PHYS 3446 – Lecture #18

Monday, Apr. 11, 2005
Dr. Jae Yu

- Symmetries
  - Local gauge symmetry
  - Gauge fields
- Parity
  - Determination of Parity
  - Conservation and violation of parity
Announcements

• Don’t forget that you have another opportunity to do your past due homework at 85% of full if you submit the by Wed., Apr. 20

• Due for your project write up is Friday, April 22
  – How are your analyses coming along?

• Individual mid-semester discussion extends till tomorrow for those who did not meet with me yet!!
Project root and macro file locations

- **W events**
  - $W \rightarrow \mu + \nu$: \\
    /data92/venkat/MC_Analysis/RootFiles/WMUNU_PHYS3446/ \\
  - $W \rightarrow e + \nu$: /data92/venkat/MC_Analysis/RootFiles/WENU_PHYS3446/

- **Z events**
  - $Z \rightarrow \mu + \mu$: \\
    /data92/venkat/MC_Analysis/RootFiles/ZMUMU_PHYS3446/ \\
  - $Z \rightarrow e + e$: /data92/venkat/MC_Analysis/RootFiles/ZEE_PHYS3446/

- **Macros are at /data92/venkat/MC_Analysis/tree_analysis/**
Output of $W \rightarrow e + \nu$ macro
Local Symmetries

• Let’s consider a local phase transformation

\[ \psi(\vec{r}) \rightarrow e^{i\alpha(\vec{r})}\psi(\vec{r}) \]

– How can we make this transformation local?

• Multiplying a phase parameter with explicit dependence on the position vector

• This does not mean that we are transforming positions but just that the phase is dependent on the position

• Thus under local x-formation, we obtain

\[
\hat{\nabla} \left[ e^{i\alpha(\vec{r})}\psi(\vec{r}) \right] = e^{i\alpha(\vec{r})} \left[ i(\hat{\nabla}\alpha(\vec{r}))\psi(\vec{r}) + \hat{\nabla}\psi(\vec{r}) \right] \neq e^{i\alpha(\vec{r})}\hat{\nabla}\psi(\vec{r})
\]
Local Symmetries

• Thus, Schrodinger equation
  \[ H\psi(\vec{r}) = \left( -\frac{\hbar^2}{2m} \nabla^2 + V(\vec{r}) \right)\psi(\vec{r}) = E\psi(\vec{r}) \]

• is not invariant (or a symmetry) under local phase transformation
  – What does this mean?
  – The energy conservation is no longer valid.

• What can we do to conserve the energy?
  – Consider an arbitrary modification of a gradient operator
    \[ \nabla \rightarrow \nabla - iA(\vec{r}) \]
Local Symmetries

• Now requiring the vector potential $\vec{A}(\vec{r})$ to change under transformation as
  $$\vec{A}(\vec{r}) \rightarrow \vec{A}(\vec{r}) + \vec{\nabla} \alpha(\vec{r})$$
  – Similar to Maxwell’s equation

• Makes
  $$(\vec{\nabla} - i\vec{A}(\vec{r}))\psi(\vec{r}) \rightarrow (\vec{\nabla} - i\vec{A}(\vec{r}) + i(\vec{\nabla}\alpha(\vec{r})))\left[e^{i\alpha(\vec{r})}\psi(\vec{r})\right] = e^{i\alpha(\vec{r})} (\vec{\nabla} - i\vec{A}(\vec{r}))\psi(\vec{r})$$

• Thus, now the local symmetry of the modified Schrodinger equation is preserved under x-formation

$$H\psi(\vec{r}) = \left[-\frac{\hbar^2}{2m}(\vec{\nabla} - i\vec{A}(\vec{r}))^2 + V(\vec{r})\right]\psi(\vec{r}) = E\psi(\vec{r})$$
Local Symmetries

• The invariance under a local phase transformation requires the introduction of additional fields
  – These fields are called gauge fields
  – Lead to the introduction of definite physical force

• The potential \( \vec{A}(\vec{r}) \) can be interpreted as the EM vector potential

