PHYS 1441 – Section 501
Lecture #5

Wednesday, June 16, 2004
*Dr. Jaehoon Yu*

- Chapter four: Newton's Laws of Motion
  - Newton's Third Law of Motion
  - Solving problems using Newton's Laws
    - Free-body diagram

- Uniform circular motion

Today's homework is homework #3, due 5pm, next Friday!!

Newton's Third Law (Law of Action and Reaction)

If two objects interact, the force, F_{21}, exerted on object 1 by object 2 is equal in magnitude and opposite in direction to the force, F_{12}, exerted on object 1 by object 2.

F_{21}

F_{12}
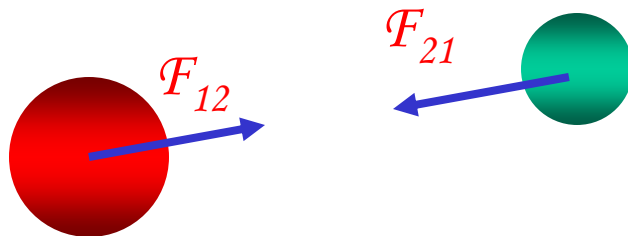
$$\vec{F}_{12} = -\vec{F}_{21}$$

The action force is equal in magnitude to the reaction force but in opposite direction. These two forces always act on different objects.

What is the reaction force to the force of a free fall object?

The force exerted by the ground when it completed the motion.

Stationary objects on top of a table has a reaction force (normal force) from table to balance the action force, the gravitational force.

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Example of Newton's 3rd Law

A large man and a small boy stand facing each other on **frictionless ice**. They put their hands together and push against each other so that they move apart. a) Who moves away with the higher speed and by how much?
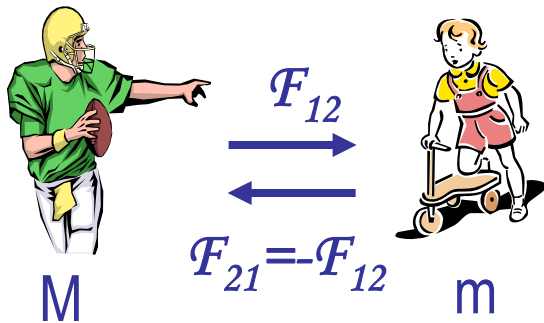
$$\vec{F}_{12} = -\vec{F}_{21} \qquad \left|\vec{F}_{12}\right| = \left|\vec{F}_{21}\right| = F$$

$$\vec{F}_{12} = m\vec{a}_b \qquad F_{12x} = ma_{bx} \qquad F_{12y} = ma_{by} = 0$$

$$\vec{F}_{21} = M\vec{a}_M \qquad F_{21x} = Ma_{Mx} \qquad F_{21y} = Ma_{My} = 0$$

$$\vec{F}_{12} = -\vec{F}_{21} \quad \left|\vec{F}_{12}\right| = \left|-\vec{F}_{21}\right| = F \qquad a_{bx} = \frac{F}{m} = \frac{M}{m}a_{Mx}$$

$$v_{Mxf} = v_{Mxi} + a_{Mx}t = a_{Mx}t$$

$$v_{bxf} = v_{bxi} + a_{bx}t = a_{bx}t = \frac{M}{m}a_{Mx}t = \frac{M}{m}v_{Mxf}$$

$$\therefore v_{bxf} > v_{Mxf} \quad \text{if } M > m \text{ by the ratio of the masses}$$

b) Who moves farther while their hands are in contact?

Given in the same time interval, since the boy has higher acceleration and thereby higher speed, he moves farther than the man.

$$x_b = v_{bxf}t + \frac{1}{2}a_{bx}t^2 = \frac{M}{m}v_{Mxf}t + \frac{M}{2m}a_{Mx}t^2$$

$$x_b = \frac{M}{m}\left(v_{Mxf}t + \frac{1}{2}a_{Mx}t^2\right) = \frac{M}{m}x_M$$

Some Basic Information

When Newton's laws are applied, *external forces* are only of interest!!

