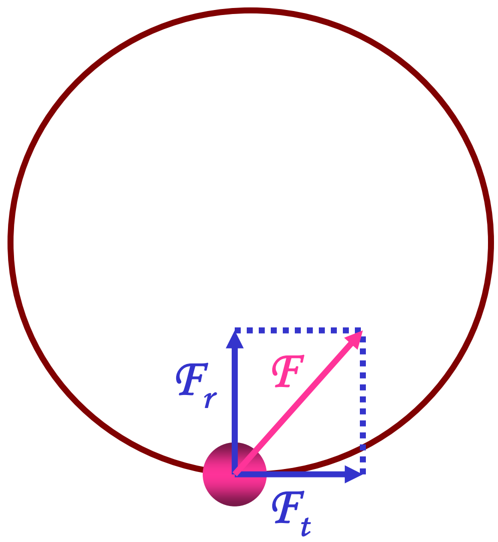
PHYS 1441 – Section 501
Lecture #6

Monday, June 21, 2004
Dr. Jaehoon Yu

- Non-uniform circular motion
- Newton's Universal Law of Gravitation
- Work done by a constant force
- Kinetic Energy and Work-Energy theorem

Today's homework is homework #6, due 1pm, next Wednesday!!

Forces in Non-uniform Circular Motion



The object has both tangential and radial accelerations.

What does this statement mean?

The object is moving under both tangential and radial forces.

$$\vec{F} = \vec{F}_r + \vec{F}_t$$

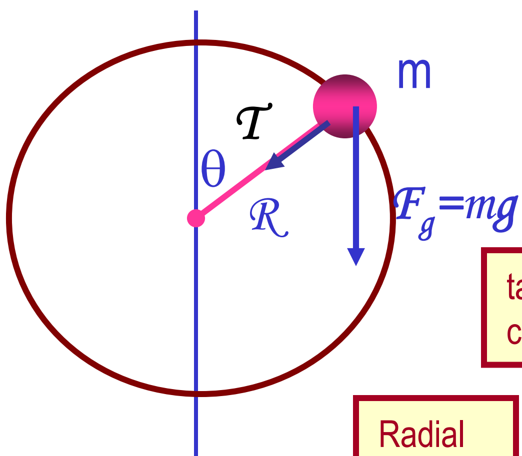
These forces cause not only the velocity but also the speed of the ball to change. The object undergoes a curved motion under the absence of constraints, such as a string.

How does the acceleration look?

$$a = \sqrt{a_r^2 + a_t^2}$$

Example of Non-Uniform Circular Motion

A ball of mass m is attached to the end of a cord of length R. The ball is moving in a vertical circle. Determine the tension of the cord at any instant when the speed of the ball is v and the cord makes an angle θ with vertical.



What are the forces involved in this motion?

The gravitational force F_g and the radial force, T, providing tension.

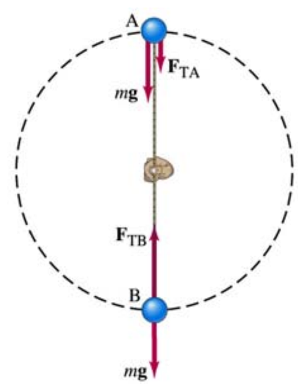
tangential comp.

$$\sum F_t = mg\sin\theta = ma_t \qquad a_t = g\sin\theta$$

Radial comp.

$$\sum F_r = T + mg\cos\theta = ma_r = m\frac{v^2}{R} \qquad T = m\left(\frac{v^2}{R} - g\cos\theta\right)$$

At what angles the tension becomes maximum and minimum. What are the tensions?



Motion in Resistive Forces

Medium can exert resistive forces on an object moving through it due to viscosity or other types frictional property of the medium.

Some examples?

Air resistance, viscous force of liquid, etc

These forces are exerted on moving objects in opposite direction of the movement.

These forces are proportional to such factors as speed. They almost always increase with increasing speed.

Two different cases of proportionality:

1. Forces linearly proportional to speed: Slowly moving or very small objects
2. Forces proportional to square of speed: Large objects w/ reasonable speed

Newton's Law of Universal Gravitation

People have been very curious about the stars in the sky, making observations for a long time. But the data people collected have not been explained until Newton has discovered the law of gravitation.

