PHYS 1441 – Section 501
Lecture #9

Wednesday, June 30, 2004
Dr. Jaehoon Yu

- Conservative and Non-conservative Forces
- Mechanical Energy Conservation

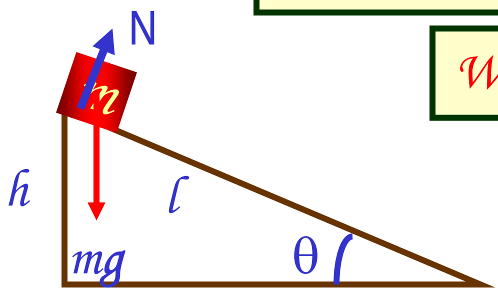
Today's Homework is #4, due at 6pm, next Wednesday, July 7!!

Announcements

- Term1 exam results
  - Average: 58.3
  - Top score: 88
  - Could do better…
- 2nd quiz next Monday, July 5
  - Beginning of the class
  - Sections 6.1 – whatever we cover today
- 2nd term exam Monday, July 19
  - Covers: Ch 6 – wherever we cover by Jul 14

Conservative and Non-conservative Forces

The work done on an object by the gravitational force does not depend on the object's path.



When directly falls, the work done on the object is $W_g = mgh$

When sliding down the hill of length l, the work is

$$W_g = F_{g-incline} \times l = mg\sin\theta \times l$$

$$W_g = mg\left(l\sin\theta\right) = mgh$$

How about if we lengthen the incline by a factor of 2, keeping the height the same??

Still the same amount of work☺ $W_g = mgh$

So the work done by the gravitational force on an object is independent on the path of the object's movements. It only depends on the difference of the object's initial and final position in the direction of the force.

The forces like gravitational or elastic forces are called conservative forces

1. *If the work performed by the force does not depend on the path*
2. *If the work performed on a closed path is 0.*

Total mechanical energy is conserved!!

$$E_M \equiv KE_i + PE_i = KE_f + PE_f$$

More Conservative and Non-conservative Forces

A potential energy can be associated with a conservative force

A work done on a object by a conservative force is the same as the potential energy change between initial and final states

$$W_c = U_i - U_f = -\Delta U$$

So what is a conservative force?

The force that conserves mechanical energy.

OK. Then what is a non-conservative force?

The force that does not conserve mechanical energy.

The work by these forces depends on the path.

Can you give me an example?

Friction

Why is it a non-conservative force?

Because the longer the path of an object's movement, the more work the friction forces perform on it.

What happens to the mechanical energy?

Kinetic energy converts to thermal energy and is not reversible.

Total mechanical energy is not conserved but the total energy is still conserved. It just exists in a different form.

$$E_T \equiv E_M + E_{Other}$$

$$KE_i + PE_i = KE_f + PE_f + W_{Friction}$$

Conservative Forces and Potential Energy

The work done on an object by a conservative force is equal to the decrease in the potential energy of the system

$$W_c = -\Delta U$$

What else does this statement tell you?

The work done by a conservative force is equal to the negative of the change of the potential energy associated with that force.

Only the changes in potential energy of a system is physically meaningful!!

We can rewrite the above equation in terms of potential energy U

$$W_c = -\Delta U = -U_f + U_i$$

So the potential energy associated with a conservative force at any given position becomes

$$U_f(x) = -W_c + U_i$$

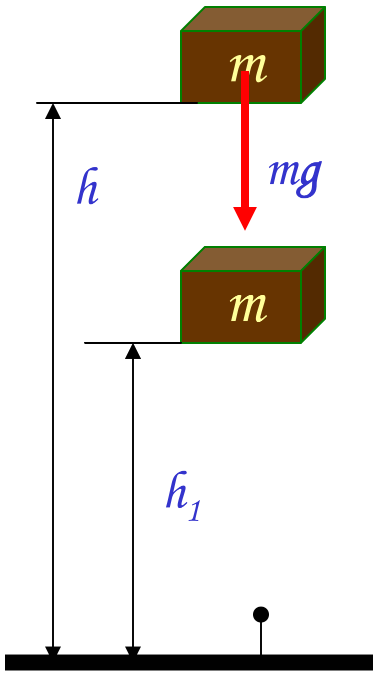
Potential energy function

What can you tell from the potential energy function above?

