PHYS 1441 – Section 501
Lecture #8

Monday, June 28, 2004
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- Work done by a constant force
- Kinetic Energy and Work-Energy theorem
- Power
- Potential Energies ➔ gravitational and elastic
- Conservative Forces
- Mechanical Energy Conservation

Newton's Law of Universal Gravitation

People have been very curious about the stars in the sky, making observations for a long time. But the data people collected have not been explained until Newton has discovered the law of gravitation.

Every object in the Universe attracts every other objects with a force that is directly ***proportional to the product of their masses*** *and* ***inversely proportional to the square of the distance*** *between them.*

How would you write this principle mathematically?

$$F_g \propto \frac{m_1 m_2}{r_{12}^2}$$

With G

$$F_g = G\frac{m_1 m_2}{r_{12}^2}$$

G is the universal gravitational constant, and its value is

$$G = 6.673 \times 10^{-11}$$

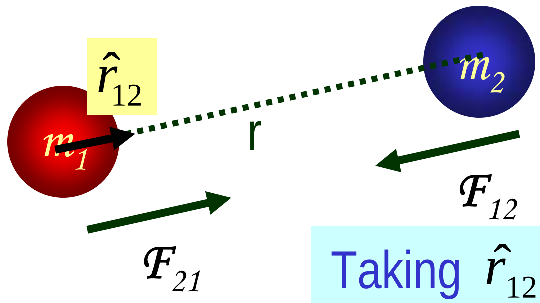
Unit? $N \cdot m^2 / kg^2$

This constant is not given by the theory but must be measured by experiment.

This form of forces is known as an inverse-square law, because the magnitude of the force is inversely proportional to the square of the distances between the objects.

More on Law of Universal Gravitation

Consider two particles exerting gravitational forces to each other.



Two objects exert gravitational force on each other following Newton's 3rd law.

Taking $\hat{r}_{12}$ as the unit vector, we can write the force m_2 experiences as

$$\vec{F}_{12} = -G\frac{m_1 m_2}{r^2}\hat{r}_{12}$$

What do you think the negative sign mean?

It means that the force exerted on the particle 2 by particle 1 is attractive force, pulling #2 toward #1.

Gravitational force is a field force: Forces act on object without physical contact between the objects at all times, independent of medium between them.

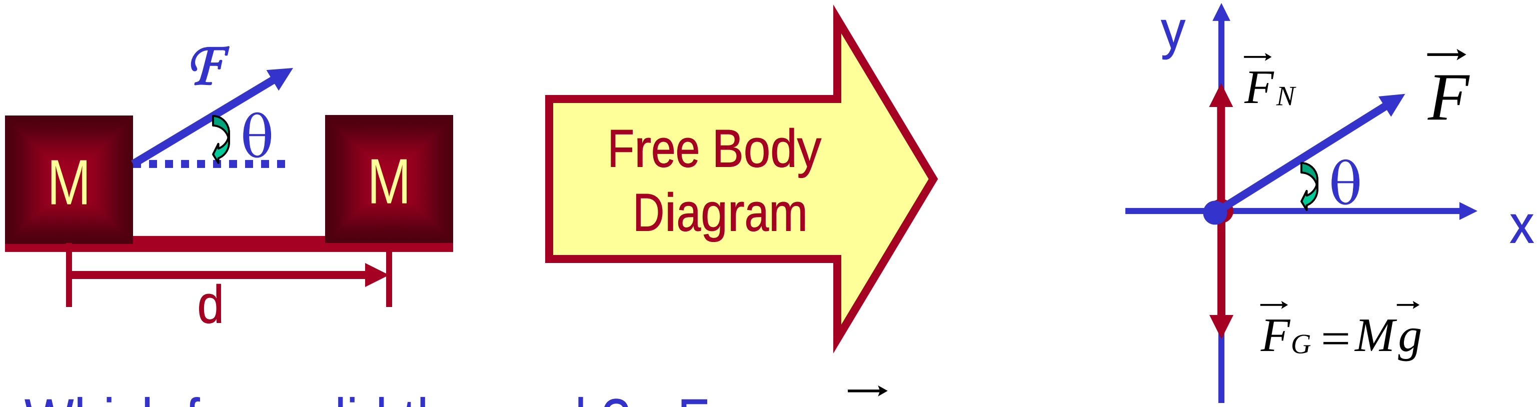
The gravitational force exerted by a finite size, spherically symmetric mass distribution on a particle outside the distribution is the same as if the entire mass of the distributions was concentrated at the center.

