PHYS 1441 – Section 501
Lecture #2

Monday, June 7, 2004
Dr. Jaehoon Yu

- Chapter two: Motion in one dimension
  - Velocity (Average and Instantaneous)
  - Acceleration (Average and instantaneous)
  - One dimensional motion at constant acceleration
    - Free Fall
  - Coordinate systems

Remember the quiz this Wednesday!!

Announcements

- Reading assignment #1: Read and follow through all sections in appendix A by Wednesday, June 9
  - A-1 through A-9
- There will be a quiz on this Wednesday, June 9, on these and Chapter 1
- E-mail distribution list: 16 of you have registered
  - Remember 5 (3) extra credit points if done by midnight tonight (Wednesday).
- Homework: You are supposed to download the homework assignment, solve it offline and input the answers back online.
  - 38 registered
  - 25 submitted
  - Must be submitted by 6pm Wednesday to get full, free credit.

Displacement, Velocity and Speed

One dimensional displacement is defined as:

$$\Delta x \equiv x_f - x_i$$

Displacement is the difference between initial and final potions of motion and is a vector quantity

Average velocity is defined as:

$$v_x \equiv \frac{x_f - x_i}{t_f - t_i} = \frac{\Delta x}{\Delta t}$$

Displacement per unit time in the period throughout the motion

Average speed is defined as:

$$v \equiv \frac{\text{Total Distance Traveled}}{\text{Total Time Interval}}$$

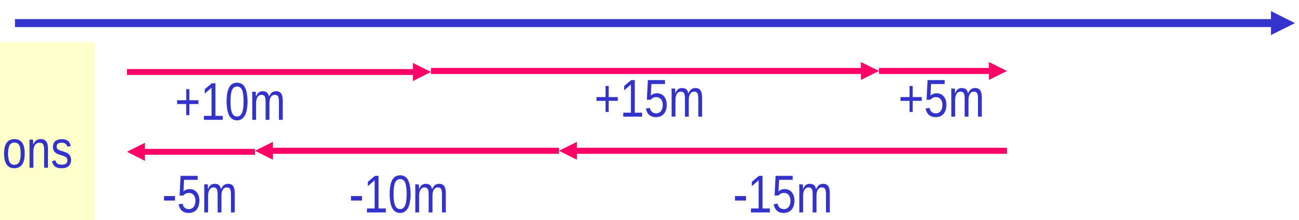
Can someone tell me what the difference between speed and velocity is?

Difference between Speed and Velocity

- Let's take a simple one dimensional translation that has many steps:

Let's call this line as X-axis

Let's have a couple of motions in a total time interval of 20 sec.



Total Displacement: $\Delta x \equiv x_f - x_i = x_i - x_i = 0(m)$

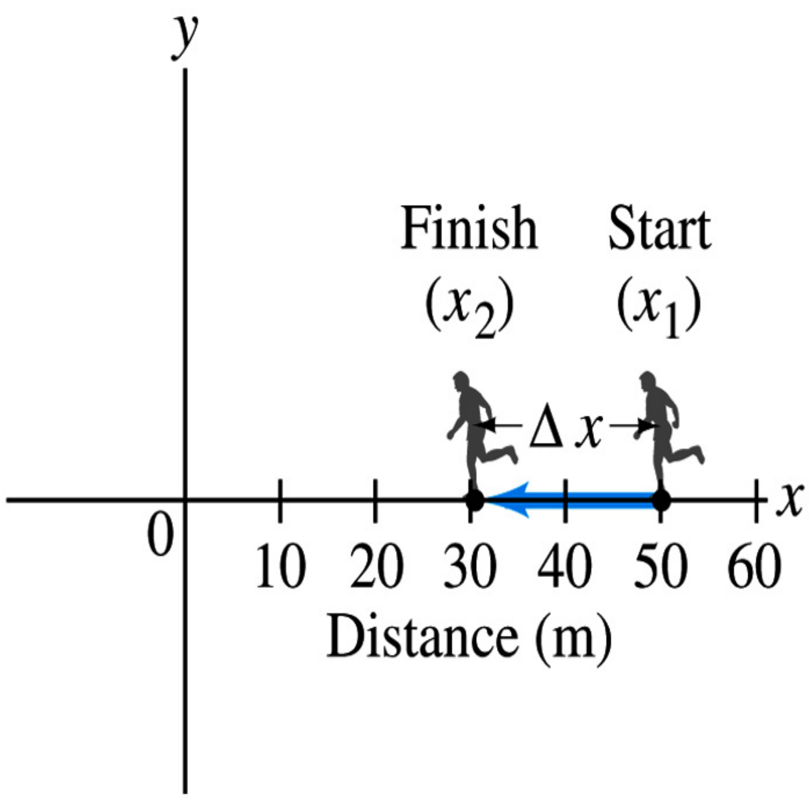
Average Velocity: $v_x \equiv \frac{x_f - x_i}{t_f - t_i} = \frac{\Delta x}{\Delta t} = \frac{0}{20} = 0(m/s)$

