PHYS 1441 – Section 501
Lecture #4

Monday, June 14, 2004
Dr. Jaehoon Yu

- Chapter three: Motion in two dimension
  - Two dimensional equation of motion
  - Projectile motion
- Chapter four: Newton's Laws of Motion
  - Force and Mass
  - Newton's Laws of Motion
  - Solving problems using Newton's Laws
  - Friction Forces

First term exam at 6pm, next Wednesday, June 23!!

Announcements

- E-mail distribution list: 36 of you have registered
  - This Wednesday is the last day of e-mail registration
  - -5 extra points if you don't register by next Wednesday, June 23
  - A test message was sent out today.
- Term exam:
  - Date: Next Wednesday, June 26
  - Time: In the class, 6 – 7:50pm
  - Covers: Chapters 1 – 6.5 and Appendix A
  - Location: Classroom, SH 125
  - Mixture of multiple choices and numeric calculations
  - There will be a total of 3 exams of which two best will be chose for your grades
  - Missing an exam is not permitted unless approved prior to the exam by me → I will only be in e-mail contact for this exam.
- Class Schedule:
  - Dr. White will teach you on June 16 and 21
  - Mr. Kaushik (Rm 133, x25700) will proctor your exam on June 23

Displacement, Velocity, and Acceleration in 2-dim

Quantity	One Dimension	Two Dimensions
Displacement	$\Delta x \equiv x_f - x_i$	$\Delta \vec{r} = \vec{r}_f - \vec{r}_i$
Average Velocity	$v_x \equiv \frac{x_f - x_i}{t_f - t_i} = \frac{\Delta x}{\Delta t}$	$\vec{v} \equiv \frac{\Delta \vec{r}}{\Delta t} = \frac{\vec{r}_f - \vec{r}_i}{t_f - t_i}$
Instantaneous Velocity	$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$	$\vec{v} \equiv \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t}$
Average Acceleration	$a_x \equiv \frac{v_{xf} - v_{xi}}{t_f - t_i} = \frac{\Delta v_x}{\Delta t}$	$\vec{a} \equiv \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i}$
Instantaneous Acceleration	$a_x \equiv \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t}$	$\vec{a} \equiv \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t}$

2-dim Motion Under Constant Acceleration

- Position vectors in x-y plane: $\vec{r}_i = x_i\vec{i} + y_i\vec{j}$ $\quad \vec{r}_f = x_f\vec{i} + y_f\vec{j}$
- Velocity vectors in x-y plane: $\vec{v}_i = v_{xi}\vec{i} + v_{yi}\vec{j}$ $\quad \vec{v}_f = v_{xf}\vec{i} + v_{yf}\vec{j}$

Velocity vectors in terms of acceleration vector

$$v_{xf} = v_{xi} + a_x t \qquad v_{yf} = v_{yi} + a_y t$$

$$\vec{v}_f = \left(v_{xi} + a_x t\right)\vec{i} + \left(v_{yi} + a_y t\right)\vec{j} = \vec{v}_i + \vec{a}t$$

- How are the position vectors written in acceleration vectors?

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2 \qquad y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$

$$\vec{r}_f = x_f\vec{i} + y_f\vec{j} = \left(x_i + v_{xi}t + \frac{1}{2}a_x t^2\right)\vec{i} + \left(y_i + v_{yi}t + \frac{1}{2}a_y t^2\right)\vec{j}$$

$$= \left(x_i\vec{i} + y_i\vec{j}\right) + \left(v_{xi}\vec{i} + v_{yi}\vec{j}\right)t + \frac{1}{2}\left(a_x\vec{i} + a_y\vec{j}\right)t^2 = \vec{r}_i + \vec{v}_i t + \frac{1}{2}\vec{a}t^2$$

Example for 2-D Kinematic Equations

A particle starts at origin when t=0 with an initial velocity **v**=(20**i**-15**j**)m/s. The particle moves in the xy plane with $a_x=4.0m/s^2$. Determine the components of velocity vector at any time, t.

$$v_{xf} = v_{xi} + a_x t = 20 + 4.0t\left(m/s\right) \quad v_{yf} = v_{yi} + a_y t = -15 + 0t = -15\left(m/s\right)$$

Velocity vector

$$\vec{v}(t) = v_x\left(t\right)\vec{i} + v_y\left(t\right)\vec{j} = \left(20 + 4.0t\right)\vec{i} - 15\vec{j}(m/s)$$

Compute the velocity and speed of the particle at t=5.0 s.

$$\vec{v}_{t=5} = v_{x,t=5}\vec{i} + v_{y,t=5}\vec{j} = \left(20 + 4.0 \times 5.0\right)\vec{i} - 15\vec{j} = \left(40\vec{i} - 15\vec{j}\right)\ m/s$$

$$speed = \left|\vec{v}\right| = \sqrt{\left(v_x\right)^2 + \left(v_y\right)^2} = \sqrt{\left(40\right)^2 + \left(-15\right)^2} = 43m/s$$

Example for 2-D Kinematic Eq. Cnt'd

Angle of the Velocity vector

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{-15}{40}\right) = \tan^{-1}\left(\frac{-3}{8}\right) = -21^\circ$$

Determine the x and y components of the particle at t=5.0 s.