• The symmetry group associated with the single parameter phase transformation in the previous slides is called Abelian or commuting symmetry and is called U(1) gauge group \( \rightarrow \) Electromagnetic force group
U(1) Local Gauge Invariance

Dirac Lagrangian for free particle of spin $\frac{1}{2}$ and mass $m$:

$$\mathcal{L} = i\hbar c \overline{\psi} \gamma^\mu \partial_\mu \psi - (mc^2) \overline{\psi} \psi$$

Is invariant under a global phase transformation (global gauge transformation) $\psi \rightarrow e^{i\theta} \psi$ since $\overline{\psi} \rightarrow e^{-i\theta} \overline{\psi}$.

However, if the phase, $\theta$, varies as a function of space-time coordinate, $x^\mu$, is $\mathcal{L}$ still invariant under the local gauge transformation, $\psi \rightarrow e^{i\theta(x)} \psi$?

No, because it adds an extra term from derivative of $\theta$. 
U(1) Local Gauge Invariance

Requiring the complete Lagrangian be invariant under $\lambda(x)$ local gauge transformation will require additional terms to free Dirac Lagrangian to cancel the extra term

$$\mathcal{L} = \left[ i \left( \frac{\hbar c}{\gamma} \right) \bar{\psi} \gamma^\mu \partial_\mu \psi - (mc^2) \bar{\psi} \psi \right] - (q \bar{\psi} \gamma^\mu \psi) A_\mu$$

Where $A_\mu$ is a new vector gauge field that transforms under local gauge transformation as follows:

$$A_\mu \rightarrow A_\mu + \partial_\mu \lambda$$

Addition of this vector field to $\mathcal{L}$ keeps $\mathcal{L}$ invariant under local gauge transformation, but…
U(1) Local Gauge Invariance

The new vector field couples with spinor through the last term. In addition, the full Lagrangian must include a “free” term for the gauge field. Thus, Proca Lagrangian needs to be added.

\[ \mathcal{L} = -\frac{1}{16\pi} F^{\mu\nu} F_{\mu\nu} + \frac{1}{8\pi} \left( \frac{m_A c}{\hbar} \right)^2 A^\nu A_\nu \]

This Lagrangian is not invariant under the local gauge transformation, \( A_\mu \rightarrow A_\mu + \partial_\mu \lambda \), because

\[
A^\nu A_\nu = (A^\nu - \partial_\mu \lambda)(A_\nu - \partial^\mu \lambda)
\]

\[
= A^\nu A_\nu - (A^\nu \partial^\mu \lambda + A_\nu \partial_\mu \lambda) + (\partial_\mu \lambda)(\partial^\mu \lambda)
\]
U(1) Local Gauge Invariance

The requirement of local gauge invariance forces the introduction of a massless vector field into the free Dirac Lagrangian.

\[
\mathcal{L} = \left[ i \left( \frac{\hbar c}{\gamma} \right) \overline{\psi} \gamma^\mu \partial_\mu \psi - \left( mc^2 \right) \overline{\psi} \psi \right] + \frac{-1}{16\pi} F_{\mu\nu} F^{\mu\nu} - \left( q \overline{\psi} \gamma^\mu \psi \right) A_\mu
\]

\( A_\mu \) is an electromagnetic potential.
And \( A_\mu \rightarrow A_\mu + \partial_\mu \lambda \) is a gauge transformation of an electromagnetic potential.
Gauge Fields and Local Symmetries

• To maintain a local symmetry, additional fields must be introduced
  – This is in general true for more complicated symmetries
  – A crucial information for modern physics theories

• A distinct fundamental forces in nature arise from local invariance of physical theories

• The associated gauge fields generate these forces
  – These gauge fields are the mediators of the given force

• This is referred as gauge principle, and such theories are gauge theories
  – Fundamental interactions are understood through this theoretical framework
Gauge Fields and Mediators

