  - Top score: 95
  - Good job!
  - This exam constitutes 15% of the total
- There are other opportunities
  - One more term exam on Mar. 21: 15%
  - Homework: 15%
  - Lab: 15%
  - Class project: Paper (20%) + Oral presentation (10%)
  - Extra credit: up to 10% of the total

Nuclear Radiation: β-Decays

- Three kinds of β-decays
  - Electron emission
    - Nucleus with large N_n
    - Proton number increases by one

$$ {}^{A}X^{Z} \rightarrow {}^{A}Y^{Z+1} + e^{-} $$

  - Positron emission
    - Nucleus with many protons
    - Proton number decreases by one

$$ {}^{A}X^{Z} \rightarrow {}^{A}Y^{Z-1} + e^{+} $$

  - Electron capture
    - Nucleus with many protons
    - Absorbs a K-shell atomic electron
    - Proton number decreases by one
    - Causes cascade x-ray emission from the transition of remaining atomic electrons

$$ {}^{A}X^{Z} + e^{-} \rightarrow {}^{A}Y^{Z-1} $$

- For β-decay: $\Delta A=0$ and $|\Delta Z|=1$

Nuclear Radiation: β-Decays

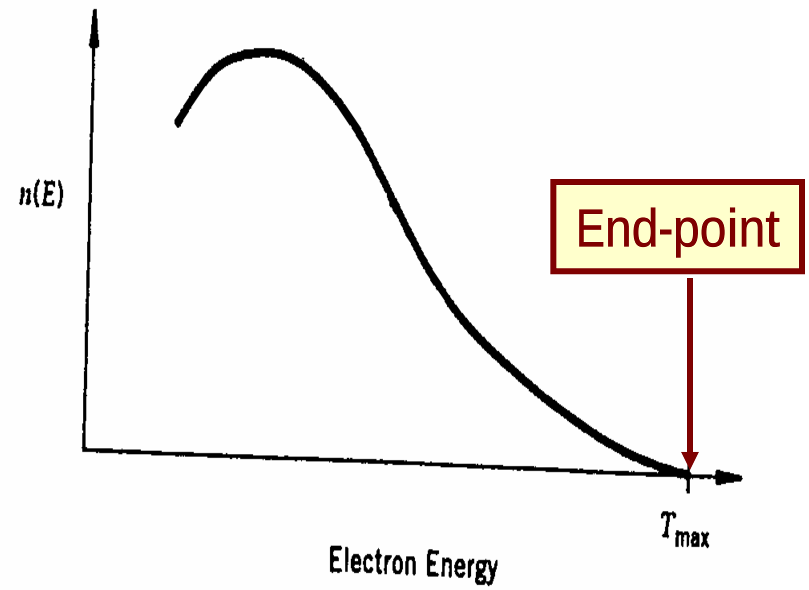
- Initially assumed to be 2-body decay
- From the conservation of energy

$$E_X = E_Y + E_{e^-} = E_Y + T_e + m_e c^2$$

- Since the lighter electron carries most the energy

$$T_e = \left(E_X - E_Y - m_e c^2\right) = \left(m_X - m_Y - m_e\right)c^2 - T_Y = Q - T_Y \approx Q$$

- Will result in a unique values as in α-decay.
- In reality, electrons emitted with continuous E spectrum with an end-point given by the formula above
- Energy conservation is violated!!!!



Nuclear Radiation: β-Decays

- Angular momentum is also in trouble
- In β-decays total number of nucleons is conserved
- Electrons are fermions with spin $\hbar/2$
- Independent of any changes of an integer orbital angular momentum, the total angular momentum cannot be conserved
- Angular momentum conservation is violated!!!

Nuclear Radiation: β-Decays

- Pauli proposed an additional particle emitted in β-decays
  - No one saw this particle in experiment
    - Difficult to detect
  - Charge is conserved in β-decays
    - Electrically neutral
  - Maximum energy of electrons is the Q values
    - Massless
  - Must conserve the angular momentum
    - Must be a fermion with spin $\hbar/2$
- This particle is called neutrino (by Feynman) and expressed as ν

Nuclear Radiation: Neutrinos

- Have anti-neutrinos $\bar{\nu}$, just like other particles
- Neutrinos and anti-neutrinos are distinguished through the spin projection on momentum
  - Helicity is used to distinguish them $H \propto \vec{p} \cdot \vec{s}$
    - Left-handed (spin and momentum opposite direction) anti-electron-neutrinos are produced in β-decays
    - Right-handed electron-neutrinos are produced in positron emission
  - e^- is a particle and e^+ is an anti-particle
  - ν_e is a particle and $\bar{\nu}_e$ is an anti-particle