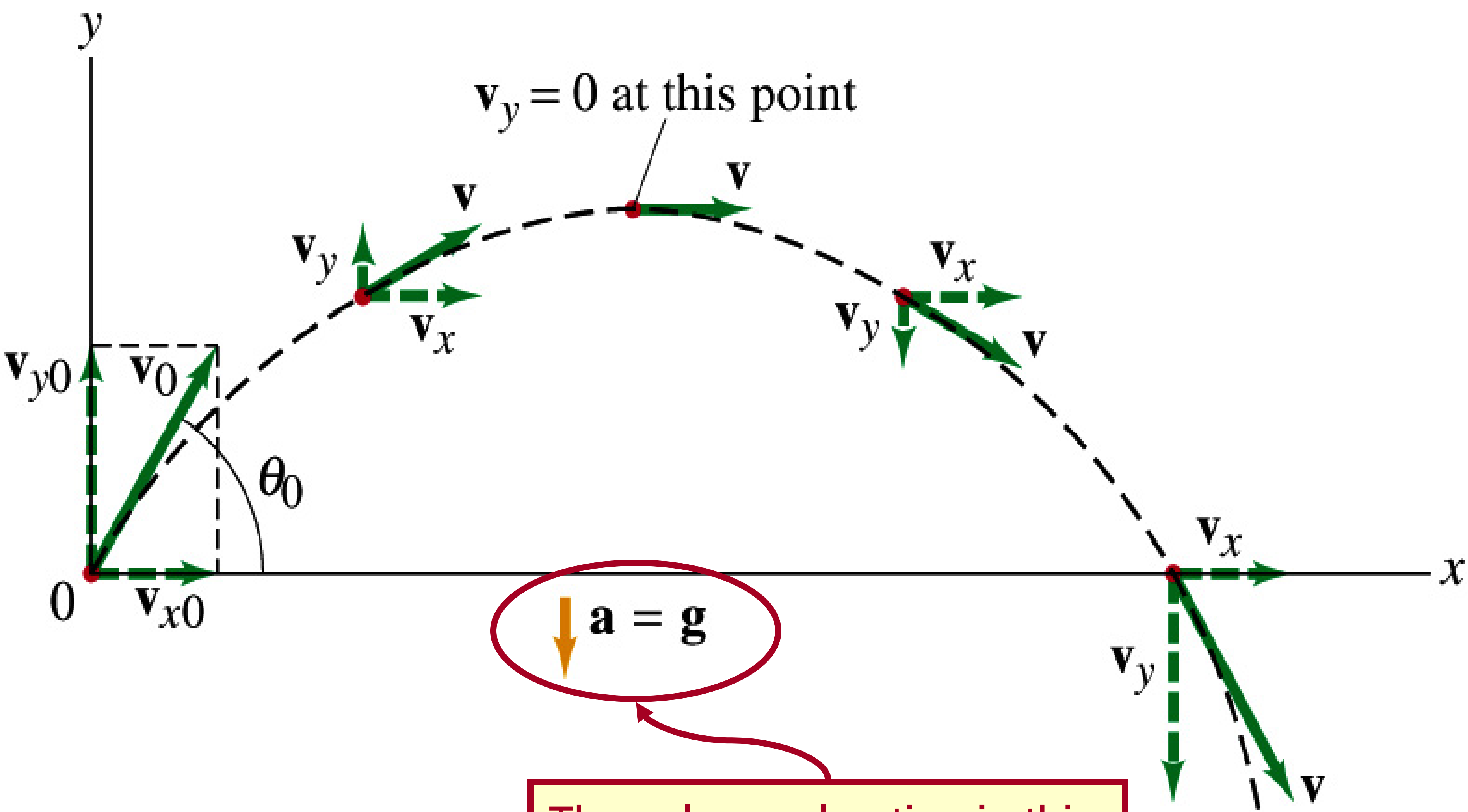
$$x_f = v_{xi}t + \frac{1}{2}a_x t^2 = 20\times5 + \frac{1}{2}\times4\times5^2 = 150(m)$$

$$y_f = v_{yi}t = -15\times5 = -75\,(m)$$

Can you write down the position vector at t=5.0s?

$$\vec{r}_f = x_f\vec{i} + y_f\vec{j} = 150\vec{i} - 75\vec{j}\,(m)$$

Projectile Motion



The only acceleration in this motion. It is a constant!!

Projectile Motion

- A 2-dim motion of an object under the gravitational acceleration with the assumptions
  - Gravitational acceleration, -*g*, is constant over the range of the motion
  - Air resistance and other effects are negligible
- A motion under constant acceleration!!!! ➔ Superposition of two motions
  - Horizontal motion with constant velocity and
  - Vertical motion under constant acceleration

In a projectile motion, the only acceleration is gravitational one whose direction is always toward the center of the earth (downward).