How do you think the gravitational force on the surface of the earth look?

$$F_g = G\frac{M_E m}{R_E^2}$$

Work Done by a Constant Force

Work in physics is done only when the SUM of forces exerted on an object caused a motion to the object.



Which force did the work? Force $\vec{F}$

How much work did it do? $W = \left|\sum\left(\vec{F}\right)\right|\left|\vec{d}\right|\cos\theta$ Unit? $N \bullet m$ $= J$ (for Joule)

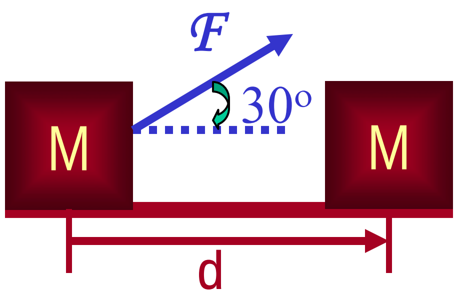
What does this mean?

Physical work is done only by the component of the force along the movement of the object.

Work is energy transfer!!

Example of Work w/ Constant Force

A man cleaning a floor pulls a vacuum cleaner with a force of magnitude F=50.0N at an angle of 30.0° with respect to East. Calculate the work done by the force on the vacuum cleaner as the vacuum cleaner is displaced by 3.00m to East.



$$W = \left|\left(\sum \vec{F}\right)\right|\left|\vec{d}\right|\cos\theta$$

$$W = 50.0 \times 3.00 \times \cos 30^{o} = 130J$$

Does work depend on mass of the object being worked on?

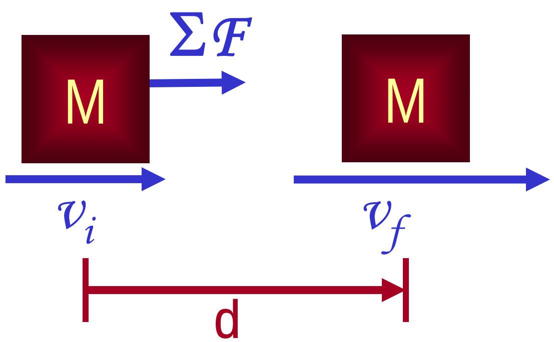
Yes!

Why don't I see the mass term in the work at all then?

It is reflected in the force. If the object has smaller mass, it would take less force to move it the same distance as the heavier object. So it would take less work. Which makes perfect sense, doesn't it?

Kinetic Energy and Work-Kinetic Energy Theorem

- Some problems are hard to solve using Newton's second law
  - The forces exerting on the object during the motion are very complicated.
  - Relate the work done on the object by the net force to the change of the speed of the object.



Suppose net force $\Sigma\mathcal{F}$ was exerted on an object for displacement d to increase its speed from v_i to v_f.

The work on the object by the net force $\Sigma\mathcal{F}$ is

$$W = Fd\cos\theta = (ma)d\cos 0 = (ma)d$$

Displacement $$d = \frac{1}{2}\left(v_f + v_i\right)t$$

Acceleration $$a = \frac{v_f - v_i}{t}$$

Work $$W = (ma)d = \left[m\left(\frac{v_f - v_i}{t}\right)\right]\frac{1}{2}\left(v_f + v_i\right)t = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

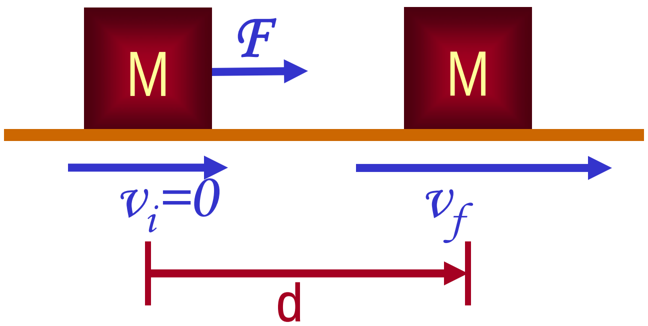
Kinetic Energy $$\boxed{KE = \frac{1}{2}mv^2}$$

Work $$W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = KE_f - KE_i = \Delta KE$$

The work done by the net force caused change of object's kinetic energy.

Example for Work-KE Theorem

A 6.0kg block initially at rest is pulled to East along a horizontal, frictionless surface by a constant horizontal force of 12N. Find the speed of the block after it has moved 3.0m.



