PHYS 3446 – Lecture #15

Monday, Mar. 28, 2005
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- Elementary Particle Properties
  - Lepton numbers
  - Strangeness
  - Isospin
  - Gell-Mann-Nishijima Relations
  - Violation of quantum numbers

Announcements

- 2nd term exam results
  - Class average: 41.1
    - What was previous average?
      - 64.8
  - Top score: 80
- Grade proportions
  - Term exams: 15% each
  - Lab: 15%
  - Homework: 15%
  - Pop quizzes: 10%
    - There will be one or two more quizzes
  - Final paper: 20%
  - Presentation: 10%
  - Extra credit: 10%
- Will have an individual mid-semester discussion next week

Lepton Numbers

- Quantum number of leptons
  - All leptons carry L=1 (particles) or L=-1 (antiparticles)
  - Photons or hadrons carry L=0
- Lepton number is a conserved quantity
  - Total lepton number must be conserved
  - Lepton numbers by species must be conserved
  - This is an empirical law necessitated by experimental observation (or lack thereof)
- Consider the decay $e^{+} + e^{-} \rightarrow \pi^{+} + \pi^{-}$
  - Does this decay process conserve energy and charge?
    - Yes
  - But it hasn't been observed, why?
    - Due to the lepton number conservation

Lepton Number Assignments

Leptons (anti-leptons)	L_e	L_μ	L_τ	$L=L_e+L_\mu+L_\tau$
e^- (e^+)	1 (-1)	0	0	1 (-1)
ν_e $(\bar{\nu}_e)$	1 (-1)	0	0	1 (-1)
μ^- (μ^+)	0	1 (-1)	0	1 (-1)
ν_μ $(\bar{\nu}_\mu)$	0	1 (-1)	0	1 (-1)
τ^- (τ^+)	0	0	1 (-1)	1 (-1)
ν_τ $(\bar{\nu}_\tau)$	0	0	1 (-1)	1 (-1)

Lepton Number Conservation

- Can the following decays occur?

Decays	$\mu^- \to e^- + \gamma$	$\mu^- \to e^- + e^+ + e^-$	$\mu^- \to e^- + \bar{\nu}_e + \nu_\mu$
L_e	$0 \to 1 + 0$	$0 \to 1 - 1 + 1$	$0 \to 1 - 1 + 0$
L_μ	$1 \to 0 + 0$	$1 \to 0 + 0 + 0$	$1 \to 0 + 0 + 1$
L_τ	$0 \to 0 + 0$	$0 \to 0 + 0 + 0$	$0 \to 0 + 0 + 0$
$L = L_e + L_\mu + L_\tau$	$1 \to 1 + 0$	$1 \to 1 - 1 + 1$	$1 \to 1 - 1 + 1$

  - Case 1: L is conserved but L_e and L_μ not conserved
  - Case 2: L is conserved but L_e and L_μ not conserved
  - Case 3: L is conserved, and L_e and L_μ are also conserved

Strangeness

- From cosmic ray observations
  - K-mesons and Σ and Λ^0 baryons are produced strongly
    - But their lifetime typical of weak interactions (~10^{-10} sec)
    - Are produced in pairs
  - Gave an indication of a new quantum number
- Consider the reaction $\pi^- + p \to K^0 + \Lambda^0$
  - K^0 and Λ^0 subsequently decay
  - $\Lambda^0 \to \pi^- + p$ and $K^0 \to \pi^+ + \pi^-$
- Observations
  - Λ^0 was always produced w/ K^0 never w/ just a π^0
  - Λ^0 was produced w/ K^+ but not w/ K^-

$$\pi^- + p \to K^+ + \pi^- + \Lambda^0$$

$$\pi^- + p \not\to K^- + \pi^+ + \Lambda^0 \qquad \pi^- + p \not\to \pi^- + \pi^+ + \Lambda^0$$

