PHYS 3446 – Lecture #10

Monday, Feb. 28, 2005

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1. Energy Deposition in Media
   - Charged particle detection
   - Ionization Process
   - Multiple scattering
   - Electron energy loss: Bremsstrahlung
   - Photon energy loss

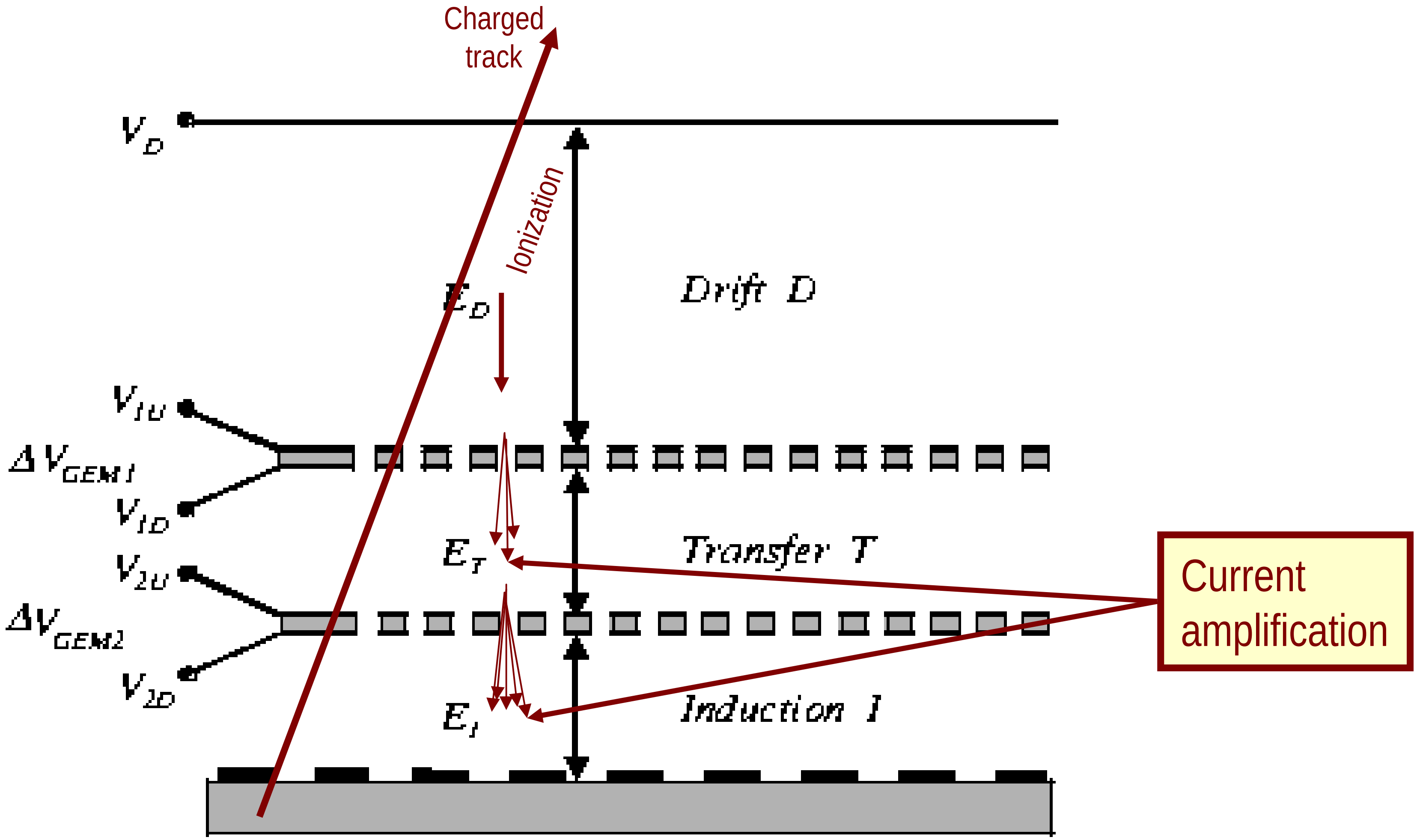
Forces in Nature

- We have learned the discovery of two additional forces
  - Gravitational force: formulated through Newton's laws
  - Electro-magnetic force: formulated through Maxwell's equations
  - Strong nuclear force: Discovered through studies of nuclei and their structure
  - Weak force: Discovered and postulated through nuclear β-decay