Every particle in the Universe attracts every other particle with a force that is directly ***proportional to the product of their masses*** *and* ***inversely proportional to the square of the distance*** *between them.*

How would you write this principle mathematically?

$$F_g \propto \frac{m_1 m_2}{r_{12}^2}$$

With G

$$F_g = G\frac{m_1 m_2}{r_{12}^2}$$

G is the universal gravitational constant, and its value is

$$G = 6.673 \times 10^{-11}$$

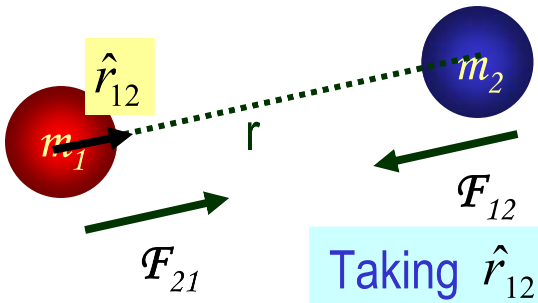
Unit? $N \cdot m^2 / kg^2$

This constant is not given by the theory but must be measured by experiment.

This form of forces is known as an inverse-square law, because the magnitude of the force is inversely proportional to the square of the distances between the objects.

More on Law of Universal Gravitation

Consider two particles exerting gravitational forces to each other.



Two objects exert gravitational force on each other following Newton's 3rd law.

Taking $\hat{r}_{12}$ as the unit vector, we can write the force m_2 experiences as

$$\vec{F}_{12} = -G\frac{m_1 m_2}{r^2}\hat{r}_{12}$$

What do you think the negative sign mean?

It means that the force exerted on the particle 2 by particle 1 is attractive force, pulling #2 toward #1.

Gravitational force is a field force: Forces act on object without physical contact between the objects at all times, independent of medium between them.

The gravitational force exerted by a finite size, spherically symmetric mass distribution on a particle outside the distribution is the same as if the entire mass of the distributions was concentrated at the center.

How do you think the gravitational force on the surface of the earth look?

$$F_g = G\frac{M_E m}{R_E^2}$$

Example for Gravitation

Using the fact that $g=9.80m/s^2$ at the Earth's surface, find the average density of the Earth.

Since the gravitational acceleration is

$$g = G\frac{M_E}{R_E^{\ 2}} = 6.67\times 10^{-11}\frac{M_E}{R_E^{\ 2}}$$

So the mass of the Earth is

$$M_E = \frac{R_E^{\ 2} g}{G}$$

Therefore the density of the Earth is

$$\rho = \frac{M_E}{V_E} = \frac{\frac{R_E^{\ 2} g}{G}}{\frac{4\pi}{3}R_E^{\ 3}} = \frac{3g}{4\pi G R_E}$$

$$= \frac{3\times 9.80}{4\pi\times 6.67\times 10^{-11}\times 6.37\times 10^{6}} = 5.50\times 10^{3} kg/m^{3}$$

Free Fall Acceleration & Gravitational Force

Weight of an object with mass m is mg. Using the force exerting on a particle of mass m on the surface of the Earth, one can get

$$mg = G\frac{M_E m}{R_E^2}$$

$$g = G\frac{M_E}{R_E^2}$$

What would the gravitational acceleration be if the object is at an altitude h above the surface of the Earth?

$$F_g = mg' = G\frac{M_E m}{r^2} = G\frac{M_E m}{(R_E + h)^2}$$

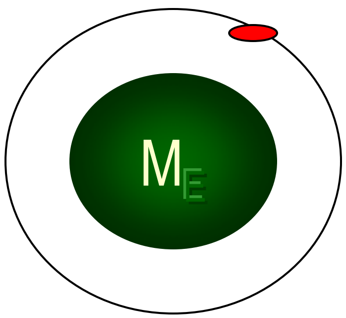
$$g' = G\frac{M_E}{(R_E + h)^2}$$

What do these tell us about the gravitational acceleration?

- The gravitational acceleration is independent of the mass of the object
- The gravitational acceleration decreases as the altitude increases
- If the distance from the surface of the Earth gets infinitely large, the weight of the object approaches 0.

Example for Gravitational Force

The international space station is designed to operate at an altitude of 350km. When completed, it will have a weight (measured on the surface of the Earth) of 4.22×10^6N. What is its weight when in its orbit?