Since U_i is a constant, it only shifts the resulting $U_f(x)$ by a constant amount. One can always change the initial potential so that U_i can be 0.

Conservation of Mechanical Energy

Total mechanical energy is the sum of kinetic and potential energies $E \equiv K + U$



Let's consider a brick of mass *m* at a height *h* from the ground

What is its potential energy?

$$U_g = mgh$$

What happens to the energy as the brick falls to the ground?

$$\Delta U = U_f - U_i$$

The brick gains speed

By how much? $v = gt$

So what? *The brick's kinetic energy increased*

$$K = \frac{1}{2}mv^2 = \frac{1}{2}mg^2t^2$$

$$= mg\left(\frac{1}{2}gt^2\right) = mgh$$

And? *The lost potential energy is converted to kinetic energy of the brick!*

What does this mean?

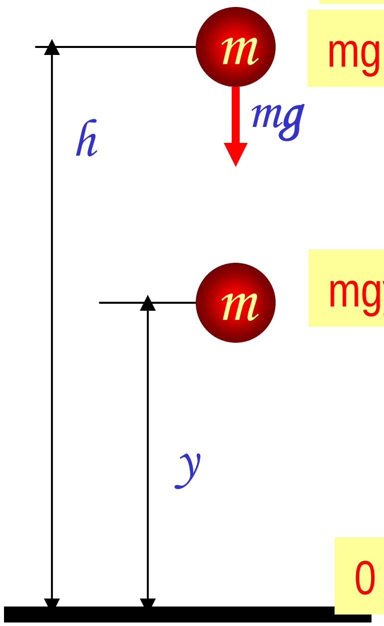
The total mechanical energy of a system remains constant in any isolated system of objects that interacts only through conservative forces: ***Principle of mechanical energy conservation***

$$E_i = E_f$$

$$K_i + \sum U_i = K_f + \sum U_f$$

Example for Mechanical Energy Conservation

A ball of mass m is dropped from a height h above the ground. Neglecting air resistance determine the speed of the ball when it is at a height y above the ground.



PE	KE	
mgh	0	$mv_i^2/2$
mgy	$mv^2/2$	$mv_f^2/2$
0		

Using the principle of mechanical energy conservation

$$K_i + U_i = K_f + U_f \quad 0 + mgh = \frac{1}{2}mv^2 + mgy$$

$$\frac{1}{2}mv^2 = mg(h-y)$$

$$\therefore v = \sqrt{2g(h-y)}$$

b) Determine the speed of the ball at y if it had initial speed v_i at the time of release at the original height h.

Again using the principle of mechanical energy conservation but with non-zero initial kinetic energy!!!

$$K_i + U_i = K_f + U_f$$

$$\frac{1}{2}mv_i^2 + mgh = \frac{1}{2}mv_f^2 + mgy$$

Reorganize terms

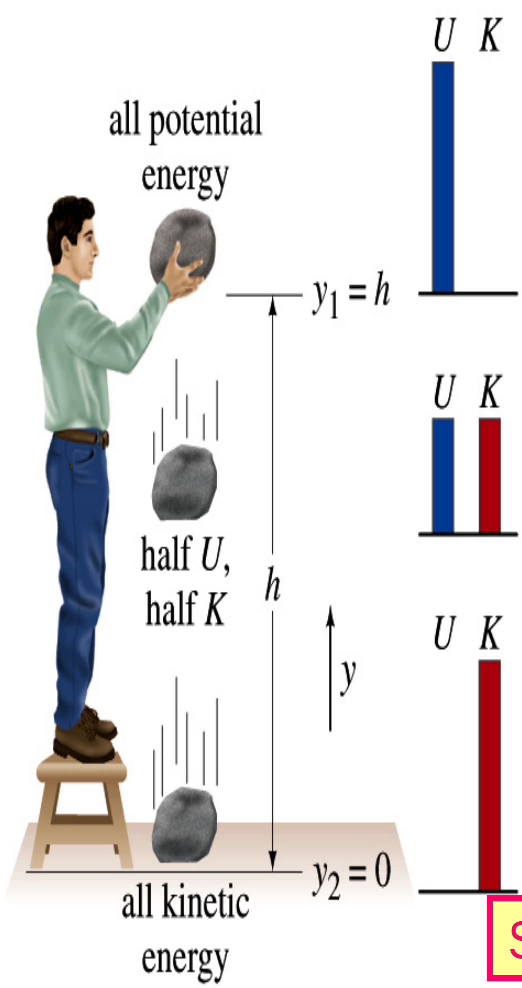
$$\frac{1}{2}m\left(v_f^2 - v_i^2\right) = mg\left(h-y\right)$$

$$\therefore v_f = \sqrt{v_i^2 + 2g(h-y)}$$

This result look very similar to a kinematic expression, doesn't it? Which one is it?