Work done by the force $\mathcal{F}$ is

$$W = \left|\vec{F}\right|\left|\vec{d}\right|\cos\theta = 12\times 3.0\cos 0 = 36(J)$$

From the work-kinetic energy theorem, we know

$$W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

Since initial speed is 0, the above equation becomes

$$W = \frac{1}{2}mv_f^2$$

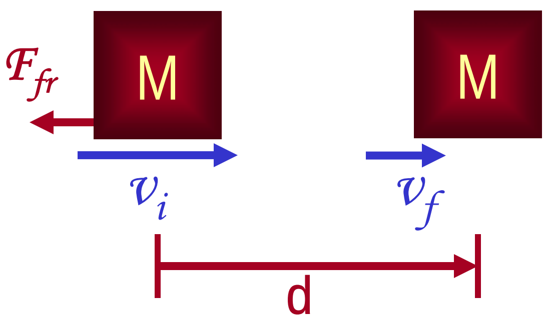
Solving the equation for v_f, we obtain

$$v_f = \sqrt{\frac{2W}{m}} = \sqrt{\frac{2\times 36}{6.0}} = 3.5m/s$$

Work and Energy Involving Kinetic Friction

- Some How do you think the work looks like if there is friction?
  - Why doesn't static friction matter?

Because it isn't there while the object is moving.



Friction force $\mathcal{F}_{fr}$ works on the object to slow down

The work on the object by the friction $\mathcal{F}_{fr}$ is

$$W_{fr} = F_{fr} d \cos(180) = -F_{fr} d \qquad \Delta KE = -F_{fr} d$$

The final kinetic energy of an object, taking into account its initial kinetic energy, friction force and other source of work, is

$$KE_f = KE_i + \sum W - F_{fr} d$$

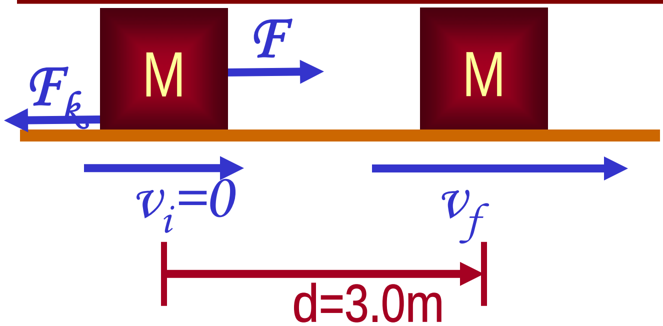
t=0, KE_i

Friction
Engine work

t=T, KE_f

Example of Work Under Friction

A 6.0kg block initially at rest is pulled to East along a horizontal surface with coefficient of kinetic friction μ_k=0.15 by a constant horizontal force of 12N. Find the speed of the block after it has moved 3.0m.



Work done by the force $\mathcal{F}$ is

$$W_F = \left|\vec{F}\right|\left|\vec{d}\right|\cos\theta = 12\times 3.0\cos 0 = 36\left(J\right)$$

Work done by friction $\mathcal{F}_k$ is

$$W_k = \left|\vec{F}_k\right|\left|\vec{d}\right|\cos\theta = \left|\mu_k mg\right|\left|\vec{d}\right|\cos\theta$$

$$= 0.15\times 6.0\times 9.8\times 3.0\cos 180 = -26\left(J\right)$$

Thus the total work is

$$W = W_F + W_k = 36 - 26 = 10(J)$$

Using work-kinetic energy theorem and the fact that initial speed is 0, we obtain

$$W = W_F + W_k = \frac{1}{2}mv_f^2$$

Solving the equation for v_f, we obtain

$$v_f = \sqrt{\frac{2W}{m}} = \sqrt{\frac{2\times 10}{6.0}} = 1.8m/s$$

Work and Kinetic Energy

Work in physics is done only when the sum of forces exerted on an object made a motion to the object.

What does this mean?

However much tired your arms feel, if you were just holding an object without moving it you have not done any physical work.

Mathematically, work is written in a product of magnitudes of the net force vector, the magnitude of the displacement vector and the angle between them,.

$$W = \left|\sum\left(\vec{F}_i\right)\right|\left|\vec{d}\right|\cos\theta$$

Kinetic Energy is the energy associated with motion and capacity to perform work. Work causes change of energy after the completion ← <u>Work-Kinetic energy theorem</u>

$$K = \frac{1}{2}mv^2$$

$$\sum W = K_f - K_i = \Delta K$$

Nm=Joule

Potential Energy

Energy associated with a system of objects ➔ *Stored energy which has Potential or possibility to work or to convert to kinetic energy*

What does this mean?

In order to describe potential energy, U, a system must be defined.

The concept of potential energy can only be used under the special class of forces called, conservative forces which results in principle of ***conservation of mechanical energy.***

$$E_M \equiv KE_i + PE_i = KE_f + PE_f$$

What are other forms of energies in the universe?