Forewords

- Physics is an experimental science
  - Understand nature through experiments
- In nuclear and particle physics, experiments are performed through scattering of particles
- In order for a particle to be detected:
  - Must leave a trace of its presence ➔ deposit energy
- The most ideal detector should
  - Detect particle without affecting them
- Realistic detectors
  - Use electromagnetic interactions of particles with matter
    - Ionization of matter by energetic particles
    - Ionization electrons can then be accelerated within an electric field to produce detectable electric current
  - Particles like neutrinos which do not interact through EM and have low cross sections, need special methods to handle

How does a charged particle get detected?



Charged Particle Detection

- What do you think is the primary interaction when a charged particle is traversing through a medium?
  - Interaction with the atomic electrons in the medium
- If the energy of the charged particle is sufficiently high
  - It deposits its energy (or loses its energy in the matter) by ionizing the atoms in the path
  - Or by exciting atoms or molecules to higher states
  - What are the differences between the above two methods?
    - The outcomes are either electrons or photons
- If the charged particle is massive, its interactions with atomic electrons will not affect the particles trajectory
- Sometimes, the particle undergoes a more catastrophic nuclear collisions

Ionization Process

- Ionization properties can be described by the stopping power S(T)
  - Amount of kinetic energy lost by any incident object per unit length of the path traversed in the medium
  - Referred as ionization energy loss or energy loss

$$S(T) = -\frac{dT}{dx} = n_{ion}\bar{I}$$

Why negative sign?

The particle's energy decreases.

- T: Kinetic energy of the incident particle
- Nion: Number of electron-ion pair formed per unit path length
- $\bar{I}$: The average energy needed to ionize an atom in the medium; for large atomic numbers ~10Z eV.

Ionization Process

- For any given medium, the stopping power is a function of incident particle's energy and the electric charge
- Since ionization is an EM process, easily calculable
  - Bethe-Bloch formula

$$S(T) = -\frac{4\pi\left(ze\right)^2 e^2 nZ}{m\beta^2 c^2}\left[\ln\left(\frac{2mc^2\gamma^2\beta^2}{\bar{I}}\right) - \beta^2\right]$$

  - z: Incident particle atomic number
  - Z: medium atomic number
  - n: number of atoms in unit volume

Ionization Process

- In natural α-decay, the formula becomes

$$S(T) = -\frac{4\pi (ze)^2 e^2 nZ}{m\beta^2 c^2} \ln\left(\frac{2mc^2\beta^2}{\bar{I}}\right)$$

  - Due to its high energy and large mass, the relativistic corrections can be ignored
- For energetic particles in accelerator experiments or beta emissions, the relativistic corrections are substantial
- Bethe-Bloch formula can be used in many media, various incident particles over a wide range of energies

Ionization Process

- Why does the interaction with atomic electrons dominate the energy loss of the incident particle?
  - Interaction with large nucleus causes large change of momentum but does not necessarily require large loss of kinetic energy
  - While momentum transfer to electrons would require large kinetic energy loss
    - Typical momentum transfer to electrons is 0.1MeV which requires 10KeV
    - The same amount of momentum transfer to nucleus would require less than 0.1eV of energy loss
- Thus Bethe-Bloch formula is inversely proportional to the mass of the medium

$$S(T) = -\frac{4\pi(ze)^2 e^2 nZ}{m\beta^2 c^2}\left[\ln\left(\frac{2mc^2\gamma^2\beta^2}{\bar{I}}\right) - \beta^2\right]$$

Ionization Process

- At low particle velocities, ionization loss is sensitive particle energy

$$S(T) = -\frac{4\pi (ze)^2 e^2 nZ}{m\beta^2 c^2}\left[\ln\left(\frac{2mc^2\gamma^2\beta^2}{\bar{I}}\right) - \beta^2\right]$$