Total Distance Traveled: $D = 10 + 15 + 5 + 15 + 10 + 5 = 60(m)$

Average Speed: $v \equiv \frac{\text{Total Distance Traveled}}{\text{Total Time Interval}} = \frac{60}{20} = 3(m/s)$

Example 2.1

The position of a runner as a function of time is plotted as moving along the x axis of a coordinate system. During a 3.00-s time interval, the runner's position changes from x_1=50.0m to x_2=30.5 m, as shown in the figure. What was the runner's average velocity? What was the average speed?



- Displacement:

$$\Delta x \equiv x_f - x_i = x_2 - x_1 = 30.5 - 50.0 = -19.5(m)$$

- Average Velocity:

$$v_x \equiv \frac{x_f - x_i}{t_f - t_i} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t} = \frac{-19.5}{3.00} = -6.50(m/s)$$

- Average Speed:

$$v \equiv \frac{\text{Total Distance Traveled}}{\text{Total Time Interval}}$$

$$= \frac{50.0 - 30.5}{3.00} = \frac{+19.5}{3.00} = +6.50(m/s)$$

Instantaneous Velocity and Speed

- Can average quantities tell you the detailed story of the whole motion?
- Instantaneous velocity is defined as:

$$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

  - What does this mean?
    - Displacement in an infinitesimal time interval
    - Average velocity over a very short amount of time

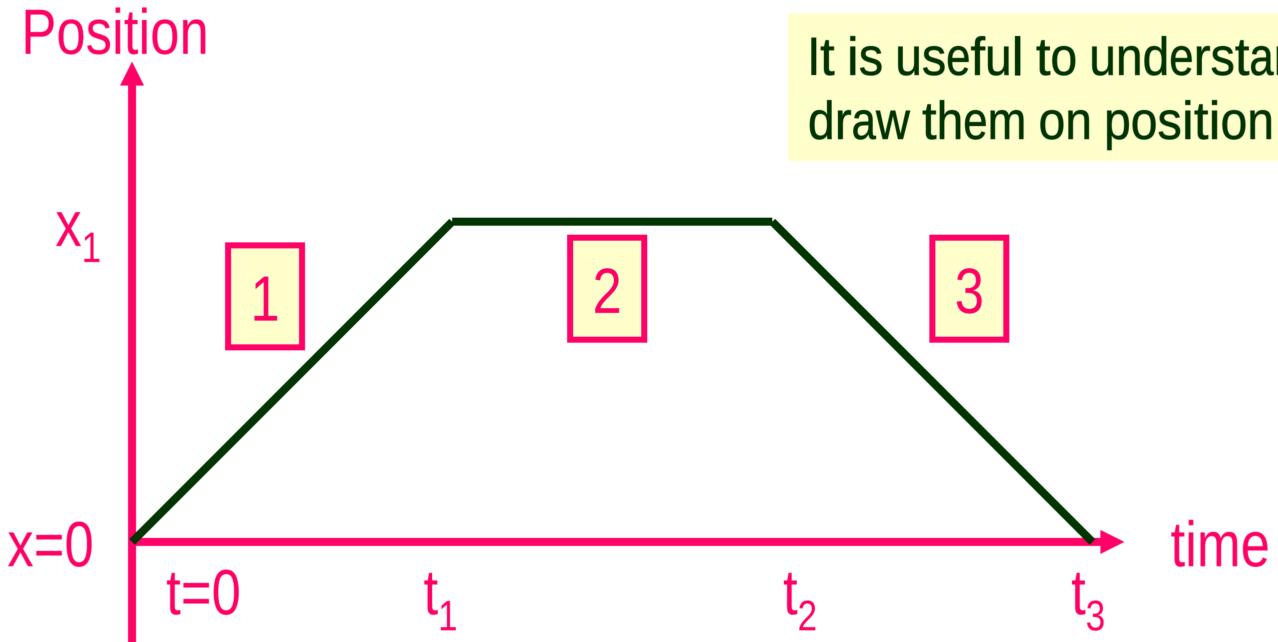
•Instantaneous speed is the size (magnitude) of the velocity vector:

$$|v_x| = \left| \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} \right|$$