Mechanical Energy

Chemical Energy

Biological Energy

Electromagnetic Energy

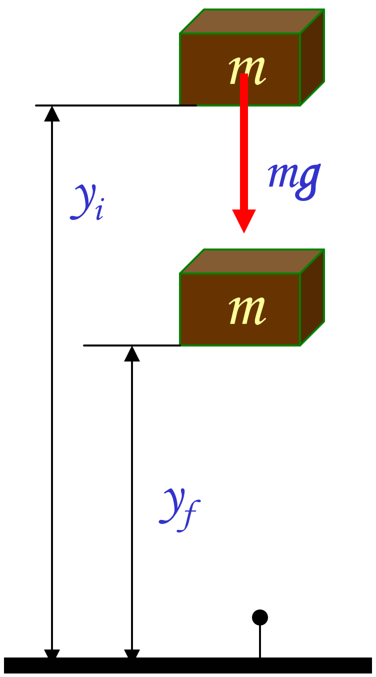
Nuclear Energy

These different types of energies are stored in the universe in many different forms!!!

If one takes into account ALL forms of energy, the total energy in the entire universe is conserved. It just transforms from one form to the other.

Gravitational Potential Energy

Potential energy given to an object by gravitational field in the system of Earth due to its height from the surface



*When an object is falling, gravitational force, M**g**, performs work on the object, increasing its kinetic energy. The potential energy of an object at a height y which is the potential to work is expressed as*

$$U_g = \left|\vec{F}_g\right|\left|\vec{y}\right|\sin\theta = \left|\vec{F}_g\right|\left|\vec{y}\right| = mgy \qquad U_g \equiv mgy$$

Work performed on the object by the gravitational force as the brick goes from y_i to y_f is:

$$W_g = U_i - U_f$$
$$= mgy_i - mgy_f = -\Delta U_g$$

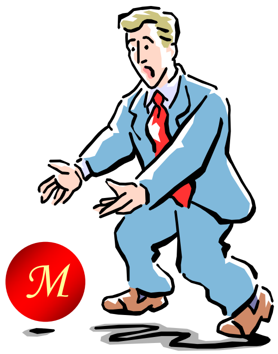
What does this mean?

Work by the gravitational force as the brick goes from y_i to y_f is negative of the change in the system's potential energy

➔ Potential energy was lost in order for gravitational force to increase the brick's kinetic energy.

Example for Potential Energy

A bowler drops bowling ball of mass 7kg on his toe. Choosing floor level as y=0, estimate the total work done on the ball by the gravitational force as the ball falls.



Let's assume the top of the toe is 0.03m from the floor and the hand was 0.5m above the floor.

$$U_i = mgy_i = 7 \times 9.8 \times 0.5 = 34.3J \quad U_f = mgy_f = 7 \times 9.8 \times 0.03 = 2.06J$$

$$W_g = -\Delta U = -\left(U_f - U_i\right) = 32.24J \cong 30J$$

b) Perform the same calculation using the top of the bowler's head as the origin.

What has to change?

First we must re-compute the positions of ball at the hand and of the toe.

Assuming the bowler's height is 1.8m, the ball's original position is –1.3m, and the toe is at –1.77m.

$$U_i = mgy_i = 7 \times 9.8 \times \left(-1.3\right) = -89.2J \quad U_f = mgy_f = 7 \times 9.8 \times \left(-1.77\right) = -121.4J$$

$$W_g = -\Delta U = -\left(U_f - U_i\right) = 32.2J \cong 30J$$

Elastic Potential Energy

Potential energy given to an object by a spring or an object with elasticity in the system consists of the object and the spring without friction.

The force spring exerts on an object when it is distorted from its equilibrium by a distance x is

$$F_s = -kx$$

The work performed on the object by the spring is

$$W_s = \int_{x_i}^{x_f} (-kx)dx = \left[-\frac{1}{2}kx^2\right]_{x_i}^{x_f} = -\frac{1}{2}kx_f^2 + \frac{1}{2}kx_i^2 = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$$

The potential energy of this system is

$$U_s \equiv \frac{1}{2}kx^2$$

What do you see from the above equations?

The work done on the object by the spring depends only on the initial and final position of the distorted spring.

Where else did you see this trend?

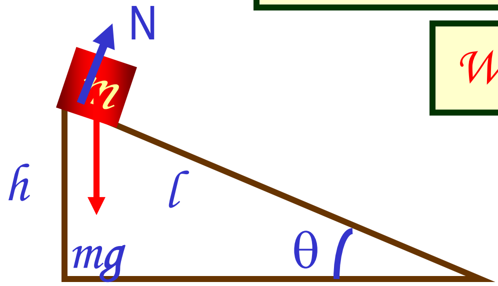
The gravitational potential energy, U_g

So what does this tell you about the elastic force?