Show that a projectile motion is a parabola!!!

$$v_{xi} = v_i \cos\theta_i \qquad v_{yi} = v_i \sin\theta_i$$

$$\vec{a} = a_x\vec{i} + a_y\vec{j} = -g\vec{j}$$

$a_x=0$ ➔

$$x_f = v_{xi}t = v_i\cos\theta_i t \qquad t = \frac{x_f}{v_i\cos\theta_i}$$

$$y_f = v_{yi}t + \frac{1}{2}(-g)\,t^2$$

$$= v_i\sin\theta_i t - \frac{1}{2}gt^2$$

Plug in the t above ➔

$$y_f = v_i\sin\theta_i\left(\frac{x_f}{v_i\cos\theta_i}\right) - \frac{1}{2}g\left(\frac{x_f}{v_i\cos\theta_i}\right)^2$$

$$y_f = x_f\tan\theta_i - \left(\frac{g}{2v_i^2\cos^2\theta_i}\right)x_f^2$$

What kind of parabola is this?

Example for Projectile Motion

A ball is thrown with an initial velocity **v**=(20**i**+40**j**)m/s. Estimate the time of flight and the distance the ball is from the original position when landed.

Which component determines the flight time and the distance?

Flight time is determined by y component, because the ball stops moving when it is on the ground after the flight.

$$y_f = 40t + \frac{1}{2}(-g)t^2 = 0m$$

$$t(80 - gt) = 0$$

$$\therefore t = 0 \text{ or } t = \frac{80}{g} \approx 8\sec$$

$$\therefore\ t \approx 8\sec$$

Distance is determined by x component in 2-dim, because the ball is at $y=0$ position when it completed it's flight.

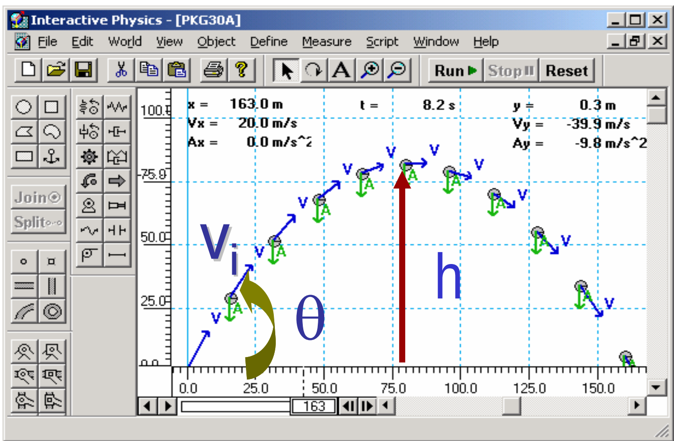
$$x_f = v_{xi}t = 20 \times 8 = 160(m)$$

Horizontal Range and Max Height

- Based on what we have learned in the previous pages, one can analyze a projectile motion in more detail
  - Maximum height an object can reach
  - Maximum range

What happens at the maximum height?

At the maximum height the object's vertical motion stops to turn around!!



$$v_{yf} = v_{yi} + a_y t = v_i \sin\theta_i - g t_A = 0$$

$$\therefore t_A = \frac{v_i \sin\theta_i}{g}$$

$$y_f = h = v_{yi} t + \frac{1}{2}(-g)t^2$$

$$y_f = v_i \sin\theta_i \left(\frac{v_i \sin\theta_i}{g}\right) - \frac{1}{2} g \left(\frac{v_i \sin\theta_i}{g}\right)^2$$

$$y_f = \left(\frac{v_i^2 \sin^2\theta_i}{2g}\right)$$

Since no acceleration in x, it still flies even if v_y=0

$$R = v_{xi} 2t_A =$$

$$2v_i \cos\theta_i \left(\frac{v_i \sin\theta_i}{g}\right)$$

$$R = \left(\frac{v_i^2 \sin 2\theta_i}{g}\right)$$

Maximum Range and Height

- What are the conditions that give maximum height and range of a projectile motion?

$$h = \left(\frac{v_i^2 \sin^2 \theta_i}{2g} \right)$$

This formula tells us that the maximum height can be achieved when $\theta_i = 90^o$!!!

$$R = \left(\frac{v_i^2 \sin 2\theta_i}{g} \right)$$

This formula tells us that the maximum range can be achieved when $2\theta_i = 90^o$, i.e., $\theta_i = 45^o$!!!