*Magnitude of Vectors are Expressed in absolute values

Position vs Time Plot

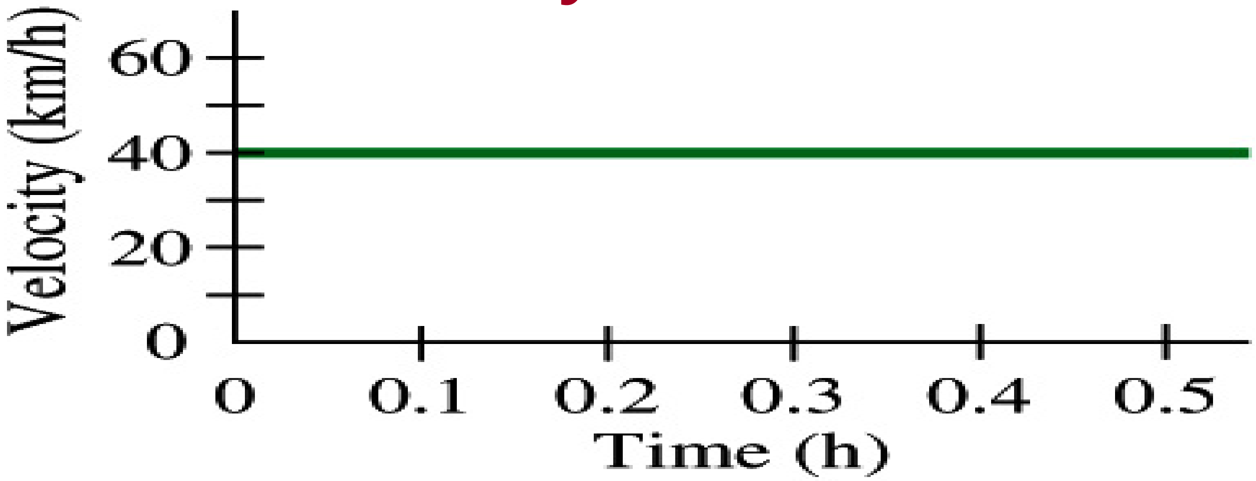
It is useful to understand motions to draw them on position vs time plots.

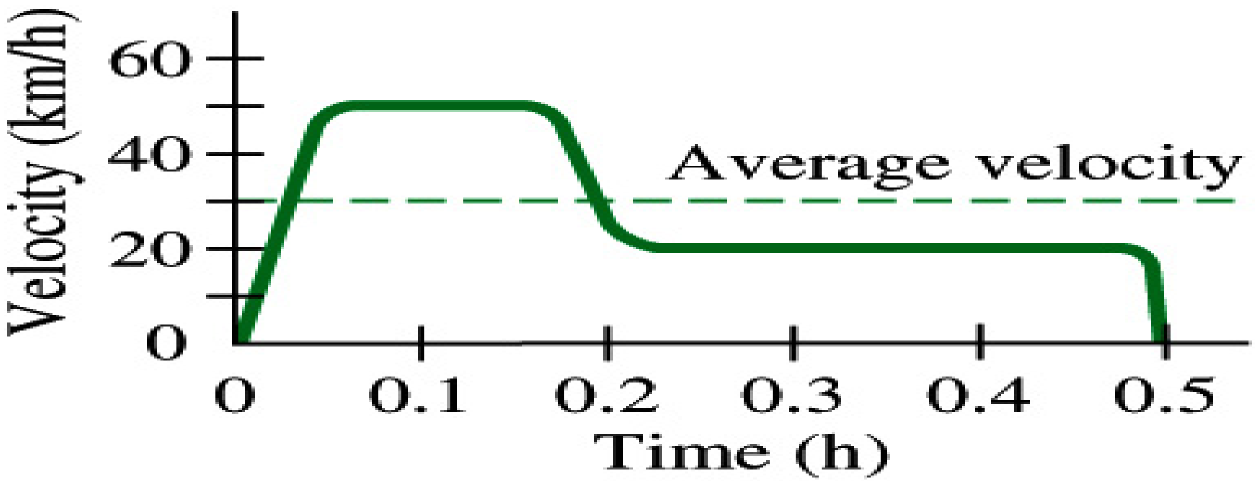


1. Running at a constant velocity (go from x=0 to $x=x_1$ in t_1, Displacement is + x_1 in t_1 time interval)
2. Velocity is 0 (go from x_1 to x_1 no matter how much time changes)
3. Running at a constant velocity but in the reverse direction as 1. (go from x_1 to x=0 in t_3-t_2 time interval, Displacement is - x_1 in t_3-t_2 time interval)

Does this motion physically make sense?