A conservative force!!!

Conservative and Non-conservative Forces

The work done on an object by the gravitational force does not depend on the object's path.



When directly falls, the work done on the object is $W_g = mgh$

When sliding down the hill of length l, the work is

$$W_g = F_{g-incline} \times l = mg\sin\theta \times l$$

$$W_g = mg\left(l\sin\theta\right) = mgh$$

How about if we lengthen the incline by a factor of 2, keeping the height the same??

Still the same amount of work☺

$$W_g = mgh$$

So the work done by the gravitational force on an object is independent on the path of the object's movements. It only depends on the difference of the object's initial and final position in the direction of the force.

The forces like gravitational or elastic forces are called conservative forces

1. *If the work performed by the force does not depend on the path*
2. *If the work performed on a closed path is 0.*

Total mechanical energy is conserved!!

$$E_M \equiv KE_i + PE_i = KE_f + PE_f$$

More Conservative and Non-conservative Forces

A potential energy can be associated with a conservative force

A work done on a object by a conservative force is the same as the potential energy change between initial and final states

$$W_c = U_i - U_f = -\Delta U$$

So what is a conservative force?

The force that conserves mechanical energy.

OK. Then what is a non-conservative force?

The force that does not conserve mechanical energy.

The work by these forces depends on the path.

Can you give me an example?

Friction

Why is it a non-conservative force?

Because the longer the path of an object's movement, the more work the friction forces perform on it.

What happens to the mechanical energy?

Kinetic energy converts to thermal energy and is not reversible.

Total mechanical energy is not conserved but the total energy is still conserved. It just exists in a different form.

$$E_T \equiv E_M + E_{Other}$$

$$KE_i + PE_i = KE_f + PE_f + W_{Friction}$$

Conservative Forces and Potential Energy

The work done on an object by a conservative force is equal to the decrease in the potential energy of the system

$$W_c = -\Delta U$$

What else does this statement tell you?

The work done by a conservative force is equal to the negative of the change of the potential energy associated with that force.

Only the changes in potential energy of a system is physically meaningful!!

We can rewrite the above equation in terms of potential energy U

$$W_c = -\Delta U = -U_f + U_i$$

So the potential energy associated with a conservative force at any given position becomes

$$U_f(x) = -W_c + U_i$$

Potential energy function

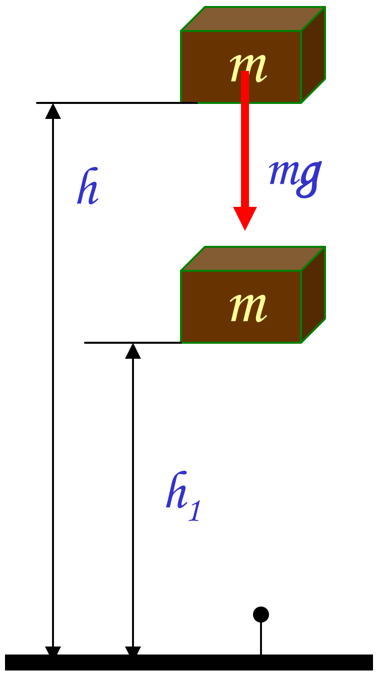
What can you tell from the potential energy function above?

Since U_i is a constant, it only shifts the resulting $U_f(x)$ by a constant amount. One can always change the initial potential so that U_i can be 0.

Conservation of Mechanical Energy

Total mechanical energy is the sum of kinetic and potential energies

$$E \equiv K + U$$



Let's consider a brick of mass *m* at a height *h* from the ground

What is its potential energy?

$$U_g = mgh$$

What happens to the energy as the brick falls to the ground?

$$\Delta U = U_f - U_i$$

The brick gains speed

By how much?

$$v = gt$$

So what?

The brick's kinetic energy increased

$$K = \frac{1}{2}mv^2 = \frac{1}{2}mg^2t^2$$

And?

The lost potential energy converted to kinetic energy

$$= mgh$$

What does this mean?

