PHYS 3446 – Lecture #18

Monday, Apr. 11, 2005
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- Symmetries
  - Local gauge symmetry
  - Gauge fields
- Parity
  - Determination of Parity
  - Conservation and violation of parity

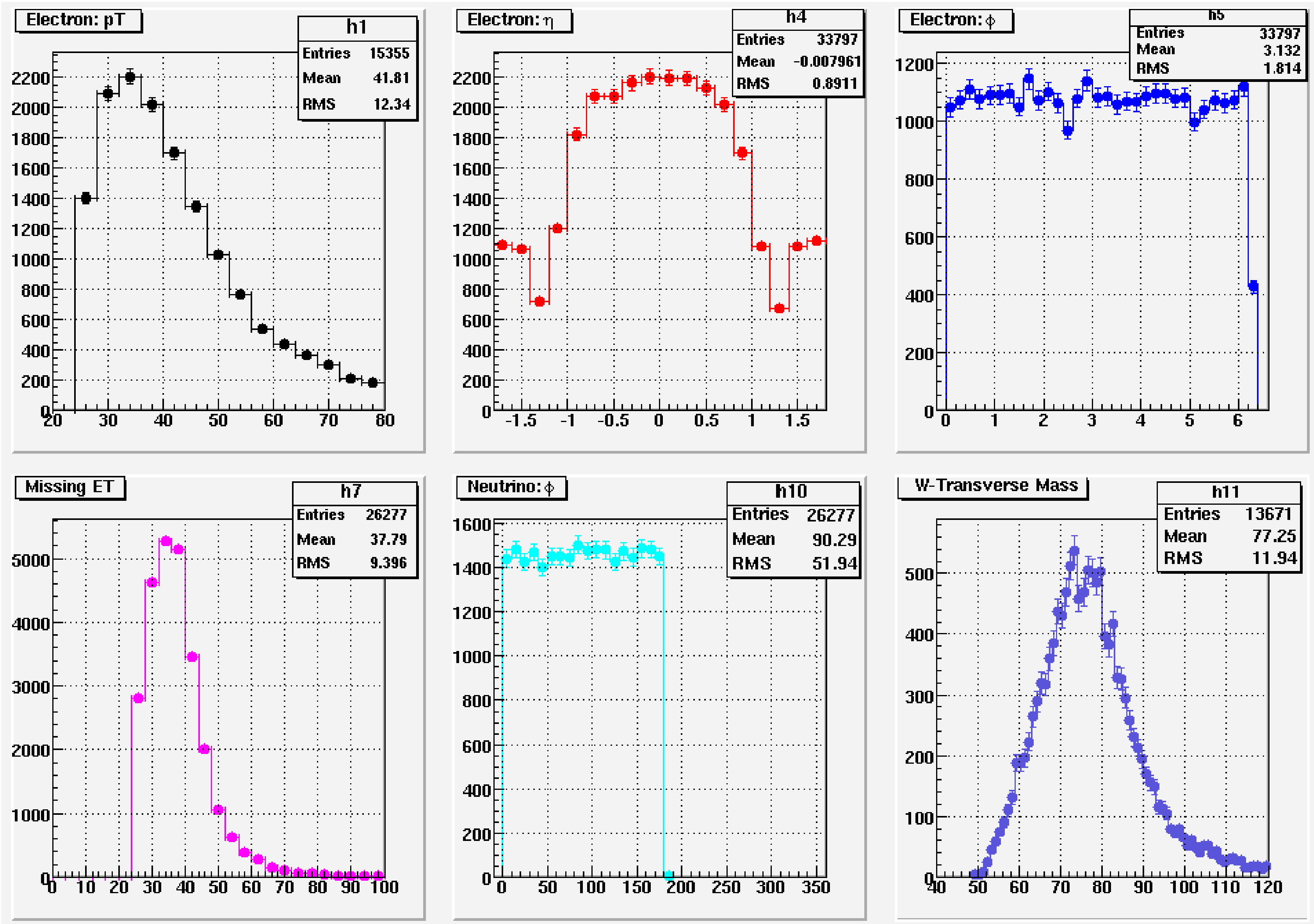
Announcements

- Don't forget that you have another opportunity to do your past due homework at 85% of full if you submit the by Wed., Apr. 20
- Due for your project write up is Friday, April 22
  – How are your analyses coming along?
- Individual mid-semester discussion extends till tomorrow for those who did not meet with me yet!!