Example 6.8

If the original height of the stone in the figure is y1=h=3.0m, what is the stone's speed when it has fallen 1.0 m above the ground? Ignore air resistance.



At y=3.0m

$$\frac{1}{2}mv_1^2 + mgy_1 = mgh = 3.0mg$$

At y=1.0m

$$\frac{1}{2}mv_2^2 + mgy_2 = \frac{1}{2}mv_2^2 + 1.0mg$$

Since Mechanical Energy is conserved

$$\frac{1}{2}mv_2^2 + 1.0mg = 3.0mg$$

$$\frac{1}{2}mv_2^2 = 2.0mg$$

Cancel m

$$\frac{1}{2}v_2^2 = 2.0g$$

Solve for v

$$v_2 = \sqrt{4.0g} = \sqrt{4.0 \times 9.8} = 6.3m/s$$

Work Done by Non-conservative Forces

Mechanical energy of a system is not conserved when any one of the forces in the system is a non-conservative force.

Two kinds of non-conservative forces:

Applied forces: Forces that are external to the system. These forces can take away or add energy to the system. So the ***mechanical energy of the system is no longer conserved.***

If you were to carry around a ball, the force you apply to the ball is external to the system of ball and the Earth. Therefore, you add kinetic energy to the ball-Earth system.

$$W_{you} + W_g = \Delta K; \quad W_g = -\Delta U$$

$$W_{you} = W_{app} = \Delta K + \Delta U$$

Kinetic Friction: Internal non-conservative force that causes irreversible transformation of energy. The friction force causes the kinetic and potential energy to transfer to internal energy

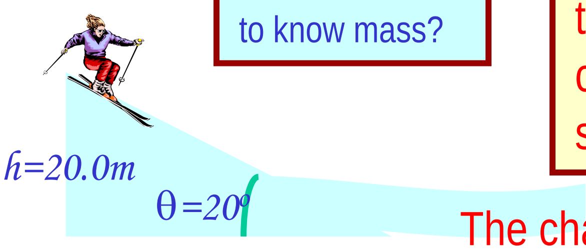
$$W_{friction} = \Delta K_{friction} = -f_k d$$

$$\Delta E = E_f - E_i = \Delta K + \Delta U = -f_k d$$

Example for Non-Conservative Force

A skier starts from rest at the top of frictionless hill whose vertical height is 20.0m and the inclination angle is 20°. Determine how far the skier can get on the snow at the bottom of the hill with a coefficient of kinetic friction between the ski and the snow is 0.210.

Don't we need to know mass?



Compute the speed at the bottom of the hill, using the mechanical energy conservation on the hill before friction starts working at the bottom

$$ME = mgh = \frac{1}{2}mv^2$$

$$v = \sqrt{2gh}$$

$$= \sqrt{2\times 9.8\times 20.0} = 19.8m/s$$

The change of kinetic energy is the same as the work done by kinetic friction.

What does this mean in this problem?

$$\Delta K = K_f - K_i = -f_k d$$

Since we are interested in the distance the skier can get to before stopping, the friction must do as much work as the available kinetic energy.

Since $K_f = 0$

$$-K_i = -f_k d; \quad f_k d = K_i$$

$$f_k = \mu_k n = \mu_k mg$$

$$d = \frac{K_i}{\mu_k mg} = \frac{\frac{1}{2}mv^2}{\mu_k mg} = \frac{v^2}{2\mu_k g} = \frac{(19.8)^2}{2\times 0.210\times 9.80} = 95.2m$$

Well, it turns out we don't need to know mass.

What does this mean?

No matter how heavy the skier is he will get as far as anyone else has gotten.