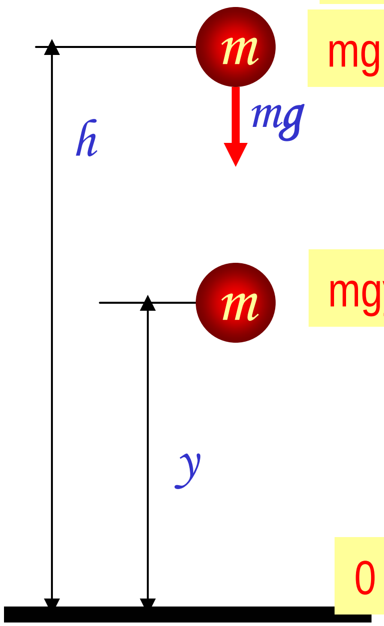
The total mechanical energy of a system remains constant in any isolated system of objects that interacts only through conservative forces: ***Principle of mechanical energy conservation***

$$E_i = E_f$$

$$K_i + \sum U_i = K_f + \sum U_f$$

Example for Mechanical Energy Conservation

A ball of mass m is dropped from a height h above the ground. Neglecting air resistance determine the speed of the ball when it is at a height y above the ground.



PE	KE	
mgh	0	$mv_i^2/2$
mgy	$mv^2/2$	$mv_i^2/2$
0		

Using the principle of mechanical energy conservation

$$K_i + U_i = K_f + U_f \qquad 0 + mgh = \frac{1}{2}mv^2 + mgy$$

$$\frac{1}{2}mv^2 = mg(h-y)$$

$$\therefore v = \sqrt{2g(h-y)}$$

b) Determine the speed of the ball at y if it had initial speed v_i at the time of release at the original height h.

Again using the principle of mechanical energy conservation but with non-zero initial kinetic energy!!!

$$K_i + U_i = K_f + U_f$$

$$\frac{1}{2}mv_i^2 + mgh = \frac{1}{2}mv_f^2 + mgy$$

Reorganize terms

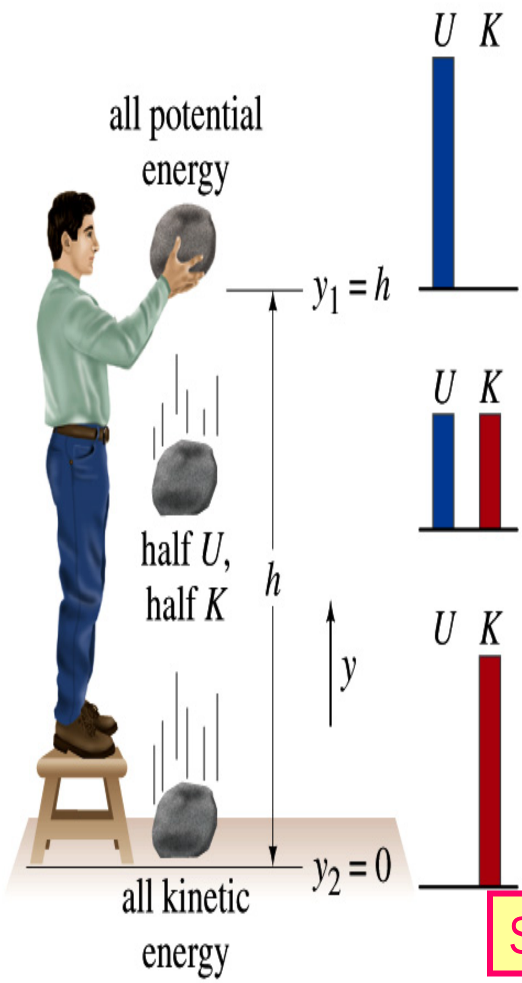
$$\frac{1}{2}m\left(v_f^2 - v_i^2\right) = mg(h-y)$$

$$\therefore v_f = \sqrt{v_i^2 + 2g(h-y)}$$

This result look very similar to a kinematic expression, doesn't it? Which one is it?

Example 6.8

If the original height of the stone in the figure is y1=h=3.0m, what is the stone's speed when it has fallen 1.0 m above the ground? Ignore air resistance.



At y=3.0m

$$\frac{1}{2}mv_1^2 + mgy_1 = mgh = 3.0mg$$

At y=1.0m

$$\frac{1}{2}mv_2^2 + mgy_2 = \frac{1}{2}mv_2^2 + 1.0mg$$

Since Mechanical Energy is conserved

$$\frac{1}{2}mv_2^2 + 1.0mg = 3.0mg$$

$$\frac{1}{2}mv_2^2 = 2.0mg$$

Cancel m

$$\frac{1}{2}v_2^2 = 2.0g$$

Solve for v

$$v_2 = \sqrt{4.0g} = \sqrt{4.0 \times 9.8} = 6.3m/s$$

Work Done by Non-conservative Forces

Mechanical energy of a system is not conserved when any one of the forces in the system is a non-conservative force.