Velocity vs Time Plot





Displacement, Velocity and Speed

Displacement

$$\Delta x \equiv x_f - x_i$$

Average velocity

$$v_x \equiv \frac{x_f - x_i}{t_f - t_i} = \frac{\Delta x}{\Delta t}$$

Average speed

$$v \equiv \frac{\text{Total Distance Traveled}}{\text{Total Time Spent}}$$

Instantaneous velocity

$$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

Instantaneous speed

$$|v_x| = \left| \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} \right|$$

Acceleration

Change of velocity in time (what kind of quantity is this?)

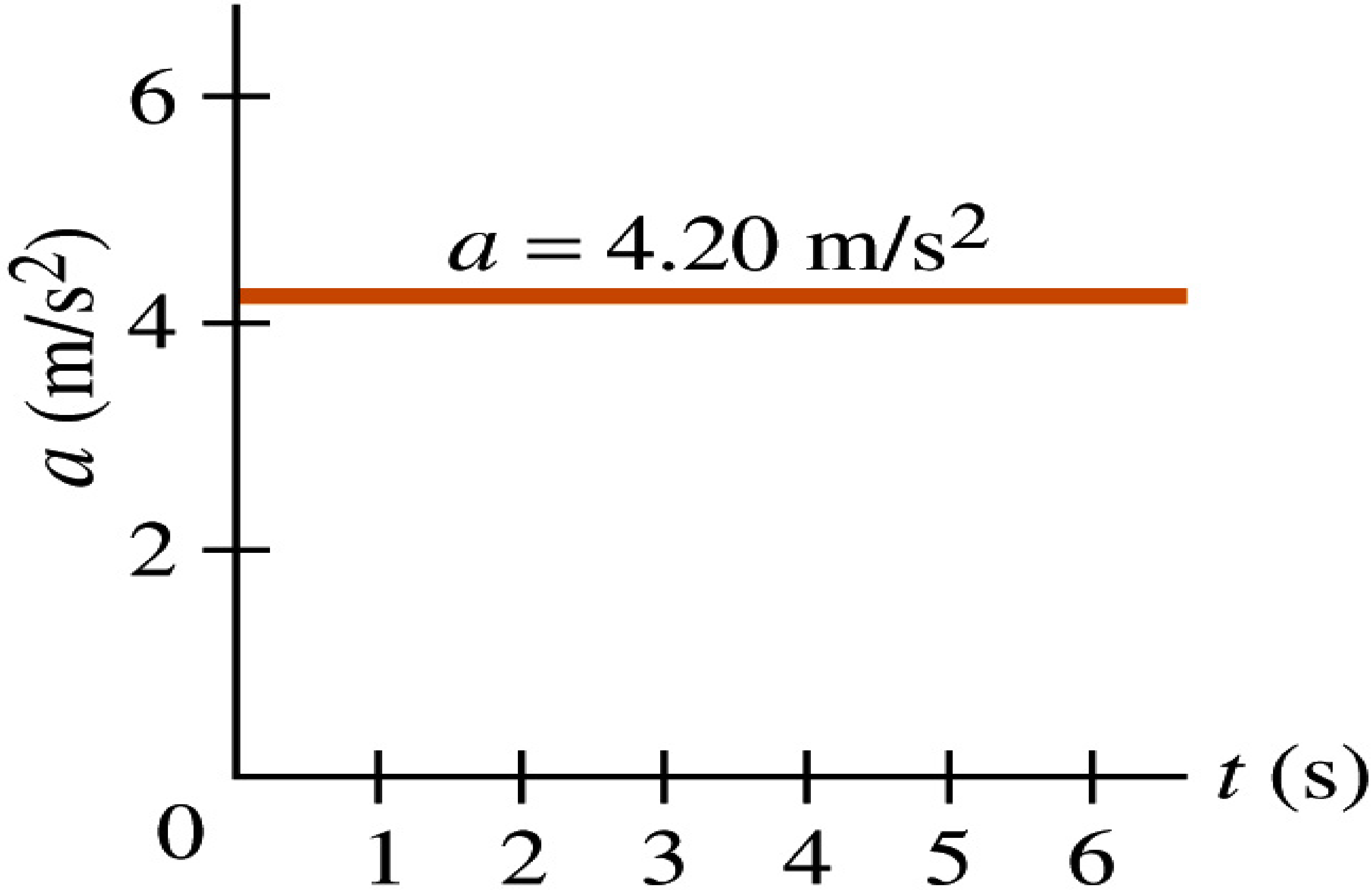
- Average acceleration:

$$a_x \equiv \frac{v_{xf} - v_{xi}}{t_f - t_i} = \frac{\Delta v_x}{\Delta t} \quad \text{analogs to} \quad v_x \equiv \frac{x_f - x_i}{t_f - t_i} = \frac{\Delta x}{\Delta t}$$

- Instantaneous acceleration: Average acceleration over a very short amount of time.

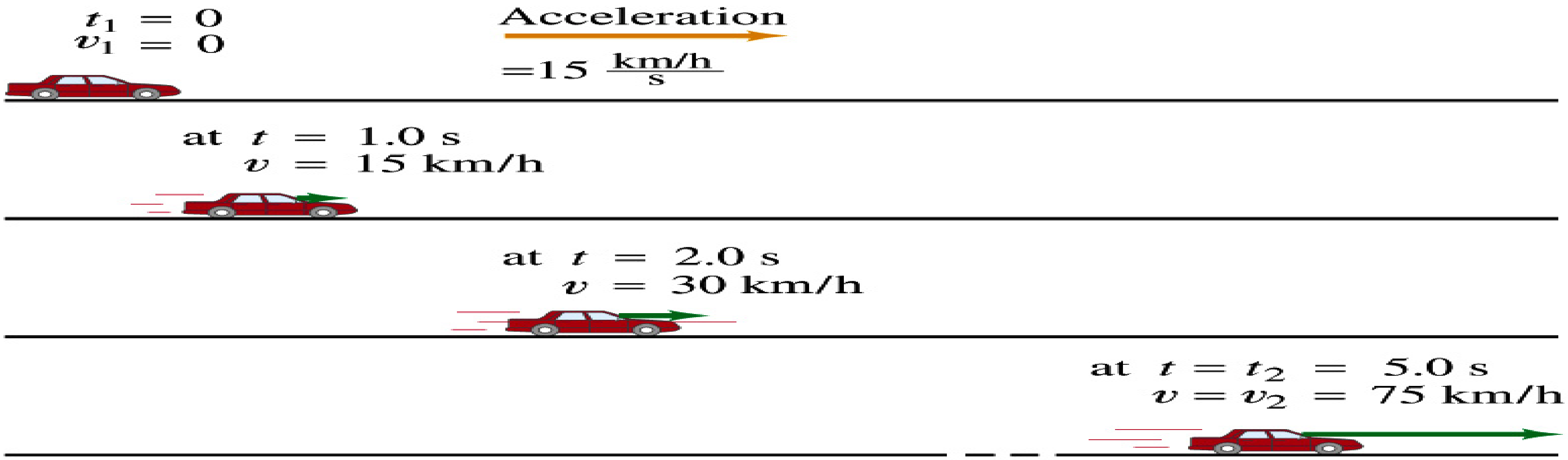
$$a_x \equiv \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} \quad \text{analogs to} \quad \boxed{v_x \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}}$$

Acceleration vs Time Plot



Example 2.4

A car accelerates along a straight road from rest to 75km/h in 5.0s.



What is the magnitude of its average acceleration?

$$v_{xi} = 0\ m/s$$

$$v_{xf} = \frac{75000m}{3600s} = 21\ m/s$$

$$\bar{a}_x = \frac{v_{xf} - v_{xi}}{t_f - t_i} = \frac{\Delta v_x}{\Delta t} = \frac{21-0}{5.0} = \frac{21}{5.0} = 4.2(m/s^2)$$

$$= \frac{4.2 \times (3600)^2}{1000} = 5.4 \times 10^4 (km/h^2)$$

Meanings of Acceleration

- When an object is moving in a constant velocity ($v=v_0$), there is no acceleration ($a=0$)
  - Is there any acceleration when an object is not moving?
- When an object is moving faster as time goes on, ($v=v(t)$), acceleration is positive ($a>0$)
  - Incorrect, since the object might be moving in negative direction initially
- When an object is moving slower as time goes on, ($v=v(t)$), acceleration is negative ($a<0$)
  - Incorrect, since the object might be moving in negative direction initially
- In all cases, velocity is positive, unless the direction of the movement changes.
  - Incorrect, since the object might be moving in negative direction initially
- Is there acceleration if an object moves in a constant speed but changes direction?