The total weight of the station on the surface of the Earth is

$$F_{GE} = mg = G\frac{M_E m}{R_E^{\,2}} = 4.22 \times 10^6 N$$

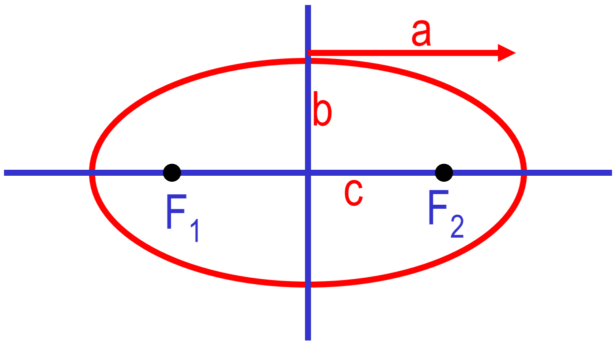
Since the orbit is at 350km above the surface of the Earth, the gravitational force at that height is

$$F_O = mg' = G\frac{M_E m}{(R_E + h)^2} = \frac{R_E^2}{(R_E + h)^2} F_{GE}$$

Therefore the weight in the orbit is

$$F_O = \frac{R_E^2}{(R_E + h)^2} F_{GE} = \frac{\left(6.37 \times 10^6\right)^2}{\left(6.37 \times 10^6 + 3.50 \times 10^5\right)^2} \times 4.22 \times 10^6 = 3.80 \times 10^6 N$$

Kepler's Laws & Ellipse



Ellipses have two different axis, major (long) and minor (short) axis, and two focal points, F_1 & F_2

a is the length of a semi-major axis

b is the length of a semi-minor axis

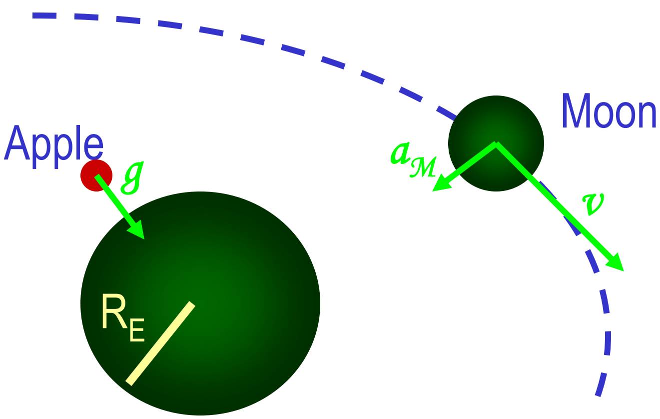
Kepler lived in Germany and discovered the law's governing planets' movement some 70 years before Newton, by analyzing data.

1. All planets move in elliptical orbits with the Sun at one focal point.
2. The radius vector drawn from the Sun to a planet sweeps out equal area in equal time intervals. *(Angular momentum conservation)*
3. The square of the orbital period of any planet is proportional to the cube of the semi-major axis of the elliptical orbit.

Newton's laws explain the cause of the above laws. Kepler's third law is the direct consequence of law of gravitation being inverse square law.

The Law of Gravity and Motions of Planets

- Newton assumed that the law of gravitation applies the same whether it is on the Moon or the apple on the surface of the Earth.
- The interacting bodies are assumed to be point like particles.



Newton predicted that the ratio of the Moon's acceleration a_M to the apple's acceleration g would be

$$\frac{a_M}{g} = \frac{(1/r_M)^2}{(1/R_E)^2} = \left(\frac{R_E}{r_M}\right)^2 = \left(\frac{6.37\times10^6}{3.84\times10^8}\right)^2 = 2.75\times10^{-4}$$