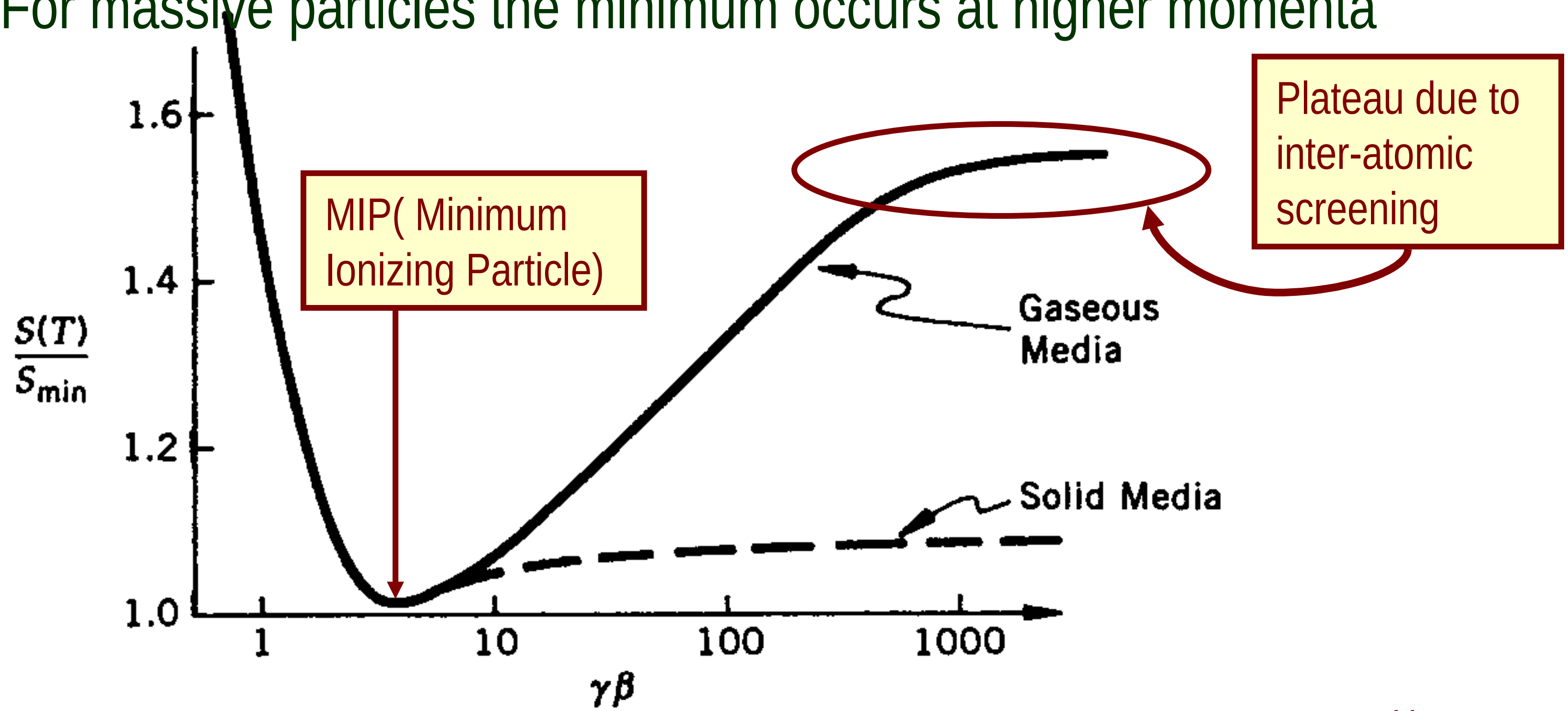
- This shows that the particles of different rest mass (M) but the same momentum (p) can be distinguished due to their different energy loss rate

$$S(T) \propto \frac{1}{v^2} = \frac{1}{(\beta c)^2} = \frac{M^2\gamma^2}{(M\gamma\beta c)^2} = \frac{M^2\gamma^2}{p^2}$$