Two kinds of non-conservative forces:

Applied forces: Forces that are external to the system. These forces can take away or add energy to the system. So the ***mechanical energy of the system is no longer conserved.***

If you were to carry around a ball, the force you apply to the ball is external to the system of ball and the Earth. Therefore, you add kinetic energy to the ball-Earth system.

$$W_{you} + W_g = \Delta K; \quad W_g = -\Delta U$$

$$W_{you} = W_{app} = \Delta K + \Delta U$$

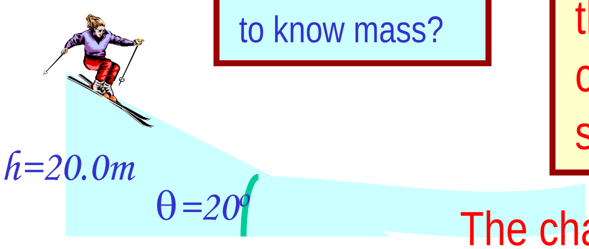
Kinetic Friction: Internal non-conservative force that causes irreversible transformation of energy. The friction force causes the kinetic and potential energy to transfer to internal energy

$$W_{friction} = \Delta K_{friction} = -f_k d$$

$$\Delta E = E_f - E_i = \Delta K + \Delta U = -f_k d$$

Example for Non-Conservative Force

A skier starts from rest at the top of frictionless hill whose vertical height is 20.0m and the inclination angle is 20°. Determine how far the skier can get on the snow at the bottom of the hill with a coefficient of kinetic friction between the ski and the snow is 0.210.



Don't we need to know mass?

Compute the speed at the bottom of the hill, using the mechanical energy conservation on the hill before friction starts working at the bottom

$$ME = mgh \quad = \frac{1}{2}mv^2$$

$$v = \sqrt{2gh}$$

$$v = \sqrt{2 \times 9.8 \times 20.0} = 19.8m/s$$

The change of kinetic energy is the same as the work done by kinetic friction.

What does this mean in this problem?

Since we are interested in the distance the skier can get to before stopping, the friction must do as much work as the available kinetic energy.

$$\Delta K = K_f - K_i = -f_k d$$

Since $K_f = 0$

$$-K_i = -f_k d; \quad f_k d = K_i$$

$$f_k = \mu_k n \quad = \mu_k mg$$

$$d = \frac{K_i}{\mu_k mg} = \frac{\frac{1}{2}mv^2}{\mu_k mg} = \frac{v^2}{2\mu_k g} = \frac{(19.8)^2}{2 \times 0.210 \times 9.80} = 95.2m$$

Well, it turns out we don't need to know mass.

What does this mean?

No matter how heavy the skier is he will get as far as anyone else has gotten.

Energy Diagram and the Equilibrium of a System

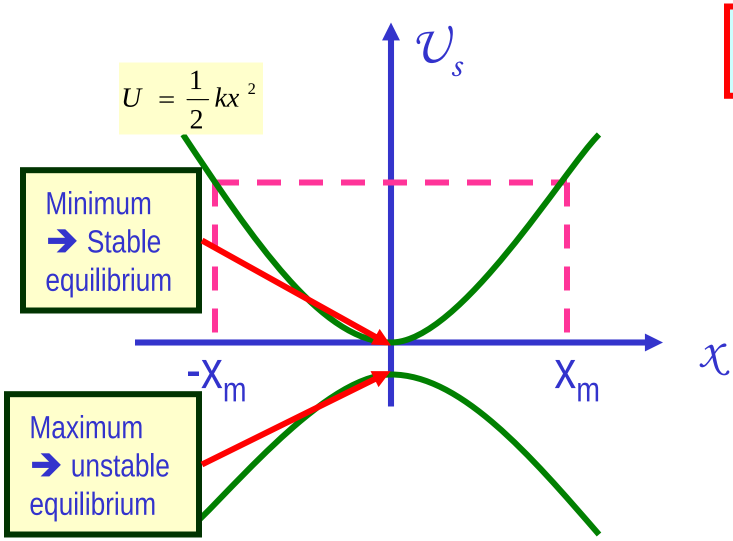
One can draw potential energy as a function of position ➔ *Energy Diagram*

Let's consider potential energy of a spring-ball system

$$U_s = \frac{1}{2}kx^2$$

What shape would this diagram be?

A Parabola



What does this energy diagram tell you?

1. *Potential energy for this system is the same independent of the sign of the position.*
2. *The force is 0 when the slope of the potential energy curve is 0 at the position.*
3. *x=0 is one of the stable or equilibrium of this system where the potential energy is minimum.*

Position of a stable equilibrium corresponds to points where potential energy is at a minimum.