Example for Projectile Motion

- A stone was thrown upward from the top of a building at an angle of 30º to horizontal with initial speed of 20.0m/s. If the height of the building is 45.0m, how long is it before the stone hits the ground?

$$v_{xi} = v_i \cos\theta_i = 20.0 \times \cos 30^\circ = 17.3 m/s$$

$$v_{yi} = v_i \sin\theta_i = 20.0 \times \sin 30^\circ = 10.0 m/s$$

$$y_f = -45.0 = v_{yi} t - \frac{1}{2} g t^2 \qquad g t^2 - 20.0 t - 90.0 = 9.80 t^2 - 20.0 t - 90.0 = 0$$

$$t = \frac{20.0 \pm \sqrt{(-20)^2 - 4 \times 9.80 \times (-90)}}{2 \times 9.80}$$

$$t = -2.18 s \quad \text{or} \quad t = 4.22 s \qquad t = 4.22 s$$

Only physical answer

- What is the speed of the stone just before it hits the ground?

$$v_{xf} = v_{xi} = v_i \cos\theta_i = 20.0 \times \cos 30^\circ = 17.3 m/s$$

$$v_{yf} = v_{yi} - g t = v_i \sin\theta_i - g t = 10.0 - 9.80 \times 4.22 = -31.4 m/s$$

$$|v| = \sqrt{v_{xf}^2 + v_{yf}^2} = \sqrt{17.3^2 + (-31.4)^2} = 35.9 m/s$$

Maximum Range and Height

- What are the conditions that give maximum height and range of a projectile motion?

$$h = \left(\frac{v_i^2 \sin^2 \theta_i}{2g} \right)$$

This formula tells us that the maximum height can be achieved when $\theta_i = 90^o$!!!

$$R = \left(\frac{v_i^2 \sin 2\theta_i}{g} \right)$$

This formula tells us that the maximum range can be achieved when $2\theta_i = 90^o$, i.e., $\theta_i = 45^o$!!!

Force

We've been learning kinematics; describing motion without understanding what the cause of the motion was. Now we are going to learn dynamics!!

Can someone tell me what FORCE is?

FORCEs are what cause an object to move

The above statement is not entirely correct. Why?

Because when an object is moving with a constant velocity no force is exerted on the object!!!

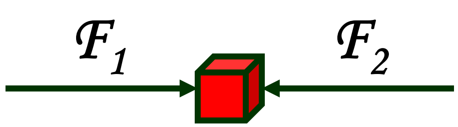
FORCEs are what cause any changes in the velocity of an object!!

What does this statement mean?

When there is force, there is change of velocity. Forces cause acceleration.

What happens if there are several forces being exerted on an object?

*Forces are **vector** quantities, so vector sum of all forces, the NET FORCE, determines the motion of the object.*



NET FORCE,
$\mathbf{F} = F_1 + F_2$

*When net force on an objectis **0**, it has constant velocity and is at its equilibrium!!*

More Force

There are various classes of forces

Contact Forces: Forces exerted by physical contact of objects

Examples of Contact Forces: Baseball hit by a bat, Car collisions

Field Forces: Forces exerted without physical contact of objects

Examples of Field Forces: Gravitational Force, Electro-magnetic force

What are possible ways to measure strength of Force?

A calibrated spring whose length changes linearly with the exerted force.

Forces are vector quantities, so addition of multiple forces must be done following the rules of vector additions.

Newton's First Law and Inertial Frames

Aristotle (384-322BC): *A natural state of a body is rest. Thus force is required to move an object. To move faster, ones needs larger force.*

Galileo's statement on natural states of matter: *Any velocity once imparted to a moving body will be rigidly maintained as long as external causes of retardation are removed!!*

> Galileo's statement is formulated by Newton into the 1st law of motion (Law of Inertia): *In the absence of external forces, an object at rest remains at rest and an object in motion continues in motion with a constant velocity.*

What does this statement tell us?

- When no force is exerted on an object, the acceleration of the object is 0.
- Any isolated object, the object that do not interact with its surroundings, is either at rest or moving at a constant velocity.
- Objects would like to keep its current state of motion, as long as there is no force that interferes with the motion. This tendency is called the Inertia.

> A frame of reference that is moving at constant velocity is called an ***Inertial Frame***

Mass

Mass: *A measure of the inertia of a body* or *quantity of matter*

1. Independent of the object's surroundings: The same no matter where you go.
2. Independent of method of measurement: The same no matter how you measure it.

The heavier an object gets the bigger the inertia!!

It is harder to make changes of motion of a heavier object than the lighter ones.

The same forces applied to two different masses result in different acceleration depending on the mass.

$$\boxed{\frac{m_1}{m_2} \equiv \frac{a_2}{a_1}}$$

Note that mass and weight of an object are two different quantities!!

Weight of an object is the magnitude of gravitational force exerted on the object.

Not an inherent property of an object!!!

Weight will change if you measure on the Earth or on the moon.

Newton's Second Law of Motion

The acceleration of an object is directly proportional to the net force exerted on it and is inversely proportional to the object's mass.

How do we write the above statement in a mathematical expression?

$$\sum_i \vec{F}_i = m\vec{a}$$

Since it's a vector expression, each component should also satisfy:

$$\sum_i F_{ix} = ma_x \qquad \sum_i F_{iy} = ma_y \qquad \sum_i F_{iz} = ma_z$$

From the above vector expression, what do you conclude the dimension and unit of force are?