β–Decays with neutrinos

- Electron emission

$$^{A}X^{Z} \rightarrow {}^{A}Y^{Z+1} + e^{-} + \bar{\nu}_{e}$$

- Positron emission

$$^{A}X^{Z} \rightarrow {}^{A}Y^{Z-1} + e^{+} + \nu_{e}$$

- Electron capture

$$^{A}X^{Z} + e^{-} \rightarrow {}^{A}Y^{Z-1} + \nu_{e}$$

β-Decays with neutrinos

- If the parent nucleus decays from rest, from the conservation of energy

$$M_p c^2 = T_D + M_D c^2 + T_{e^-} + m_e c^2 + T_{\bar{\nu}_e} + m_{\bar{\nu}_e} c^2$$

- Thus the Q-value of a β-decay can be written

$$T_D + T_{e^-} + T_{\bar{\nu}_e} = \left(M_p - M_D - m_e - m_{\bar{\nu}_e}\right)c^2 = \Delta M c^2 = Q$$

- Electron emission can only occur if Q>0
- Neglecting all small atomic BE, e emission can occur if

$$Q = \left(M(A,Z) - M(A,Z+1) - m_{\bar{\nu}_e}\right)c^2$$

$$\approx \left(M(A,Z) - M(A,Z+1)\right)c^2 \geq 0$$

β-Decays with neutrinos

- Daughter nucleus is much heavier than e or ν, the small recoil energy of daughter can be ignored.
- Thus we can obtain $T_{e^-} + T_{\bar{\nu}_e} \approx Q$
- This means that the energy of electron is not unique and can be any value in the range $0 \le T_{e^-} \le Q$
- The maximum electron kinetic energy can be Q.
- The same can apply to the other two β-decays

Particle Numbers

- Baryon numbers: A quantum number assigned to baryons (particles consists of quarks)
  - Mostly conserved in many interactions
  - Baryons: +1
  - Anti-baryons: -1
  - Protons and neutrons are baryons with baryon number +1 each
- Lepton numbers: A quantum number assigned to leptons (electrons, muons, taus and their corresponding neutrinos)
  - Leptons: +1
  - Anti-leptons: -1
  - Must be conserved at all times under SM in each species

Lepton Numbers

- Three charged leptons exist in nature with their own associated neutrinos

$$\begin{pmatrix} e^- \\ \nu_e \end{pmatrix} \quad \begin{pmatrix} \mu^- \\ \nu_\mu \end{pmatrix} \quad \begin{pmatrix} \tau^- \\ \nu_\tau \end{pmatrix}$$

- These three types of neutrinos are distinct from each other
  - muon neutrinos never produce other leptons than muons or anti-muons

$$\nu_\mu + {}^A X^Z \rightarrow {}^A Y^{Z+1} + \mu^-$$

$$\nu_\mu + {}^A X^Z \not\rightarrow {}^A Y^{Z+1} + e^-$$

$$\nu_\mu + {}^A X^Z \not\rightarrow {}^A Y^{Z+1} + \tau^-$$

Lepton Numbers

For electron neutrinos

$$\nu_e + {}^A X^Z \rightarrow {}^A Y^{Z+1} + e^-$$

$$\nu_e + {}^A X^Z \not\rightarrow {}^A Y^{Z+1} + \mu^-$$

$$\nu_e + {}^A X^Z \not\rightarrow {}^A Y^{Z+1} + \tau^-$$

For tau neutrinos

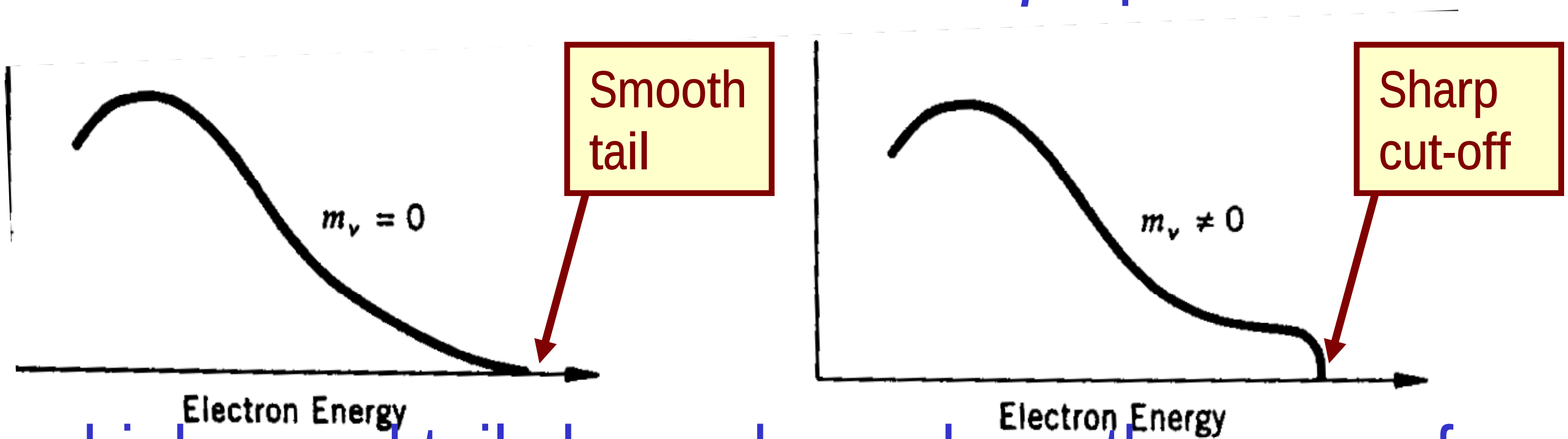
$$\nu_\tau + {}^A X^Z \rightarrow {}^A Y^{Z+1} + \tau^-$$