Why? — *Because, as described in Newton's first law, an object will keep its current motion unless non-zero net external force is applied.*

Normal Force, *n*: — *Reaction force that reacts to gravitational force due to the surface structure of an object. Its direction is perpendicular to the surface.*

Tension, *T*: — *The reactionary force by a stringy object against an external force exerted on it.*

Free-body diagram — *A graphical tool which is a **diagram of external forces on an object** and is extremely useful analyzing forces and motion!! Drawn only on an object.*

Free Body Diagrams and Solving Problems

- Free-body diagram: A diagram of vector forces acting on an object

$\Rightarrow$ A great tool to solve a problem using forces or using dynamics

1. Select a point on an object and w/ information given
2. Identify all the forces acting only on the selected object
3. Define a reference frame with positive and negative axes specified
4. Draw arrows to represent the force vectors on the selected point
5. Write down net force vector equation
6. Write down the forces in components to solve the problems

$\Rightarrow$ No matter which one we choose to draw the diagram on, the results should be the same, as long as they are from the same motion

$\vec{F}_N$

M

$\vec{F}_G = M\vec{g}$

$\vec{F}_N$

$\vec{F}_G = M\vec{g}$
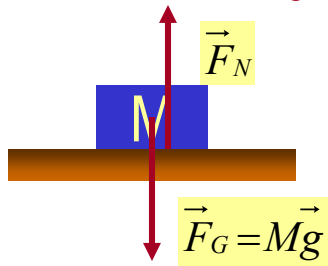
Which one would you like to select to draw FBD?

What do you think are the forces acting on this object?

Gravitational force   A force supporting the object exerted by the floor

Which one would you like to select to draw FBD?

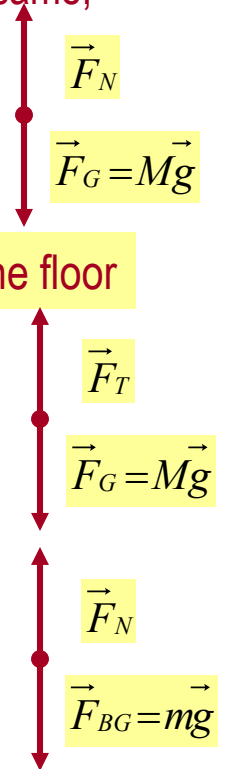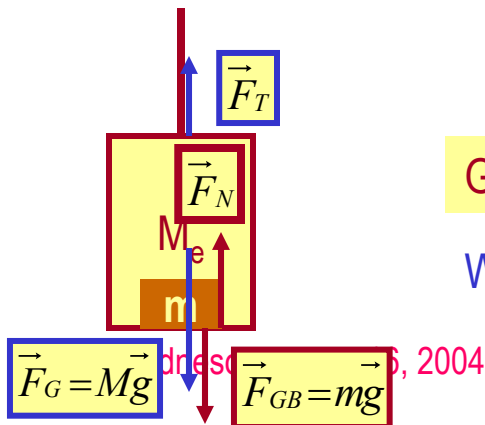What do you think are the forces acting on this elevator?

$\vec{F}_T$

$\vec{F}_G = M\vec{g}$

$\vec{F}_T$

$\vec{F}_N$

M_e

m

Gravitational force   The force pulling the elevator (Tension)

What about the box in the elevator?

Gravitational force   Normal force

$\vec{F}_N$

$\vec{F}_G = M\vec{g}$ $\vec{F}_{GB} = m\vec{g}$

$\vec{F}_{BG} = m\vec{g}$

Applications of Newton's Laws

Suppose you are pulling a box on frictionless ice, using a rope.

M T

Free-body diagram

$n = -F_g$

$F_g = Mg$

$n = -F_g$

T

$F_g = Mg$
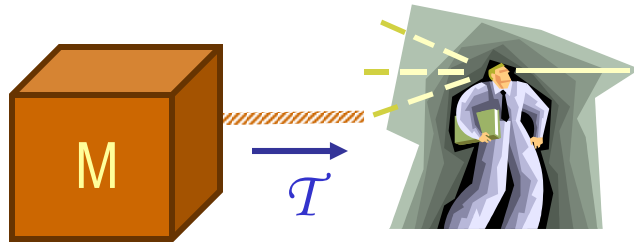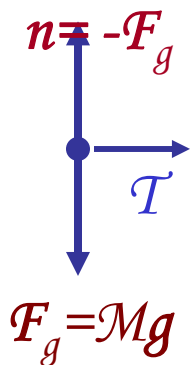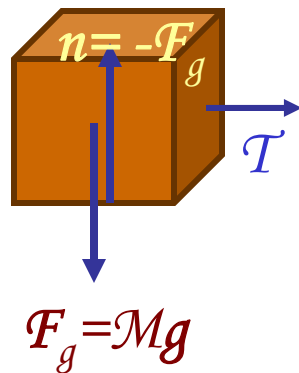
What are the forces being exerted on the box?