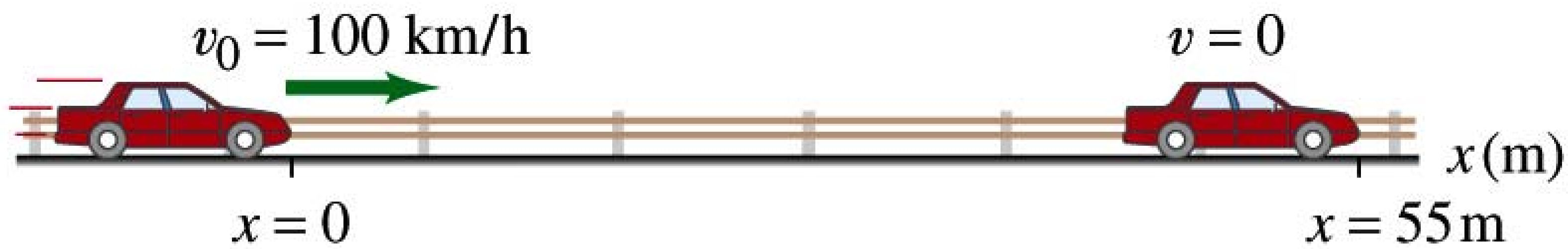
The unit of force in SI is $[Force] = [m][a] = kg \cdot m/s^2$

For ease of use, we define a new derived unit called, a Newton (N)

$$1N \equiv 1kg \cdot m/s^2 \approx \frac{1}{4} lbs$$

Example 4.2

What constant net force is required to bring a 1500kg car to rest from a speed of 100km/h within a distance of 55m?



What do we need to know to figure out the force? **Acceleration!!**

What are given? Initial speed: $v_{xi} = 100\ km/h = 28\ m/s$ Final speed: $v_{xf} = 0\ m/s$

Displacement: $\Delta x = x_f - x_i = 55\ m$

This is a one dimensional motion. Which kinetic formula do we use to find acceleration?

$$v_{xf}^2 = v_{xi}^2 + 2a_x\left(x_f - x_i\right)$$

Acceleration

$$a_x = \frac{v_{xf}^2 - v_{xi}^2}{2\left(x_f - x_i\right)} = \frac{-\left(28\ m/s\right)^2}{2\left(55\ m\right)} = -7.1\ m/s^2$$

Thus, the force needed to stop the car is

$$F_x = ma_x = 1500kg \times \left(-7.1m/s^2\right) = -1.1\times10^4\ N$$

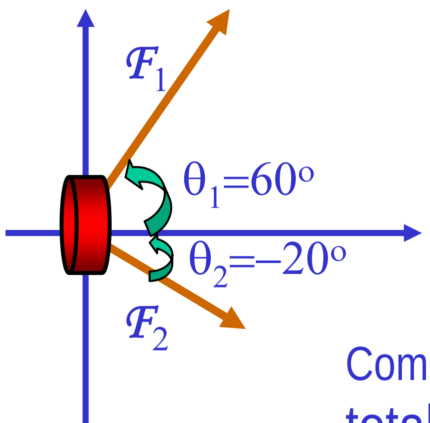
Given the force how far does the car move till it stops?

$$\Delta x = x_f - x_i = \frac{v_{xf}^2 - v_{xi}^2}{2a_x} = \frac{m\left(v_{xf}^2 - v_{xi}^2\right)}{2ma_x} = \frac{m\left(v_{xf}^2 - v_{xi}^2\right)}{2F_x}$$

- Linearly proportional to the mass of the car
- Squarely proportional to the speed of the car
- Inversely proportional to the force by the brake

Example for Newton's 2nd Law of Motion

Determine the magnitude and direction of acceleration of the puck whose



Components of $\mathcal{F}_1$

$$F_{1x} = \left|\vec{F}_1\right| \cos\theta_1 = 8.0 \times \cos\left(60^\circ\right) = 4.0N$$

$$F_{1y} = \left|\vec{F}_1\right| \sin\theta_1 = 8.0 \times \sin\left(60^\circ\right) = 6.9N$$

Components of $\mathcal{F}_2$

$$F_{2x} = \left|\vec{F}_2\right| \cos\theta_2 = 5.0 \times \cos\left(-20^\circ\right) = 4.7N$$

$$F_{2y} = \left|\vec{F}_2\right| \sin\theta_2 = 5.0 \times \sin\left(-20^\circ\right) = -1.7N$$

Components of total force $\mathcal{F}$

$$F_x = F_{1x} + F_{2x} = 4.0 + 4.7 = 8.7N = ma_x$$

$$F_y = F_{1y} + F_{2y} = 6.9 - 1.7 = 5.2N = ma_y$$

Magnitude and direction of acceleration a

$$a_x = \frac{F_x}{m} = \frac{8.7}{0.3} = 29m/s^2 \quad a_y = \frac{F_y}{m} = \frac{5.2}{0.3} = 17m/s^2 \quad \left|\vec{a}\right| = \sqrt{\left(a_x\right)^2 + \left(a_y\right)^2} = \sqrt{\left(29\right)^2 + \left(17\right)^2} = 34m/s^2$$

$$\theta = \tan^{-1}\left(\frac{a_y}{a_x}\right) = \tan^{-1}\left(\frac{17}{29}\right) = 30^\circ$$