$$\nu_\tau + {}^A X^Z \not\rightarrow {}^A Y^{Z+1} + e^-$$

$$\nu_\tau + {}^A X^Z \not\rightarrow {}^A Y^{Z+1} + \mu^-$$

Neutrino Mass

- What does neutrino mass do to the β-spectrum?



- The higher end tail shape depends on the mass of neutrinos
  - β-spectrum could be used to measure the mass of neutrinos
    - Very sensitive to the resolution on the device
  - Most stringent limit on neutrino mass is $m_\nu < 2eV/c^2$
- Non-zero mass of neutrino means
  - Neutrino Oscillation: Mixing of neutrino species

Weak Interactions

- β-decay can be written in a nucleon level as:

$$n \rightarrow p + e^{-} + \bar{\nu}_e \qquad p \rightarrow n + e^{+} + \nu_e \qquad p + e^{-} \rightarrow n + \nu_e$$

- Since neutrons are heavier than protons, they can decay to a proton in a free space
  - On the other hand, protons are lighter than neutrons therefore they can only undergo a β-decay within a nucleus
  - Life time of a neutron is about 900sec
  - This life time is a lot longer than nuclear reaction time scale 10^{-23} s or EM scale 10^{-16} s.
- This means that a β-decay is a nuclear phenomenon that does not involve strong nuclear or EM forces
- Fermi postulated a new weak force responsible for β-decay

Weak Interactions

- Weak forces are short ranged
  - Occurs in nuclear domain
  - Weakness of the strength is responsible for long life time
- Nucleus does not contain electrons
  - Electrons in β-decays must come from somewhere else
  - The electron must come at the time of decay just like the photons from the transition of atomic electrons
  - β-decay can be considered to be induced by a weak force
- The transition probability per unit time, the width, can be calculated from perturbation theory using Fermi's Golden rule

$$P = \frac{2\pi}{\hbar} \left| H_{fi} \right|^2 \rho\left(E_f\right)$$

- Where the weak interaction Hamiltonian is

$$H_{fi} = \left\langle f \left| H_{wk} \right| i \right\rangle = \int d^3 x \psi_f^*\left(x\right) H_{wk} \psi_i\left(x\right)$$

Weak Interactions

- Based on β-decay reaction equations, the H_{wk} must be a four fermionic states
  - H_{wk} proposed by Fermi is a four-fermion interaction or current-current interaction
  - Relativistic
  - Agreed rather well with experiments for low energy β-decays
- Parity violation
  - There are only left-handed neutrinos and right-handed anti-neutrinos
  - A system is parity invariant if it does not change under reflection of spatial coordinates
  - The spin $\vec{r} \rightarrow -\vec{r},\ \vec{p} \rightarrow -\vec{p} \quad \Rightarrow \vec{L} = \vec{r} \times \vec{p} = (-\vec{r}) \times (-\vec{p}) = \vec{L}$
  - The handedness, helicity, changes upon the spatial reflection since direction of motion changes while the spin does not
  - Since there is no right handed neutrinos, parity must be violated in weak interactions

Gamma Decays

- When a heavy nuclei undergo alpha and beta decays, the daughters get into an excited state
  - Must either break apart
  - Or emit another particle
  - To bring the daughter into its ground state
- Typical energies of photons in γ-decays are a few MeV's
  - These decays are EM interactions thus the life time is on the order of 10^{-16}sec.
- Photons carry one unit of angular momentum
  - Parity is conserved in this decay

Assignments

1. End of the chapter problems: 4.4 and 4.5
2. Reading assignment: Chapter 5
3. Due for these assignments is Wednesday, Mar. 2