Project root and macro file locations

- W events
  - W$\rightarrow\mu+\nu$: /data92/venkat/MC_Analysis/RootFiles/WMUNU_PHYS3446/
  - W$\rightarrow e+\nu$: /data92/venkat/MC_Analysis/RootFiles/WENU_PHYS3446/
- Z events
  - Z$\rightarrow\mu+\mu$: /data92/venkat/MC_Analysis/RootFiles/ZMUMU_PHYS3446/
  - Z$\rightarrow e+e$: /data92/venkat/MC_Analysis/RootFiles/ZEE_PHYS3446/
- Macros are at /data92/venkat/MC_Analysis/tree_analysis/

Output of W→e+nu macro



Local Symmetries

- Let's consider a local phase transformation

$$\psi(\vec{r}) \to e^{i\alpha(\vec{r})}\psi(\vec{r})$$

  – How can we make this transformation local?
    - Multiplying a phase parameter with explicit dependence on the position vector
    - This does not mean that we are transforming positions but just that the phase is dependent on the position
- Thus under local x-formation, we obtain

$$\vec{\nabla}\left[e^{i\alpha(\vec{r})}\psi(\vec{r})\right] = e^{i\alpha(\vec{r})}\left[i\left(\vec{\nabla}\alpha(\vec{r})\right)\psi(\vec{r}) + \vec{\nabla}\psi(\vec{r})\right] \neq e^{i\alpha(\vec{r})}\vec{\nabla}\psi(\vec{r})$$

Local Symmetries

- Thus, Schrodinger equation

$$H\psi(\vec{r})=\left(-\frac{\hbar^2}{2m}\vec{\nabla}^2+V(\vec{r})\right)\psi(\vec{r})=E\psi(\vec{r})$$

- is not invariant (or a symmetry) under local phase transformation
  - What does this mean?
  - The energy conservation is no longer valid.
- What can we do to conserve the energy?
  - Consider an arbitrary modification of a gradient operator

$$\vec{\nabla}\rightarrow\vec{\nabla}-i\vec{A}(\vec{r})$$

Local Symmetries

- Now requiring the vector potential $\vec{A}(\vec{r})$ to change under transformation as

$$\vec{A}(\vec{r}) \rightarrow \vec{A}(\vec{r}) + \vec{\nabla}\alpha(\vec{r})$$

Additional Field

  – Similar to Maxwell's equation

- Makes

$$\left(\vec{\nabla} - i\vec{A}(\vec{r})\right)\psi(\vec{r}) \rightarrow \left(\vec{\nabla} - i\vec{A}(\vec{r}) + i\left(\vec{\nabla}\alpha(\vec{r})\right)\right)\left[e^{i\alpha(\vec{r})}\psi(\vec{r})\right] = e^{i\alpha(\vec{r})}\left(\vec{\nabla} - i\vec{A}(\vec{r})\right)\psi(\vec{r})$$

- Thus, now the local symmetry of the modified Schrodinger equation is preserved under x-formation

$$H\psi(\vec{r}) = \left[-\frac{\hbar^2}{2m}\left(\vec{\nabla} - i\vec{A}(\vec{r})\right)^2 + V(\vec{r})\right]\psi(\vec{r}) = E\psi(\vec{r})$$

Local Symmetries

- The invariance under a local phase transformation requires the introduction of additional fields
  - These fields are called gauge fields
  - Lead to the introduction of definite physical force
- The potential $\vec{A}(\vec{r})$ can be interpreted as the EM vector potential
- The symmetry group associated with the single parameter phase transformation in the previous slides is called Abelian or commuting symmetry and is called U(1) gauge group ➔ Electromagnetic force group