Therefore the centripetal acceleration of the Moon, a_M, is

$$a_M = 2.75\times10^{-4}\times9.80 = 2.70\times10^{-3}\,m/s^2$$

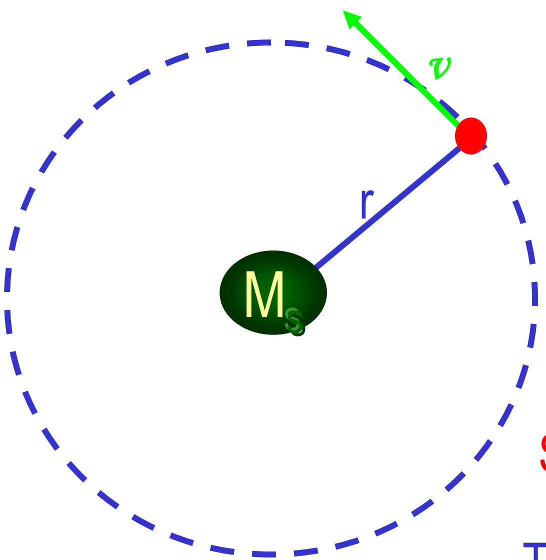
Newton also calculated the Moon's orbital acceleration a_M from the knowledge of its distance from the Earth and its orbital period, T=27.32 days=2.36×10^6s

$$a_M = \frac{v^2}{r_M} = \frac{(2\pi r_M/T)^2}{r_M} = \frac{4\pi^2 r_M}{T^2} = \frac{4\pi^2\times3.84\times10^8}{\left(2.36\times10^6\right)^2} = 2.72\times10^{-3}\,m/s^2 \approx \frac{9.80}{(60)^2}$$

This means that the Moon's distance is about 60 times that of the Earth's radius, its acceleration is reduced by the square of the ratio. This proves that the inverse square law is valid.

Kepler's Third Law

It is crucial to show that Keper's third law can be predicted from the inverse square law for circular orbits.



Since the gravitational force exerted by the Sun is radially directed toward the Sun to keep the planet circle, we can apply Newton's second law

$$\frac{GM_sM_P}{r^2} = \frac{M_p v^2}{r}$$

Since the orbital speed, v, of the planet with period T is $v = \frac{2\pi r}{T}$

The above can be written $\frac{GM_sM_P}{r^2} = \frac{M_p(2\pi r/T)^2}{r}$

Solving for T one can obtain $T^2 = \left(\frac{4\pi^2}{GM_s}\right)r^3 = K_s r^3$ and $K_s = \left(\frac{4\pi^2}{GM_s}\right) = 2.97\times10^{-19}\, s^2/m^3$

This is Kepler's third law. It's also valid for ellipse for r being the length of the semi-major axis. The constant K_s is independent of mass of the planet.

Example of Kepler's Third Law

Calculate the mass of the Sun using the fact that the period of the Earth's orbit around the Sun is 3.16×10^{7}s, and its distance from the Sun is 1.496×10^{11}m.

Using Kepler's third law.

$$T^2 = \left(\frac{4\pi^2}{GM_s}\right) r^3 = K_s r^3$$

The mass of the Sun, M_s, is

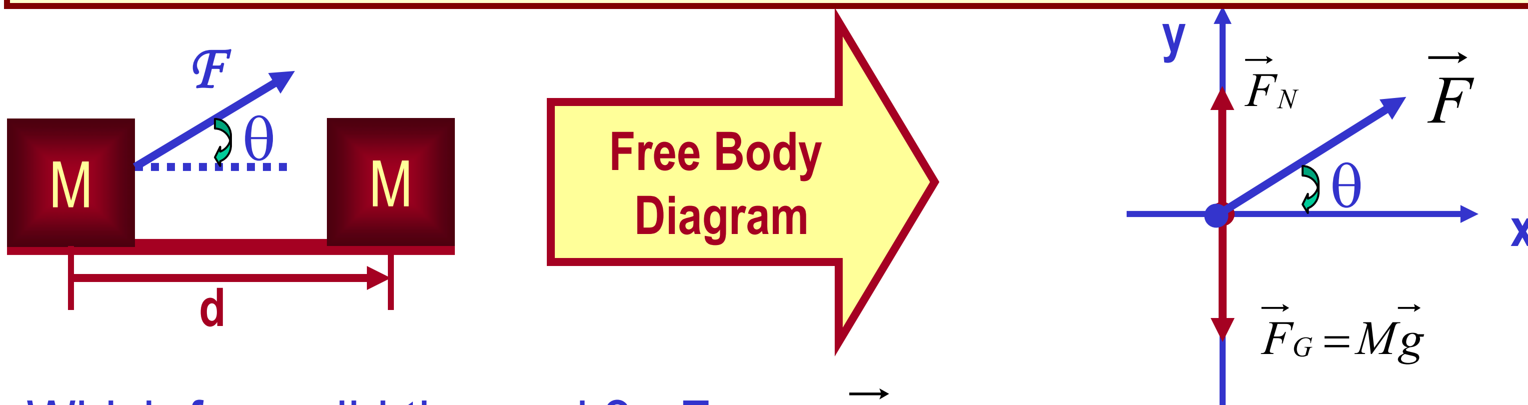
$$M_s = \left(\frac{4\pi^2}{GT}\right) r^3$$