Strangeness

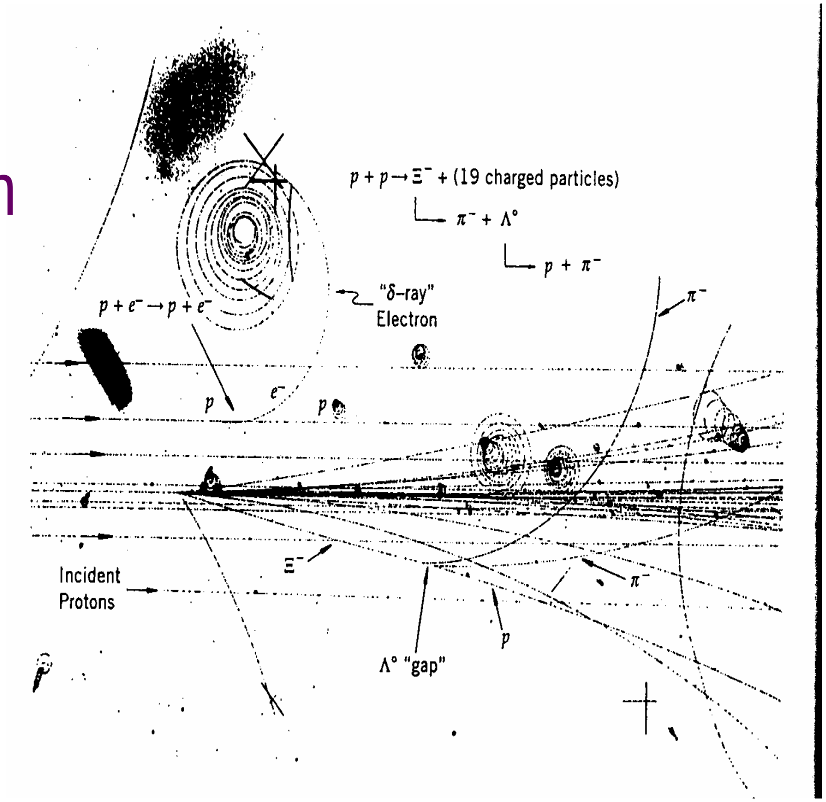
- Consider the reaction $\pi^+ + p \rightarrow \Sigma^+ + K^+$ and $\pi^- + p \rightarrow \Sigma^- + K^+$
  - With the decay $\Sigma^{+(-)} \rightarrow n + \pi^{+(-)}$ and $K^+ \rightarrow \pi^+ + \pi^0$
- Observations from Σ^+
  - Σ^+ is always produced w/ a K^+ never w/ just a π^+
  - Σ^+ is also produced w/ a K^0 but w/ an additional π^+ for charge conservation
- Observations from Σ^-
  - Σ^- is always produced w/ a K^+ never w/ K^-
- Thus,
  - Observed: $\pi^+ + p \rightarrow \Sigma^+ + \pi^+ + K^0$ $\pi^- + p \rightarrow \Sigma^- + K^+$
  - Not observed: $\pi^- + p \not\rightarrow \Sigma^+ + K^-$ $\pi^- + p \not\rightarrow \Sigma^- + \pi^+$

Strangeness

- Further observation of cross section measurements
  - Cross sections for the allow reactions w/ 1GeV/c pion momenta are ~ 1mb
    - Total pion cross section is ~ 30mb
    - The interactions are strong
  - Λ^0 at v~0.1c decays in about 0.3cm
    - Lifetime of Λ^0 baryon is

