PHYS 3446 – Lecture #17

Wednesday, Apr. 6, 2005

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- Symmetries
  - Local gauge symmetry
  - Gauge fields

Announcements

- Don't forget that you have another opportunity to do your past due homework at 85% of full if you submit the by Wed., Apr. 20
- Due for your project write up is Friday, April 22
  - You must generate the plots by early next week!!
  - You must start the write up now!!
- Individual mid-semester discussion today after the lecture

Project root and macro file locations

- W events
  - W$\rightarrow\mu+\nu$: /data92/venkat/MC_Analysis/RootFiles/WMUNU_PHYS3446/
  - W$\rightarrow$e+ν: /data92/venkat/MC_Analysis/RootFiles/WENU_PHYS3446/
- Z events
  - Z$\rightarrow\mu+\mu$: /data92/venkat/MC_Analysis/RootFiles/ZMUMU_PHYS3446/
  - Z$\rightarrow$e+e: /data92/venkat/MC_Analysis/RootFiles/ZEE_PHYS3446/
- Macros are at /data92/venkat/MC_Analysis/tree_analysis/

Quantum Numbers

- We've learned about various newly introduced quantum numbers as a patch work to explain experimental observations
  - Lepton numbers
  - Baryon numbers
  - Isospin
  - Strangeness
- Some of these numbers are conserved in certain situation but not in others
  - Very frustrating indeed….
- These are due to lack of quantitative description by an elegant theory

Why symmetry?

- When does a quantum number conserved?
  - When there is an underlying symmetry in the system
  - When the quantum number is not affected (or is conserved) by (under) changes in the physical system
- Noether's theorem: If there is a conserved quantity associated with a physical system, there exists an underlying invariance or symmetry principle responsible for this conservation.
- Symmetries provide critical restrictions in formulating theories

Symmetries in Translation and Conserved quantities

- The translational symmetries of a physical system give invariance in the corresponding physical quantities
  - Symmetry under linear translation
    - Linear momentum conservation
  - Symmetry under spatial rotation
    - Angular momentum conservation
  - Symmetry under time translation
    - Energy conservation
  - Symmetry under isospin space rotation
    - Isospin conservation

Local Symmetries

- All continuous symmetries can be classified as
  - Global symmetry: Parameter of transformation can be constant
    - Transformation is the same throughout the entire space-time points
    - All continuous transformations we discussed so far are global symmetries
  - Local symmetry: Parameters of transformation depend on space-time coordinates
    - The magnitude of transformation is different from point to point
    - How do we preserve symmetry in this situation?
      - Real forces must be introduced!!

Local Symmetries

- Let's consider time-independent Schrodinger Eq.

$$H\psi(\vec{r}) = \left(-\frac{\hbar^2}{2m}\vec{\nabla}^2 + V(\vec{r})\right)\psi(\vec{r}) = E\psi(\vec{r})$$

- If $\psi(\vec{r})$ is a solution, $e^{i\alpha}\psi(\vec{r})$ should also be for a constant α
  - Any QM wave functions can be defined up to a constant phase
  - A transformation involving a constant phase is a symmetry of any QM system
  - Conserves probability density ➔ Conservation of electrical charge is associated w/ this kind of global transformation.

Local Symmetries

- Let's consider a local phase transformation

$$\psi(\vec{r}) \to e^{i\alpha(\vec{r})}\psi(\vec{r})$$

  – How can we make this transformation local?
    - Multiplying a phase parameter with explicit dependence on the position vector
    - This does not mean that we are transforming positions but just that the phase is dependent on the position
- Thus under local x-formation, we obtain

$$\vec{\nabla}\left[e^{i\alpha(\vec{r})}\psi(\vec{r})\right] = e^{i\alpha(\vec{r})}\left[i\left(\vec{\nabla}\alpha(\vec{r})\right)\psi(\vec{r}) + \vec{\nabla}\psi(\vec{r})\right] \neq e^{i\alpha(\vec{r})}\vec{\nabla}\psi(\vec{r})$$

Local Symmetries

- Thus, Schrodinger equation

$$H\psi(\vec{r}) = \left(-\frac{\hbar^2}{2m}\vec{\nabla}^2 + V(\vec{r})\right)\psi(\vec{r}) = E\psi(\vec{r})$$

- is not invariant (or a symmetry) under local phase transformation
  - What does this mean?
  - The energy conservation is no longer valid.
- What can we do to conserve the energy?
  - Consider an arbitrary modification of a gradient operator

$$\vec{\nabla} \rightarrow \vec{\nabla} - i\vec{A}(\vec{r})$$

Local Symmetries

- Now requiring the vector potential $\vec{A}(\vec{r})$ to change under transformation as

$$\vec{A}(\vec{r}) \rightarrow \vec{A}(\vec{r}) + \vec{\nabla}\alpha(\vec{r})$$

Additional Field

  – Similar to Maxwell's equation

- Makes

$$\left(\vec{\nabla} - i\vec{A}(\vec{r})\right)\psi(\vec{r}) \rightarrow \left(\vec{\nabla} - i\vec{A}(\vec{r}) + i\left(\vec{\nabla}\alpha(\vec{r})\right)\right)\left[e^{i\alpha(\vec{r})}\psi(\vec{r})\right] = e^{i\alpha(\vec{r})}\left(\vec{\nabla} - i\vec{A}(\vec{r})\right)\psi(\vec{r})$$

- Thus, now the local symmetry of the modified Schrodinger equation is preserved under x-formation

$$H\psi(\vec{r}) = \left[-\frac{\hbar^2}{2m}\left(\vec{\nabla} - i\vec{A}(\vec{r})\right)^2 + V(\vec{r})\right]\psi(\vec{r}) = E\psi(\vec{r})$$

Assignments

1. No homework today!!!