$$= \left(\frac{4\pi^2}{6.67\times10^{-11}\times3.16\times10^{7}}\right)\times\left(1.496\times10^{11}\right)^3$$

$$= 1.99\times10^{30}\,kg$$

Work Done by a Constant Force

Work in physics is done only when a sum of forces exerted on an object made a motion to the object.



Which force did the work? Force $\vec{F}$

How much work did it do? $W = \left(\sum \vec{F}\right) \cdot \vec{d} = Fd\cos\theta$ Unit? $N \bullet m$ $= J$ (for Joule)

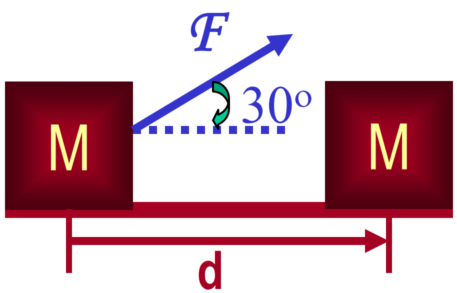
What does this mean?

Physical work is done only by the component of the force along the movement of the object.

Work is energy transfer!!

Example of Work w/ Constant Force

A man cleaning a floor pulls a vacuum cleaner with a force of magnitude F=50.0N at an angle of 30.0° with East. Calculate the work done by the force on the vacuum cleaner as the vacuum cleaner is displaced by 3.00m to East.



$$W = \left(\sum \vec{F}\right) \cdot \vec{d} = \left|\left(\sum \vec{F}\right)\right| \left|\vec{d}\right| \cos\theta$$

$$W = 50.0 \times 3.00 \times \cos 30^{\circ} = 130J$$

Does work depend on mass of the object being worked on?

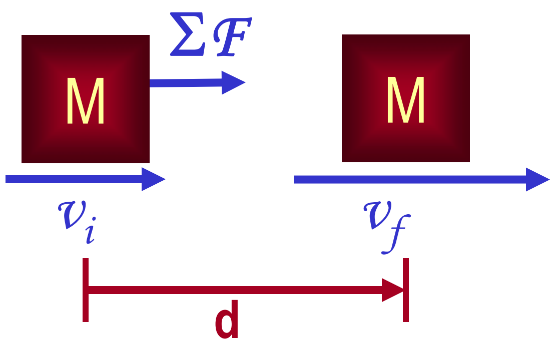
Yes

Why don't I see the mass term in the work at all then?

It is reflected in the force. If the object has smaller mass, its would take less force to move it the same distance as the heavier object. So it would take less work. Which makes perfect sense, doesn't it?

Kinetic Energy and Work-Kinetic Energy Theorem

- Some problems are hard to solve using Newton's second law
  - If forces exerting on the object during the motion are so complicated
  - Relate the work done on the object by the net force to the change of the speed of the object



Suppose net force ΣF was exerted on an object for displacement d to increase its speed from v_i to v_f

The work on the object by the net force ΣF is

$$W = Fd\cos\theta = (ma)d\cos 0 = (ma)d$$

Displacement $d = \frac{1}{2}(v_f + v_i)t$ Acceleration $a = \frac{v_f - v_i}{t}$

Work $W = (ma)d = \left[m\left(\frac{v_f - v_i}{t}\right)\right]\frac{1}{2}(v_f + v_i)t = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$

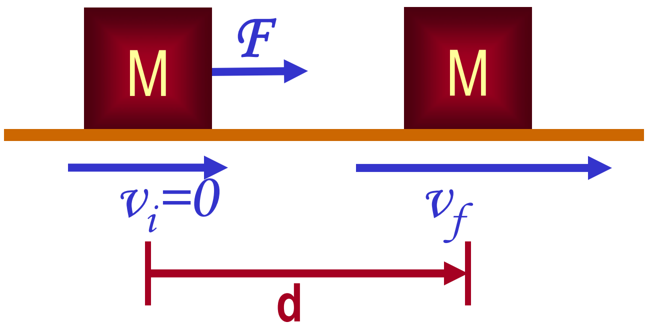
Kinetic Energy $KE = \frac{1}{2}mv^2$

Work $W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = KE_f - KE_i = \Delta KE$

The work done by the net force caused change of object's kinetic energy.