- At low velocities (γ~1), particles can be distinguished

Properties of Ionization Process

- Stopping power decreases with increasing particle velocity independent of particle mass
  - Minimum occurs when $\gamma\beta \sim 3$ ($v > 0.96c$)
    - Particle is minimum ionizing when $v \sim 0.96c$
    - For massive particles the minimum occurs at higher momenta



Ionization Process

- At very high energies
  - Relativistic rise becomes an energy independent constant rate
  - Cannot be used to distinguish particle-types purely using ionization
  - Except for gaseous media, the stopping power at high energies can be approximated by the value at $\gamma\beta \sim 3$.
- At low energies, the stopping power expectation becomes unphysical
  - Ionization loss is very small when the velocity is very small
  - Detailed atomic structure becomes important

Ranges of Ionization Process

- Once the stopping power is known, we can compute the expected range of any particle in the medium
  - The distance the incident particle can travel in the medium till its kinetic energy runs out

$$R = \int_0^R dx = \int_T^0 \frac{dx}{dT} dT = \int_0^T \frac{dT}{S(T)}$$

Units of Energy Loss and Range

- What would be the sensible unit for energy loss?
  - MeV/cm
  - Equivalent thickness of g/cm^2: MeV/(g/cm^2)
- Range is expressed in
  - cm or g/cm^2
- Minimum value of S(T) for z=1 at $\gamma\beta$=3 is

$$S(T)_{\min} \approx -\frac{4\pi e^4 A_0 \left(\rho Z/A\right)}{m\beta^2 c^2} \ln\left(\frac{2mc^2\gamma^2\beta^2}{\bar{I}}\right) \approx 5.2\times 10^{-7}\left(13.7-\ln Z\right)\rho Z/A \text{ erg/cm}$$

- Using <Z>=20 we can approximate

$$S(T)_{\min} \approx 3.5 Z/A \text{ MeV}/\left(\text{g/cm}^2\right)$$

Straggling, Multiple Scattering and Statistical process

- Phenomenological calculations can describe average behavior but large fluctuations are observed in an event-by-event bases
  - This is due to the statistical nature of scattering process
    - Finite dispersion of energy deposit or scattering angular distributions is measured
- Statistical effect of angular deviation experienced in Rutherford scattering off atomic electrons in the medium
  - Consecutive collisions add up in a random fashion and provide net deflection of any incident particles from its original path
  - Called "Multiple Coulomb Scattering" ➔ Increases as a function of path length

$$\theta_{rms} \approx \frac{20MeV}{\beta pc} z\sqrt{\frac{L}{X_0}}$$

    - z: charge of the incident particle, L: material thickness, X_0: radiation length of the medium

Energy Loss Through Bremsstrahlung

- Energy loss of incident electrons
  - Bethe-Bloch formula works well (up to above 1MeV for electrons)
  - But due to the small mass, electron's energy loss gets complicated
    - Relativistic corrections take large effect
    - Electron projectiles can transfer large fractions of energies to the atomic electrons they collide
      - Produce δ-rays or knock-on electrons ➔ Which have the same properties as the incident electrons
  - Electrons suffer large acceleration as a result of interaction with electric field by nucleus. What do these do?
  - Causes electrons to radiate or emit photons
    - Bremsstrahlung ➔ An important mechanism of relativistic electron energy loss

Total Electron Energy Loss

- The electron energy loss can be written

$$\left(-\frac{dT}{dx}\right)_{tot} = \left(-\frac{dT}{dx}\right)_{ion} + \left(-\frac{dT}{dx}\right)_{brem}$$