$$\tau_{\Lambda^0} \approx \frac{0.3cm}{3\times10^9 cm/s} = 10^{-10}\sec$$

    - These short lifetime of these strange particles indicate weak decay



Strangeness

- Strangeness quantum number
  - Murray Gell-Mann and Abraham Pais proposed a new additive quantum number that are carried by these particles
  - Conserved in strong interactions
  - Violated in weak decays
  - All ordinary mesons and baryons as well as photons and leptons have strangeness 0 (S=0)
  - For any strong associated-production reaction w/ the initial state S=0, the total strangeness of particles in the final state should add up to 0.
- Based on experimental observations of reactions and w/ an arbitrary choice of $S(K^0)=1$, we obtain
  - $S(K^+)=S(K^0)=1$ and $\Sigma(K^-)=\Sigma(\bar{K}^0)=-1$
  - $S(\Lambda^0)=S(\Sigma^+)=S(\Sigma^0)=S(\Sigma^-)=-1$
- For strong production reactions $K^- + p \rightarrow \Xi^- + K^+$ and $\bar{K}^0 + p \rightarrow \Xi^0 + K^+$
  - cascade particles $S(\Xi^-) = S(\Xi^0) = -2$ if $S(\bar{K}^0) = S(K^-) = -1$

More on Strangeness

- Let's look at the reactions again

$$\pi^- + p \rightarrow K^0 + \Lambda^0$$

  - This is a strong interaction
    - Strangeness must be conserved
    - S: 0 + 0 → +1 -1
- How about the decays of the final state particles?
  - $\Lambda^0 \rightarrow \pi^- + p$ and $K^0 \rightarrow \pi^+ + \pi^-$
  - These decays are weak interactions so S is not conserved
  - S: -1 → 0 + 0 and +1 → 0 + 0
- A not-really-elegant solution
- Leads into the necessity of strange quarks

Isospin Quantum Number

- Strong force does not depend on the charge of the particle
  - Nuclear properties of protons and neutrons are very similar
  - From the studies of mirror nuclei, p-p, p-n and n-n strong interactions are essentially the same
  - If corrected by EM interactions, the x-sec between n-n and p-p are the same
- Since strong force is much stronger than any other forces, we could imagine a new quantum number that applies to all particles
  - Protons and neutrons are two orthogonal mass eigenstates of the same particle like spin up and down states

$$p = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \text{ and } \mathrm{n} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Isospin Quantum Number

- Protons and neutrons are degenerate in mass because of some symmetry of the strong force
  - Isospin symmetry ➔ Under the strong force these two particles appear identical
  - Presence of Electromagnetic or Weak forces breaks this symmetry, distinguishing p from n.
- Isospin works just like spins
  - Protons and neutrons have isospin ½ ➔ Isospin doublet
  - Three pions, π+, π- and π^0, have almost the same masses
  - X-sec by these particles are almost the same after correcting for EM effects
  - Strong force does not distinguish these particles ➔ Isospin triplet

$$\pi^+ = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad \pi^0 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad \text{and} \quad \pi^- = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Isospin Quantum Number

- This QN is found to be conserved in strong interactions
- But not conserved in EM or Weak interactions
- Third component of isospin QN is assigned to be positive for the particles with larger electric charge
- Isospin is not a space-time symmetry
- Cannot be assigned uniquely to leptons and photons since they are not involved in strong interactions
  - There is something called weak-isospin for weak interactions

Gell-Mann-Nishijima Relation

- Strangeness assignment is based on Gell-Mann-Nishijima relation
  - Electric charge of a hadron can be related to its other quantum numbers

$$Q = I_3 + \frac{Y}{2} = I_3 + \frac{B+S}{2}$$

  - Where Q: hadron electrical charge, I_3: third component of isospin and Y=B+S, strong hypercharge
  - Quantum numbers of several long lived particles follow this rule
- With the discovery of new flavor quantum numbers, charm and bottom, this relationship was modified to include these new additions (Y=B+S+C+B)
  - Since charge and isospin are conserved in strong interactions, strong hypercharge, Y, is also conserved in strong interactions
- This relationship holds in all strong interactions