U(1) Local Gauge Invariance

Dirac Lagrangian for free particle of spin ½ and mass m;

$$\mathcal{L} = i(\hbar c)\overline{\psi}\gamma^{\mu}\partial_{\mu}\psi - (mc^2)\overline{\psi}\psi$$

Is invariant under a global phase transformation (global gauge transformation) $\psi \to e^{i\theta}\psi$ since $\overline{\psi} \to e^{-i\theta}\overline{\psi}$.

However, if the phase, θ, varies as a function of space-time coordinate, x^{μ}, is $\mathcal{L}$ still invariant under the local gauge transformation, $\psi \to e^{i\theta(x)}\psi$?

No, because it adds an extra term from derivative of θ.

U(1) Local Gauge Invariance

Requiring the complete Lagrangian be invariant under $\lambda(x)$ local gauge transformation will require additional terms to free Dirac Lagrangian to cancel the extra term

$$\mathcal{L} = \left[i(\hbar c)\overline{\psi}\gamma^{\mu}\partial_{\mu}\psi - (mc^{2})\overline{\psi}\psi \right] - \left(q\overline{\psi}\gamma^{\mu}\psi\right)A_{\mu}$$

Where A_{μ} is a new vector gauge field that transforms under local gauge transformation as follows:

$$A_{\mu} \rightarrow A_{\mu} + \partial_{\mu}\lambda$$

Addition of this vector field to $\mathcal{L}$ keeps $\mathcal{L}$ invariant under local gauge transformation, but…

U(1) Local Gauge Invariance

The new vector field couples with spinor through the last term. In addition, the full Lagrangian must include a "free" term for the gauge field. Thus, Proca Largangian needs to be added.

$$\mathcal{L} = -\frac{1}{16\pi} F^{\mu\nu} F_{\mu\nu} + \frac{1}{8\pi}\left(\frac{m_A c}{\hbar}\right)^2 A^\nu A_\nu$$

This Lagrangian is not invariant under the local gauge transformation, $A_\mu \rightarrow A_\mu + \partial_\mu \lambda$, because

$$A^\nu A_\nu = \left(A^\nu - \partial_\mu \lambda\right)\left(A_\nu - \partial^\mu \lambda\right)$$

$$= A^\nu A_\nu - \left(A^\nu \partial^\mu \lambda + A_\nu \partial_\mu \lambda\right) + \left(\partial_\mu \lambda\right)\left(\partial^\mu \lambda\right)$$

U(1) Local Gauge Invariance

The requirement of local gauge invariance forces the introduction of a massless vector field into the free Dirac Lagrangian.

$$\mathcal{L} = \left[i\left(\hbar c\right)\overline{\psi}\gamma^{\mu}\partial_{\mu}\psi - \left(mc^{2}\right)\overline{\psi}\psi \right] + \left[\frac{-1}{16\pi}F^{\mu\nu}F_{\mu\nu} \right] - \left(q\overline{\psi}\gamma^{\mu}\psi\right)A_{\mu}$$

Free $\mathcal{L}$ for gauge field.

Vector field for gauge invariance

A_{μ} is an electromagnetic potential.

And $A_{\mu} \rightarrow A_{\mu} + \partial_{\mu}\lambda$ is a gauge transformation of an electromagnetic potential.