- Relative magnitude between the two is

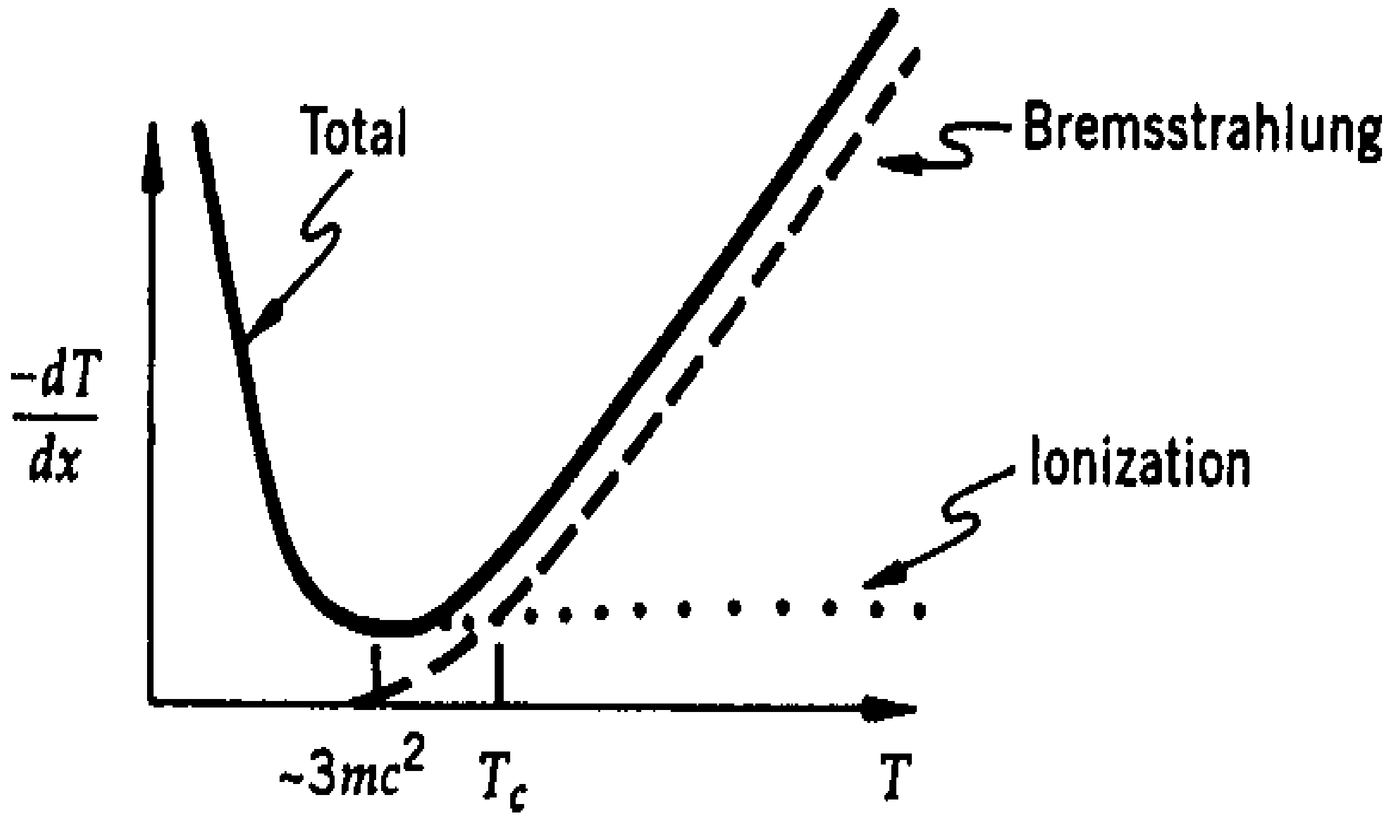
$$\left(-\frac{dT}{dx}\right)_{brem} \Big/ \left(-\frac{dT}{dx}\right)_{ion} \approx \frac{TZ}{1200mc^2}$$

  - Z: Atomic number of the medium, m: rest mass of the electron, T: Kinetic energy of electron in MeV

- At high energies, ionization loss is constant
  - Radiation dominates the energy loss
  - The energy loss is directly proportional to incident energy

Total Electron Energy Loss

- Above the critical energy (T_c) the brem process dominates $$\left(\frac{dT}{dx}\right)_{brem} = \left(\frac{dT}{dx}\right)_{ion} = -\frac{T_c}{X_0}$$



Assignments

1. Performed the detailed calculations in examples 1 – 4
2. What is the radiation length, X_0?
3. Due for these assignments is Monday, Mar. 7