- To keep local gauge invariance, new particles had to be introduced in gauge theories
  - U(1) gauge introduced a new field (particle) that mediates the electromagnetic force: Photon
  - SU(2) gauge introduces three new fields that mediates weak force
    - Charged current mediator: \( W^+ \) and \( W^- \)
    - Neutral current: \( Z^0 \)
  - SU(3) gauge introduces 8 mediators for the strong force
- Unification of electromagnetic and weak force SU(2)\( \times \)U(1) introduces a total of four mediators
  - Neutral current: Photon, \( Z^0 \)
  - Charged current: \( W^+ \) and \( W^- \)
Parity

• The space inversion transformation (mirror image) ➔ Switch right-handed coordinate system to left-handed

\[
\begin{pmatrix}
  c & t \\
  x \\
  y \\
  z
\end{pmatrix} \quad \text{Parity} \quad \begin{pmatrix}
  c & t \\
  \quad -x \\
  \quad -y \\
  \quad -z
\end{pmatrix}
\]

• How is this different than spatial rotation?
  – Rotation is continuous in a given coordinate system
    • Quantum numbers related rotational transformation are continuous
  – Space inversion cannot be obtained through any set of rotational transformation
    • Quantum numbers related to space inversion is discrete
Properties of Parity

- Position and momentum vectors change sign under space inversion
  \[ \vec{r} \xrightarrow{P} -\vec{r} \]
  \[ \vec{p} = m\vec{r} \xrightarrow{P} -m\vec{r} = -\vec{p} \]
- Where as their magnitudes do not change signs
  \[ r = \sqrt{\vec{r} \cdot \vec{r}} \xrightarrow{P} \sqrt{(-\vec{r}) \cdot (-\vec{r})} = \sqrt{\vec{r} \cdot \vec{r}} = r \]
  \[ p = \sqrt{\vec{p} \cdot \vec{p}} \xrightarrow{P} \sqrt{(-\vec{p}) \cdot (-\vec{p})} = \sqrt{\vec{p} \cdot \vec{p}} = p \]
- Vectors change signs under space-inversion while the scalars do not.
Properties of Parity

• Some vectors, however, behave like a scalar
  – Angular momentum
    \[ \vec{L} = \vec{r} \times \vec{p} \quad \text{P.} \quad (-\vec{r}) \times (-\vec{p}) = \vec{r} \times \vec{p} = \vec{L} \]
  – These are called pseudo-vectors or axial vectors

• Likewise some scalars behave like vectors
  \[ \vec{a} \cdot (\vec{b} \times \vec{c}) \quad \text{P.} \quad (-\vec{a}) \cdot \left( (-\vec{b}) \times (-\vec{c}) \right) = -\vec{a} \cdot (\vec{b} \times \vec{c}) \]
  – These are called pseudo-scalars

• Two successive application of parity operations must turn the coordinates back to original
  – The possible values (eigen values) of parity are +1 (even) or -1 (odd).

• Parity is a multiplicative quantum number
Parity

- Two parity quantum numbers
  - Intrinsic parity: Bosons have the same intrinsic parities as their anti-particles while fermions have opposite parity than its anti-particle (odd)
  - Parity under spatial transformation that follows the rule: \( P = (-1)^l \)
    - \( l \) is the orbital angular momentum quantum number

- Are electromagnetic and gravitational forces invariant under parity operation or space inversion?
  - Newton’s equation of motion for a point-like particle \( \frac{d^2 \vec{r}}{dt^2} = \vec{F} \)
  - For electromagnetic and gravitational forces we can write the forces \( m \frac{d^2 \vec{r}}{dt^2} = \vec{F} = \frac{C}{r^2} \hat{r} \), and thus are invariant under parity.
Determination of Parity Quantum Numbers

- How do we find out the intrinsic parity of particles?
  - Use observation of decays and production processes
  - Absolute determination of parity is not possible, just like electrical charge or other quantum numbers.
  - Thus the accepted convention is to assign \(+1\) intrinsic parity to proton, neutron and the \(\Lambda\) hyperon.
- The parities of other particles are determined relative to these assignments through the analysis of parity conserving interactions involving these particles.
Assignments

1. No homework today!!!