Gauge Fields and Local Symmetries

- To maintain a local symmetry, additional fields must be introduced
  - This is in general true for more complicated symmetries
  - A crucial information for modern physics theories
- A distinct fundamental forces in nature arise from local invariance of physical theories
- The associated gauge fields generate these forces
  - These gauge fields are the mediators of the given force
- This is referred as gauge principle, and such theories are gauge theories
  - Fundamental interactions are understood through this theoretical framework

Gauge Fields and Mediators

- To keep local gauge invariance, new particles had to be introduced in gauge theories
  - U(1) gauge introduced a new field (particle) that mediates the electromagnetic force: Photon
  - SU(2) gauge introduces three new fields that mediates weak force
    - Charged current mediator: W^+ and W^-
    - Neutral current: Z^0
  - SU(3) gauge introduces 8 mediators for the strong force
- Unification of electromagnetic and weak force SU(2)xU(1) introduces a total of four mediators
  - Neutral current: Photon, Z^0
  - Charged current: W^+ and W^-

Parity

- The space inversion transformation (mirror image) ➔ Switch right- handed coordinate system to left-handed

$$\begin{pmatrix} c\,t \\ x \\ y \\ z \end{pmatrix} \xrightarrow{Parity} \begin{pmatrix} c\,t \\ -\,x \\ -\,y \\ -\,z \end{pmatrix}$$

- How is this different than spatial rotation?
  - Rotation is continuous in a given coordinate system
    - Quantum numbers related rotational transformation are continuous
  - Space inversion cannot be obtained through any set of rotational transformation
    - Quantum numbers related to space inversion is discrete

Properties of Parity

- Position and momentum vectors change sign under space inversion

$$\vec{r} \xrightarrow{P} -\vec{r}$$

$$\vec{p} = m\dot{\vec{r}} \xrightarrow{P} -m\dot{\vec{r}} = -\vec{p}$$

- Where as their magnitudes do not change signs

$$r = \sqrt{\vec{r}\cdot\vec{r}} \xrightarrow{P} \sqrt{\left(-\vec{r}\right)\cdot\left(-\vec{r}\right)} = \sqrt{\vec{r}\cdot\vec{r}} = r$$

$$p = \sqrt{\vec{p}\cdot\vec{p}} \xrightarrow{P} \sqrt{\left(-\vec{p}\right)\cdot\left(-\vec{p}\right)} = \sqrt{\vec{p}\cdot\vec{p}} = p$$

- Vectors change signs under space-inversion while the scalars do not.

Properties of Parity

- Some vectors, however, behave like a scalar
  - Angular momentum
    $$\vec{L} = \vec{r} \times \vec{p} \xrightarrow{P} (-\vec{r}) \times (-\vec{p}) = \vec{r} \times \vec{p} = \vec{L}$$
  - These are called pseudo-vectors or axial vectors
- Likewise some scalars behave like vectors
  $$\vec{a} \cdot (\vec{b} \times \vec{c}) \xrightarrow{P} (-\vec{a}) \cdot \left((-\vec{b}) \times (-\vec{c}) \right) = -\vec{a} \cdot (\vec{b} \times \vec{c})$$
  - These are called pseudo-scalars
- Two successive application of parity operations must turn the coordinates back to original
  - The possible values (eigen values) of parity are +1 (even) or -1 (odd).
- Parity is a multiplicative quantum number

Parity

- Two parity quantum numbers
  - Intrinsic parity: Bosons have the same intrinsic parities as their anti-particles while fermions have opposite parity than its anti-particle (odd)
  - Parity under spatial transformation that follows the rule: P=$(-1)^{\ell}$
    - ℓ is the orbital angular momentum quantum number
- Are electromagnetic and gravitational forces invariant under parity operation or space inversion?
  - Newton's equation of motion for a point-like particle $m\frac{d^2\vec{r}}{dt^2}=\vec{F}$
  - For electromagnetic and gravitational forces we can write the forces $m\frac{d^2\vec{r}}{dt^2}=\vec{F}=\frac{C}{r^2}\hat{r}$, and thus are invariant under parity.

Determination of Parity Quantum Numbers

- How do we find out the intrinsic parity of particles?
  - Use observation of decays and production processes
  - Absolute determination of parity is not possible, just like electrical charge or other quantum numbers.
  - Thus the accepted convention is to assign **+1 intrinsic parity to proton, neutron and the Λ hyperon**.
    - The parities of other particles are determined relative to these assignments through the analysis of parity conserving interactions involving these particles.

Assignments

1. No homework today!!!