Example of Work-KE Theorem

A 6.0kg block initially at rest is pulled to East along a horizontal, frictionless surface by a constant horizontal force of 12N. Find the speed of the block after it has moved 3.0m.



Work done by the force $\mathcal{F}$ is

$$W = \vec{F} \cdot \vec{d} = \left|\vec{F}\right|\left|\vec{d}\right| \cos\theta = 12 \times 3.0 \cos 0 = 36(J)$$

From the work-kinetic energy theorem, we know

$$W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

Since initial speed is 0, the above equation becomes

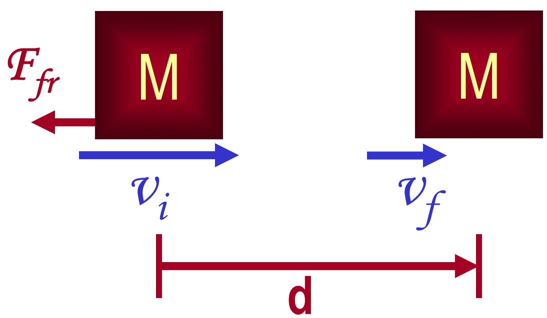
$$W = \frac{1}{2}mv_f^2$$

Solving the equation for v_f, we obtain

$$v_f = \sqrt{\frac{2W}{m}} = \sqrt{\frac{2 \times 36}{6.0}} = 3.5m/s$$

Work and Energy Involving Kinetic Friction

- Some How do you think the work looks like if there is friction?
  - Why doesn't static friction matter?



Friction force $\mathcal{F}_{fr}$ works on the object to slow down

The work on the object by the friction $\mathcal{F}_{fr}$ is

$$W_{fr} = F_{fr} d \cos(180) = -F_{fr} d$$

$$\Delta KE = -F_{fr} d$$

The final kinetic energy of an object with initial kinetic energy, friction force and other source of work is

$$KE_f = KE_i + \sum W - F_{fr} d$$

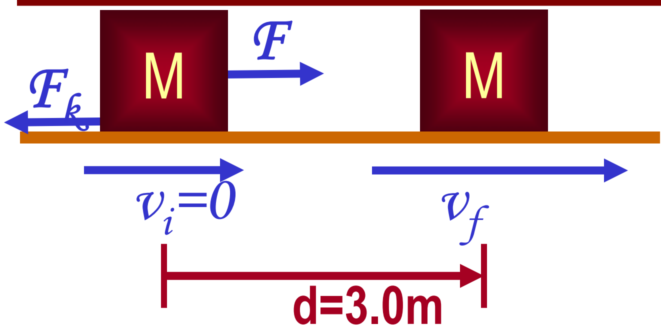
t=0, KE_i

Friction
Engine work

t=T, KE_f

Example of Work Under Friction

A 6.0kg block initially at rest is pulled to East along a horizontal surface with coefficient of kinetic friction μ_k=0.15 by a constant horizontal force of 12N. Find the speed of the block after it has moved 3.0m.



Work done by the force $\mathcal{F}$ is

$$W_F = \vec{F}\cdot\vec{d} = \left|\vec{F}\right|\left|\vec{d}\right|\cos\theta = 12\times 3.0\cos 0 = 36\,(J)$$

Work done by friction $\mathcal{F}_k$ is

$$W_k = \vec{F}_k\cdot\vec{d} = \left|\mu_k mg\right|\left|\vec{d}\right|\cos\theta$$

$$= 0.15\times 6.0\times 9.8\times 3.0\cos 180 = -26\,(J)$$

Thus the total work is

$$W = W_F + W_k = 36 - 26 = 10(J)$$

Using work-kinetic energy theorem and the fact that initial speed is 0, we obtain

$$W = W_F + W_k = \frac{1}{2}mv_f^2$$

Solving the equation for v_f, we obtain

$$v_f = \sqrt{\frac{2W}{m}} = \sqrt{\frac{2\times 10}{6.0}} = 1.8m/s$$