Gravitational force: F_g

Normal force: n

Tension force: T

Total force:
$F = F_g + n + T = T$

If T is a constant force, a_x, is constant

$$\sum F_x = T = Ma_x$$

$$\sum F_y = -F_g + n = Ma_y = 0$$

$$a_x = \frac{T}{M}$$

$$a_y = 0$$

$$v_{xf} = v_{xi} + a_x t = v_{xi} + \left(\frac{T}{M}\right)t$$

$$\Delta x = x_f - x_i = v_{xi}t + \frac{1}{2}\left(\frac{T}{M}\right)t^2$$

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What happened to the motion in y-direction?

Example of Using Newton's Laws

A traffic light weighing 125 N hangs from a cable tied to two other cables fastened to a support. The upper cables make angles of 37.0° and 53.0° with the horizontal. Find the tension in the three cables.

Free-body Diagram
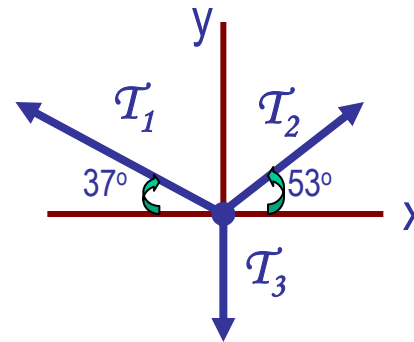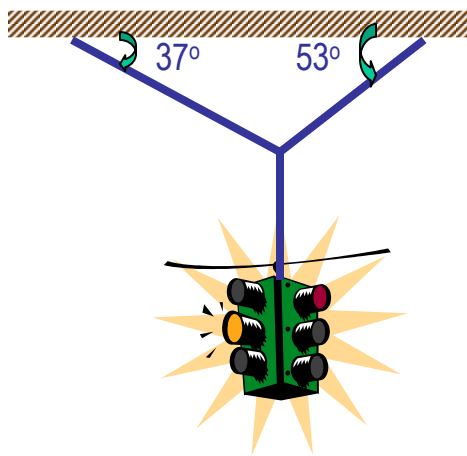
$$\vec{F} = \vec{T}_1 + \vec{T}_2 + \vec{T}_3 = m\vec{a} = 0$$

Newton's 2nd law

x-comp. of net force

$$F_x = \sum_{i=1}^{i=3} T_{ix} = 0 \qquad -T_1 \cos\left(37°\right) + T_2 \cos\left(53°\right) = 0 \quad \therefore T_1 = \frac{\cos\left(53°\right)}{\cos\left(37°\right)} T_2 = 0.754\, T_2$$

y-comp. of net force

$$F_y = \sum_{i=1}^{i=3} T_{iy} = 0$$

$$T_1 \sin\left(37°\right) + T_2 \sin\left(53°\right) - mg = 0$$

$$T_2\left[\sin\left(53°\right) + 0.754 \times \sin\left(37°\right)\right] = 1.25\, T_2 = 125N$$

$$T_2 = 100\, N; \quad T_1 = 0.754\, T_2 = 75.4\, N$$
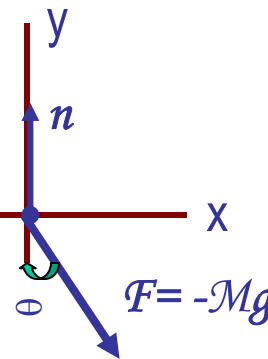
Example w/o Friction

A crate of mass M is placed on a frictionless inclined plane of angle θ.