The answer is YES!!

One Dimensional Motion

- Let's start with the simplest case: acceleration is a constant ($a=a_0$)
- Using definitions of average acceleration and velocity, we can derive equations of motion (description of motion, velocity and position as a function of time)

$$\bar{a}_x = a_x = \frac{v_{xf} - v_{xi}}{t_f - t_i}$$ (If $t_f=t$ and $t_i=0$) $$a_x = \frac{v_{xf} - v_{xi}}{t} \Rightarrow v_{xf} = v_{xi} + a_x t$$

For constant acceleration, average velocity is a simple numeric average

$$\bar{v}_x = \frac{v_{xi} + v_{xf}}{2} = \frac{2v_{xi} + a_x t}{2} = v_{xi} + \frac{1}{2}a_x t$$

$$\bar{v}_x = \frac{x_f - x_i}{t_f - t_i}$$ (If $t_f=t$ and $t_i=0$) $$\bar{v}_x = \frac{x_f - x_i}{t} \Rightarrow x_f = x_i + \bar{v}_x t$$

Resulting Equation of Motion becomes

$$x_f = x_i + \bar{v}_x t = x_i + v_{xi} t + \frac{1}{2}a_x t^2$$

One Dimensional Motion cont'd

Average velocity $\bar{v}_x = \frac{v_{xi} + v_{xf}}{2}$ ⇒ $x_f = x_i + \bar{v}_x t = x_i + \left(\frac{v_{xi} + v_{xf}}{2}\right)t$

Since $a_x = \frac{v_{xf} - v_{xi}}{t}$ Solving for t ⇒ $t = \frac{v_{xf} - x_{xi}}{a_x}$

Substituting t in the above equation,

$$x_f = x_i + \left(\frac{v_{xf} + v_{xi}}{2}\right)\left(\frac{v_{xf} - v_{xi}}{a_x}\right) = x_i + \frac{v_{xf}^2 - v_{xi}^2}{2a_x}$$

Resulting in

$$v_{xf}^2 = v_{xi}^2 + 2a_x\left(x_f - x_i\right)$$

Kinematic Equations of Motion on a Straight Line Under Constant Acceleration

$$v_{xf}(t) = v_{xi} + a_x t$$

Velocity as a function of time

$$x_f - x_i = \frac{1}{2}\bar{v}_x t = \frac{1}{2}\left(v_{xf} + v_{xi}\right) t$$

Displacement as a function of velocities and time

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$$

Displacement as a function of time, velocity, and acceleration

$$v_{xf}^2 = v_{xi}^2 + 2a_x\left(x_f - x_i\right)$$

Velocity as a function of Displacement and acceleration

You may use different forms of Kinetic equations, depending on the information given to you for specific physical problems!!

How do we solve a problem using a kinematic formula for constant acceleration?

- Identify what information is given in the problem.
  - Initial and final velocity?
  - Acceleration?
  - Distance?
  - Time?
- Identify what the problem wants.
- Identify which formula is appropriate and easiest to solve for what the problem wants.
  - Frequently multiple formulae can give you the answer for the quantity you are looking for. ➔ Do not just use any formula but use the one that can be easiest to solve.
- Solve the equation for the quantity wanted

Example 2.10

Suppose you want to design an air-bag system that can protect the driver in a head-on collision at a speed 100km/s (~60miles/hr). Estimate how fast the air-bag must inflate to effectively protect the driver. Assume the car crumples upon impact over a distance of about 1m. How does the use of a seat belt help the driver?

How long does it take for the car to come to a full stop? ⇨ As long as it takes for it to crumple.

The initial speed of the car is $v_{xi} = 100km/h = \frac{100000m}{3600s} = 28m/s$

We also know that $v_{xf} = 0m/s$ and $x_f - x_i = 1m$

Using the kinematic formula $v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$

The acceleration is $a_x = \frac{v_{xf}^2 - v_{xi}^2}{2(x_f - x_i)} = \frac{0-(28m/s)^2}{2\times 1m} = -390m/s^2$

Thus the time for air-bag to deploy is $t = \frac{v_{xf} - v_{xi}}{a} = \frac{0 - 28m/s}{-390m/s^2} = 0.07s$