Acceleration Vector a

$$\vec{a} = a_x \hat{i} + a_y \hat{j} = \left(29\hat{i} + 17\hat{j}\right) m/s^2$$

Gravitational Force and Weight

Gravitational Force, F_g

The attractive force exerted on an object by the Earth

$$\vec{F}_G = m\vec{a} = m\vec{g}$$

Weight of an object with mass M is

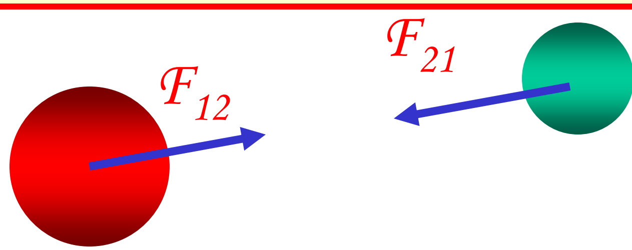
$$W = \left|\vec{F}_G\right| = M\left|\vec{g}\right| = Mg$$

*Since weight depends on the magnitude of gravitational acceleration, **g**, it varies depending on geographical location.*

By measuring the forces one can determine masses. This is why you can measure mass using spring scale.

Newton's Third Law (Law of Action and Reaction)

If two objects interact, the force, F_{21}, exerted on object 1 by object 2 is equal in magnitude and opposite in direction to the force, F_{12}, exerted on object 1 by object 2.



$$\boxed{\vec{F}_{12} = -\vec{F}_{21}}$$

The action force is equal in magnitude to the reaction force but in opposite direction. These two forces always act on different objects.

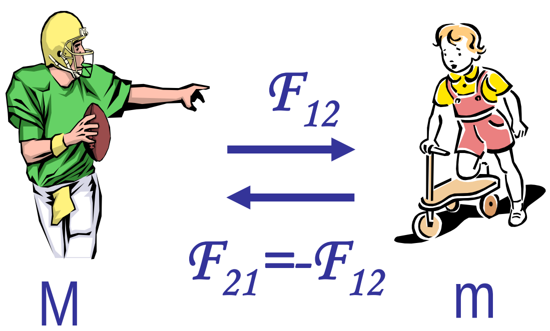
What is the reaction force to the force of a free fall object?

The force exerted by the ground when it completed the motion.

Stationary objects on top of a table has a reaction force (normal force) from table to balance the action force, the gravitational force.

Example of Newton's 3rd Law

A large man and a small boy stand facing each other on frictionless ice. They put their hands together and push against each other so that they move apart. a) Who moves away with the higher speed and by how much?



$$\vec{F}_{12} = -\vec{F}_{21} \qquad \left|\vec{F}_{12}\right| = \left|\vec{F}_{21}\right| = F$$

$$\vec{F}_{12} = m\vec{a}_b \qquad F_{12x} = ma_{bx} \qquad F_{12y} = ma_{by} = 0$$

$$\vec{F}_{21} = M\vec{a}_M \qquad F_{21x} = Ma_{Mx} \qquad F_{21y} = Ma_{My} = 0$$

$$\vec{F}_{12} = -\vec{F}_{21} \qquad \left|\vec{F}_{12}\right| = \left|-\vec{F}_{21}\right| = F \qquad a_{bx} = \frac{F}{m} = \frac{M}{m}a_{Mx}$$

$$v_{Mxf} = v_{Mxi} + a_{Mx}t = a_{Mx}t$$

$$v_{bxf} = v_{bxi} + a_{bx}t = a_{bx}t = \frac{M}{m}a_{Mx}t = \frac{M}{m}v_{Mxf}$$

$$\therefore\ v_{bxf} \rangle v_{Mxf} \ \text{ if } \ M \rangle m \ \text{ by the ratio of the masses}$$

b) Who moves farther while their hands are in contact?

Given in the same time interval, since the boy has higher acceleration and thereby higher speed, he moves farther than the man.

$$x_b = v_{bxf}t + \frac{1}{2}a_{bx}t^2 = \frac{M}{m}v_{Mxf}t + \frac{M}{2m}a_{Mx}t^2$$

$$x_b = \frac{M}{m}\left(v_{Mxf}t + \frac{1}{2}a_{Mx}t^2\right) = \frac{M}{m}x_M$$

Some Basic Information

When Newton's laws are applied, *external forces* are only of interest!!

Why?

Because, as described in Newton's first law, an object will keep its current motion unless non-zero net external force is applied.

Normal Force, ***n:***

Reaction force that reacts to gravitational force due to the surface structure of an object. Its direction is perpendicular to the surface.

Tension, ***T:***

The reactionary force by a stringy object against an external force exerted on it.