a) Determine the acceleration of the crate after it is released.
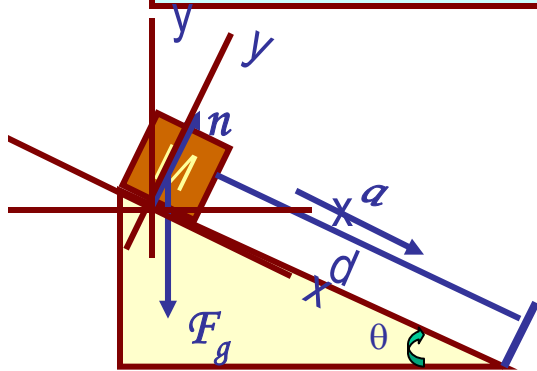
Free-body Diagram

$$\vec{F} = \vec{F_g} + \vec{n} = m\vec{a}$$

$$F_x = Ma_x = F_{gx} = Mg\sin\theta$$

$$\boxed{a_x = g\sin\theta}$$

$$F_y = Ma_y = n - F_{gy} = n - mg\cos\theta = 0$$

Supposed the crate was released at the top of the incline, and the length of the incline is d. How long does it take for the crate to reach the bottom and what is its speed at the bottom?

$$d = v_{ix}t + \frac{1}{2}a_x t^2 = \frac{1}{2}g\sin\theta\, t^2 \qquad \therefore t = \sqrt{\frac{2d}{g\sin\theta}}$$

$$v_{xf} = v_{ix} + a_x t = g\sin\theta\sqrt{\frac{2d}{g\sin\theta}} = \sqrt{2dg\sin\theta}$$

$$\boxed{\therefore v_{xf} = \sqrt{2dg\sin\theta}}$$

Forces of Friction

Resistive force exerted on a moving object due to viscosity or other types frictional property of the medium in or surface on which the object moves.

These forces are either proportional to velocity or normal force

Force of static friction, f_s:

The resistive force exerted on the object until just before the beginning of its movement

Empirical Formula

$$\left|\vec{f_s}\right| \le \mu_s \left|\vec{n}\right|$$

What does this formula tell you?

Frictional force increases till it reaches to the limit!!

Beyond the limit, there is no more static frictional force but kinetic frictional force takes it over.

Force of kinetic friction, f_k

$$\left|\vec{f_k}\right| = \mu_k \left|\vec{n}\right|$$

The resistive force exerted on the object during its movement

Example w/ Friction

Suppose a block is placed on a rough surface inclined relative to the horizontal. The inclination angle is increased till the block starts to move. Show that by measuring this critical angle, θ_c, one can determine coefficient of static friction, μ_s.
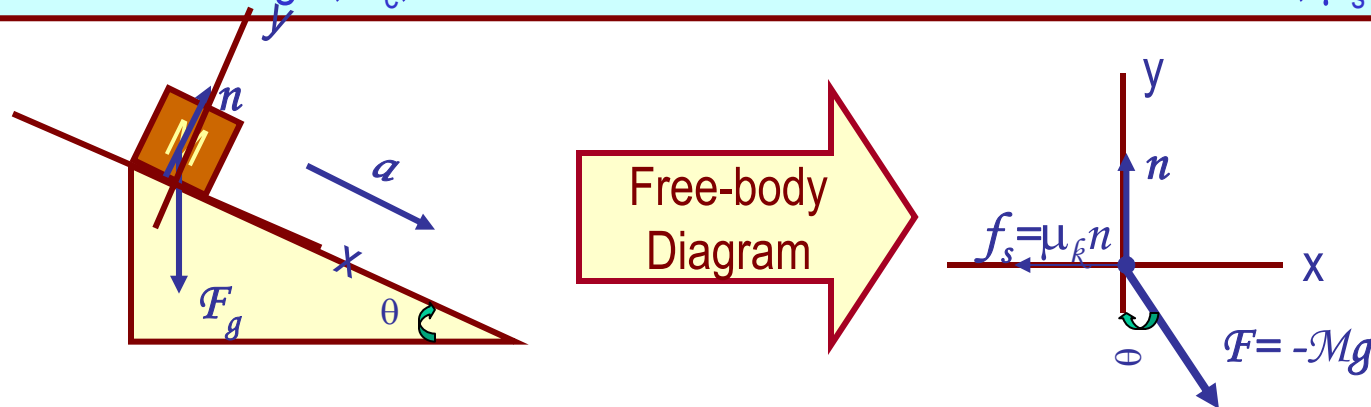
Net force