Position of an unstable equilibrium corresponds to points where potential energy is a maximum.

General Energy Conservation and Mass-Energy Equivalence

General Principle of Energy Conservation

The total energy of an isolated system is conserved as long as all forms of energy are taken into account.

What about friction?

Friction is a non-conservative force and causes mechanical energy to change to other forms of energy.

However, if you add the new form of energy altogether, the system as a whole did not lose any energy, as long as it is self-contained or isolated.

In the grand scale of the universe, no energy can be destroyed or created but just transformed or transferred from one place to another.
Total energy of universe is constant.

Principle of Conservation of Mass

In any physical or chemical process, mass is neither created nor destroyed. Mass before a process is identical to the mass after the process.

Einstein's Mass-Energy equality.

$$\boxed{E_R = mc^2}$$

How many joules does your body correspond to?

Power

- Rate at which work is done
  - What is the difference for the same car with two different engines (4 cylinder and 8 cylinder) climbing the same hill? ➔ 8 cylinder car climbs up faster

Is the total amount of work done by the engines different? NO

Then what is different? The rate at which the same amount of work performed is higher for 8 cylinder than 4.

Average power $\boxed{\overline{P} = \frac{\Delta W}{\Delta t}}$

Instantaneous power $P \equiv \lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t} = \lim_{\Delta t \to 0} \left|\sum \vec{F}\right| \left|\frac{\Delta \vec{s}}{\Delta t}\right| \cos\theta = \left|\sum \vec{F}\right| \left|\vec{v}\right| \cos\theta$

Unit? $\boxed{J/s = Watts}$ $\boxed{1HP = 746\, Watts}$

What do power companies sell? $1kWH = 1000Watts \times 3600s = 3.6 \times 10^6 J$

$\boxed{\text{Energy}}$

Energy Loss in Automobile

Automobile uses only at 13% of its fuel to propel the vehicle.

Why?

67% in the engine:

1. *Incomplete burning*
2. *Heat*
3. *Sound*

16% in friction in mechanical parts

4% in operating other crucial parts such as oil and fuel pumps, etc

13% used for balancing energy loss related to moving vehicle, like air resistance and road friction to tire, etc

Two frictional forces involved in moving vehicles

$$m_{car} = 1450kg \quad Weight = mg = 14200N$$

Coefficient of Rolling Friction; μ=0.016

$$\mu n = \mu mg = 227N$$

Air Drag

$$f_a = \frac{1}{2} D\rho A v^2 = \frac{1}{2} \times 0.5 \times 1.293 \times 2v^2 = 0.647v^2$$

Total Resistance

$$f_t = f_r + f_a$$

Total power to keep speed v=26.8m/s=60mi/h

$$P = f_t v = (691N) \cdot 26.8 = 18.5kW$$

Power to overcome each component of resistance

$$P_r = f_r v = (227) \cdot 26.8 = 6.08kW$$

$$P_a = f_a v = (464.7) \cdot 26.8 = 12.5kW$$

Linear Momentum

The principle of energy conservation can be used to solve problems that are harder to solve just using Newton's laws. It is used to describe motion of an object or a system of objects.

A new concept of linear momentum can also be used to solve physical problems, especially the problems involving collisions of objects.

*Linear momentum of an object whose mass is m and is moving at a velocity of **v** is defined as*

$$\boxed{\vec{p} = m\vec{v}}$$

What can you tell from this definition about momentum?

1. *Momentum is a vector quantity.*
2. *The heavier the object the higher the momentum*
3. *The higher the velocity the higher the momentum*
4. *Its unit is kg.m/s*

What else can use see from the definition? Do you see force?

The change of momentum in a given time interval

$$\frac{\Delta \vec{p}}{\Delta t} = \frac{m\vec{v} - m\vec{v}_0}{\Delta t} = \frac{m\left(\vec{v} - \vec{v}_0\right)}{\Delta t} = m\frac{\Delta \vec{v}}{\Delta t} = m\vec{a} = \sum \vec{F}$$

Linear Momentum and Forces

$$\sum \vec{F} = \frac{\Delta \vec{p}}{\Delta t}$$

What can we learn from this Force-momentum relationship?

- The rate of the change of particle's momentum is the same as the net force exerted on it.
- When net force is 0, the particle's linear momentum is constant as a function of time.
- If a particle is isolated, the particle experiences no net force, therefore its momentum does not change and is conserved.

Something else we can do with this relationship. What do you think it is?

The relationship can be used to study the case where the mass changes as a function of time.

Can you think of a few cases like this?

Motion of a meteorite

Motion of a rocket