Free-body diagram

A graphical tool which is a ***diagram of external forces on an object*** *and is extremely useful analyzing forces and motion!! Drawn only on an object.*

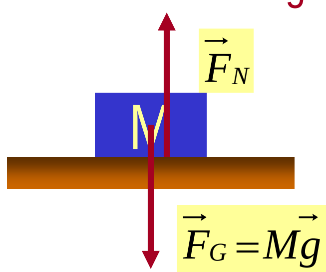
Free Body Diagrams and Solving Problems

- Free-body diagram: A diagram of vector forces acting on an object

⇒ A great tool to solve a problem using forces or using dynamics

1. Select a point on an object in the problem
2. Identify all the forces acting only on the selected object
3. Define a reference frame with positive and negative axes specified
4. Draw arrows to represent the force vectors on the selected point
5. Write down net force vector equation
6. Write down the forces in components to solve the problems

⇒ No matter which one we choose to draw the diagram on, the results should be the same, as long as they are from the same motion

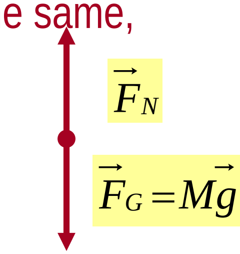


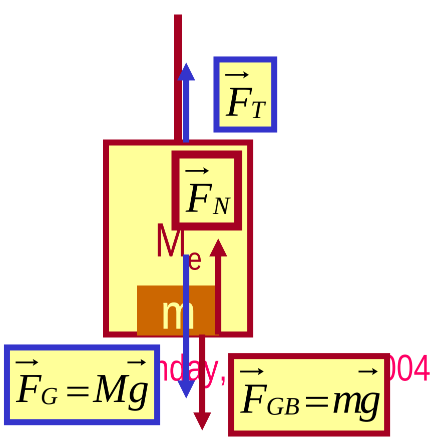
Which one would you like to select to draw FBD?

What do you think are the forces acting on this object?

Gravitational force

A force supporting the object exerted by the floor



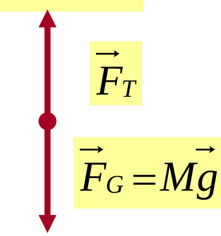


Which one would you like to select to draw FBD?

What do you think are the forces acting on this elevator?

Gravitational force

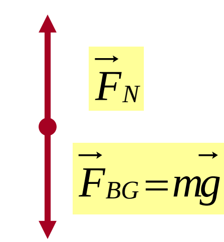
The force pulling the elevator (Tension)



What about the box in the elevator?

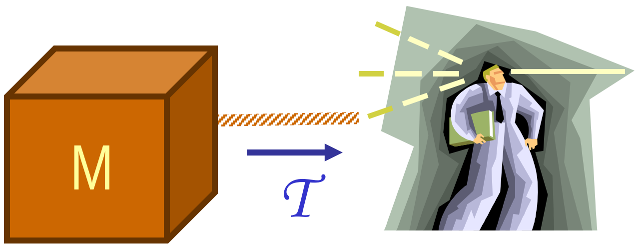
Gravitational force

Normal force



Applications of Newton's Laws

Suppose you are pulling a box on frictionless ice, using a rope.



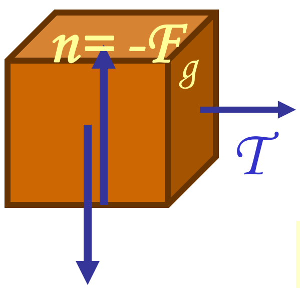
What are the forces being exerted on the box?

Gravitational force: $\mathcal{F}_g$

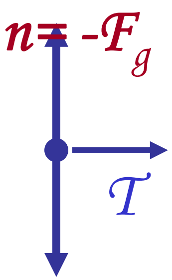
Normal force: $\boldsymbol{n}$

Tension force: $\mathcal{T}$

Free-body diagram



$F_g = Mg$



$F_g = Mg$

Total force:
$F = F_g + n + T = T$

$$\sum F_x = T = Ma_x \qquad \boxed{a_x = \frac{T}{M}}$$

$$\sum F_y = -F_g + n = Ma_y = 0 \qquad \boxed{a_y = 0}$$

If T is a constant force, a_x, is constant

$$v_{xf} = v_{xi} + a_x t = v_{xi} + \left(\frac{T}{M}\right)t$$

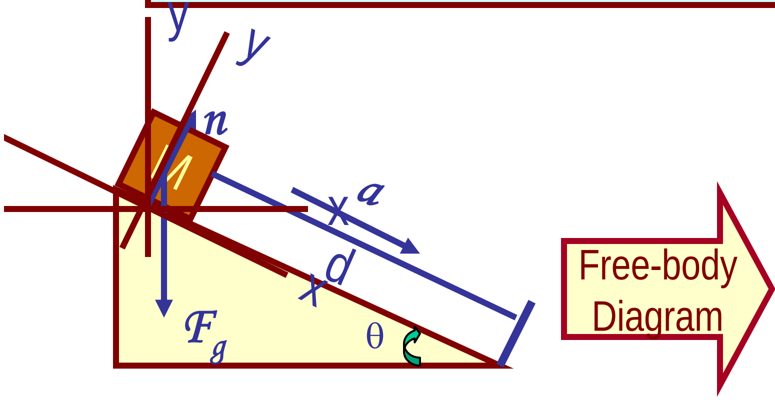
$$\Delta x = x_f - x_i = v_{xi}t + \frac{1}{2}\left(\frac{T}{M}\right)t^2$$