Quantum numbers for a few hadrons

$Hadron$	Q	I_3	B	S	$Y=(B+S)$
π^+	1	1	0	0	0
π^0	0	0	0	0	0
π^-	-1	-1	0	0	0
K^+	1	$1/2$	0	1	1
K^0	0	$-1/2$	0	1	1
η^0	0	0	0	0	0
p	1	$1/2$	1	0	1
n	0	$-1/2$	1	0	1
Σ^+	1	1	1	-1	0
Λ^0	0	0	1	-1	0
Ξ^-	-1	$-1/2$	1	-2	-1
Ω^-	-1	0	1	-3	-2

Violation of Quantum Numbers

- The QN we learned are conserved in strong interactions are but many of them are violated in EM or weak interactions
- Three types of weak interactions
  - Hadron decays with only hadrons in the final state

$$\Lambda^0 \to \pi^- + p$$

  - Semi-leptonic: both hadrons and leptons are present

$$n \to p + e^- + \overline{\nu}_e$$

  - Leptonic: only leptons are present

$$\mu^- \to e^- + \overline{\nu}_e + \nu_\mu$$

Hadronic Weak Decays

- These decays follow selection rules: $|\Delta I_3|=1/2$ and $|\Delta S|=1$

QN	$\Lambda^0 \rightarrow$	π^-	p	$\|\Delta\|$
I_3	0	-1	½	1/2
S	-1	0	0	1

QN	$\Sigma^+ \rightarrow$	π^0	p	
I_3	1	0	½	1/2
S	-1	0	0	1

QN	$K^0 \rightarrow$	π^+	π^-	
I_3	- ½	1	-1	1/2
S	1	0	0	1

QN	$\Xi^- \rightarrow$	Λ^0	π^-	
I_3	- ½	0	-1	1/2
S	-2	-1	0	1

Semi-leptonic Weak Decays

These decays follow selection rules: $|\Delta I_3|=1$ and $|\Delta S|=0$ or $|\Delta I_3|=½$ and $|\Delta S|=1$

QN	$n \rightarrow$	p	$e^- + \bar{\nu}_e$	$\lvert\Delta\rvert$
I_3	-1/2	1/2		1
S	0	0		0

QN	$\pi^- \rightarrow$	μ^-	$\bar{\nu}_\mu$	
I_3	-1			1
S	0			0

QN	$K^+ \rightarrow$	π^0	$\mu^+ + \nu_\mu$	
I_3	½	0		1/2
S	1	0		1

QN	$\Sigma^- \rightarrow$	n	$e^- + \bar{\nu}_e$	
I_3	-1	-1/2		1/2
S	-1	0		1

Summary of Weak Decays

- Hadronic weak-decay
  - Selection rules are $|\Delta I_3|=1/2$ and $|\Delta S|=1$
  - $|\Delta I_3|=3/2$ and $|\Delta S|=2$ exists but heavily suppressed
- Semi-leptonic weak-decays
  - Type 1: Strangeness conserving
    - Selection rules are: $|\Delta S|=0$, $|\Delta I_3|=1$ and $\Delta I=1$
  - Type 2: Strangeness non-conserving
    - Selection rules are: $|\Delta S|=1$, $|\Delta I_3|=$ ½ and $\Delta I=$ ½ or 3/2
    - $\Delta I=3/2$ and $|\Delta S|=1$ exist but heavily suppressed

EM Processes

QN	$\pi^0 \rightarrow$	γ	γ	$\|\Delta\|$
I_3	0			0
S	0			0
QN	$\eta^0 \rightarrow$	γ	γ	
I_3	0			0
S	0			0
QN	$\Sigma^0 \rightarrow$	Λ^0	γ	
I_3	0	0		0
S	-1	-1		0

- Strangeness is conserved but total isospin is not
  - Selection rules are: $|\Delta S|=0$, $|\Delta I_3|=0$ and $\Delta I= 1$

Assignments

1. Reading assignments: 9.6 and 9.7
2. End of chapter problems 9.1, 9.2 and 9.3
3. Due for these assignments is next Monday, Apr. 4