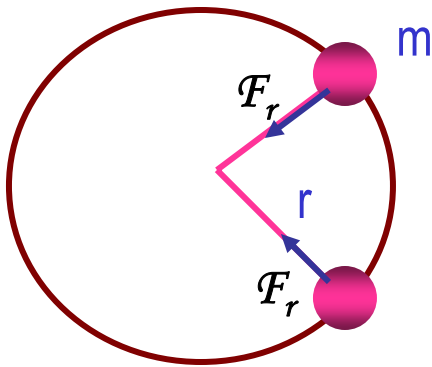$$\vec{F} = M\vec{a} = \vec{F}_g + \vec{n} + \vec{f}_s$$

x comp.

$$F_x = F_{gx} - f_s = Mg\sin\theta - f_s = 0 \qquad f_s = \mu_s n = Mg\sin\theta_c$$

y comp.

$$F_y = Ma_y = n - F_{gy} = n - Mg\cos\theta_c = 0 \qquad n = F_{gy} = Mg\cos\theta_c$$

$$\mu_s = \frac{Mg\sin\theta_c}{n} = \frac{Mg\sin\theta_c}{Mg\cos\theta_c} = \tan\theta_c$$

Newton's Second Law & Uniform Circular Motion

The centripetal acceleration is always perpendicular to velocity vector, *v*, for uniform circular motion.

$$a_r = \frac{v^2}{r}$$

Are there forces in this motion? If so, what do they do?

The force that causes the centripetal acceleration acts toward the center of the circular path and causes a change in the direction of the velocity vector. This force is called **centripetal force.**
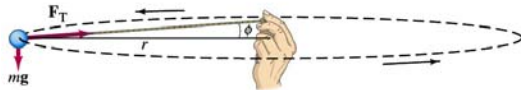
$$\sum F_r = ma_r = m\frac{v^2}{r}$$

What do you think will happen to the ball if the string that holds the ball breaks? Why?

Based on Newton's 1st law, since the external force no longer exist, the ball will continue its motion without change and will fly away following the tangential direction to the circle.

Example of Uniform Circular Motion

A ball of mass 0.500kg is attached to the end of a 1.50m long cord. The ball is moving in a horizontal circle. If the string can withstand maximum tension of 50.0 N, what is the maximum speed the ball can attain before the cord breaks?

Centripetal acceleration:

$$a_r = \frac{v^2}{r}$$

When does the string break?

$$\sum F_r = ma_r = m\frac{v^2}{r} > T$$

*when the centripetal force is greater than the sustainable tension.*

$$m\frac{v^2}{r} = T \qquad v = \sqrt{\frac{Tr}{m}} = \sqrt{\frac{50.0 \times 1.5}{0.500}} = 12.2\,(m/s)$$

Calculate the tension of the cord when speed of the ball is 5.00m/s.

$$T = m\frac{v^2}{r} = 0.500 \times \frac{(5.00)^2}{1.5} = 8.33\,(N)$$

Example of Banked Highway

(a) For a car traveling with speed v around a curve of radius r, determine a formula for the angle at which a road should be banked so that no friction is required to keep the car from skidding.
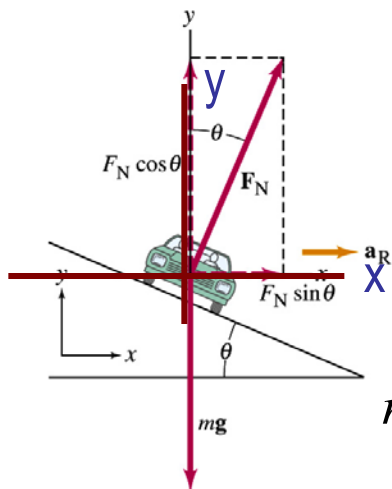
x comp. $\sum F_x = n\sin\theta - ma_r = n\sin\theta - \dfrac{mv^2}{r} = 0$

$$n\sin\theta = \frac{mv^2}{r}$$

y comp. $\sum F_y = n\cos\theta - mg = 0 \qquad n\cos\theta = mg$

$$n = \frac{mg}{\sin\theta} \qquad n\sin\theta = \frac{mg\sin\theta}{\cos\theta} = mg\tan\theta = \frac{mv^2}{r} \qquad \boxed{\tan\theta = \frac{v^2}{gr}}$$

(b) What is this angle for an expressway off-ramp curve of radius 50m at a design speed of 50km/h?

$$v = 50\,km/hr = 14\,m/s \qquad \tan\theta = \frac{(14)^2}{50 \times 9.8} = 0.4 \qquad \theta = \tan^{-1}(0.4) = 22°$$