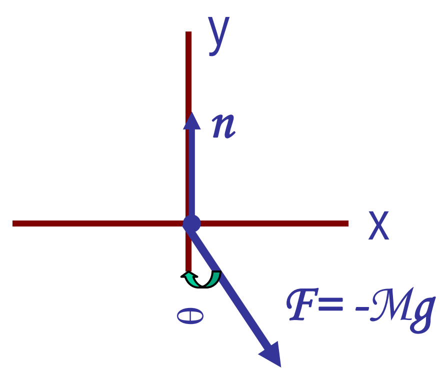
What happened to the motion in y-direction?

Example w/o Friction

A crate of mass M is placed on a frictionless inclined plane of angle θ.

a) Determine the acceleration of the crate after it is released.





$$\vec{F} = \vec{F}_g + \vec{n} = m\vec{a}$$

$$F_x = Ma_x = F_{gx} = Mg\sin\theta$$

$$\boxed{a_x = g\sin\theta}$$

$$F_y = Ma_y = n - F_{gy} = n - mg\cos\theta = 0$$

Supposed the crate was released at the top of the incline, and the length of the incline is d. How long does it take for the crate to reach the bottom and what is its speed at the bottom?

$$d = v_{ix}t + \frac{1}{2}a_x t^2 = \frac{1}{2}g\sin\theta\, t^2$$

$$\boxed{\therefore t = \sqrt{\frac{2d}{g\sin\theta}}}$$

$$v_{xf} = v_{ix} + a_x t = g\sin\theta\sqrt{\frac{2d}{g\sin\theta}} = \sqrt{2dg\sin\theta}$$

$$\boxed{\therefore v_{xf} = \sqrt{2dg\sin\theta}}$$

Forces of Friction

Resistive force exerted on a moving object due to viscosity or other types frictional property of the medium in or surface on which the object moves.

These forces are either proportional to velocity or normal force

Force of static friction, f_s: *The resistive force exerted on the object until just before the beginning of its movement*

Empirical Formula

$$\left|\vec{f}_s\right| \le \mu_s \left|\vec{n}\right|$$

What does this formula tell you?

Frictional force increases till it reaches to the limit!!

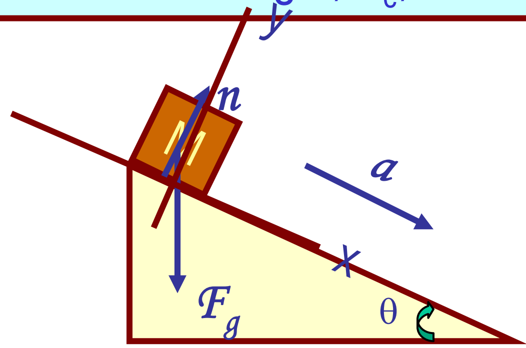
Beyond the limit, there is no more static frictional force but kinetic frictional force takes it over.

Force of kinetic friction, f_k: *The resistive force exerted on the object during its movement*

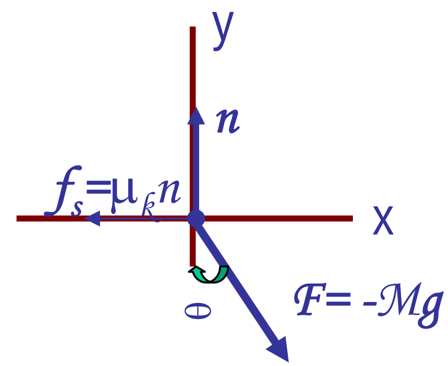
$$\left|\vec{f}_k\right| = \mu_k \left|\vec{n}\right|$$

Example w/ Friction

Suppose a block is placed on a rough surface inclined relative to the horizontal. The inclination angle is increased till the block starts to move. Show that by measuring this critical angle, θ_c, one can determine coefficient of static friction, μ_s.



Free-body Diagram



Net force $\vec{F} = M\vec{a} = \vec{F}_g + \vec{n} + \vec{f}_s$

x comp. $F_x = F_{gx} - f_s = Mg\sin\theta - f_s = 0 \qquad f_s = \mu_s n = Mg\sin\theta_c$

y comp. $F_y = Ma_y = n - F_{gy} = n - Mg\cos\theta_c = 0 \qquad n = F_{gy} = Mg\cos\theta_c$

$$\mu_s = \frac{Mg\sin\theta_c}{n} = \frac{Mg\sin\theta_c}{Mg\cos\theta_c} = \tan\theta_c$$