PHYS 1441 – Section 501
Lecture #10

Monday, July 5, 2004
Dr. Jaehoon Yu

- Energy Diagrams
- Power
- Linear Momentum & its conservation
- Impulse & Collisions
- Center of Mass
- CM of a group of particles

Remember the second term exam, Monday, July 19!!

Energy Diagram and the Equilibrium of a System

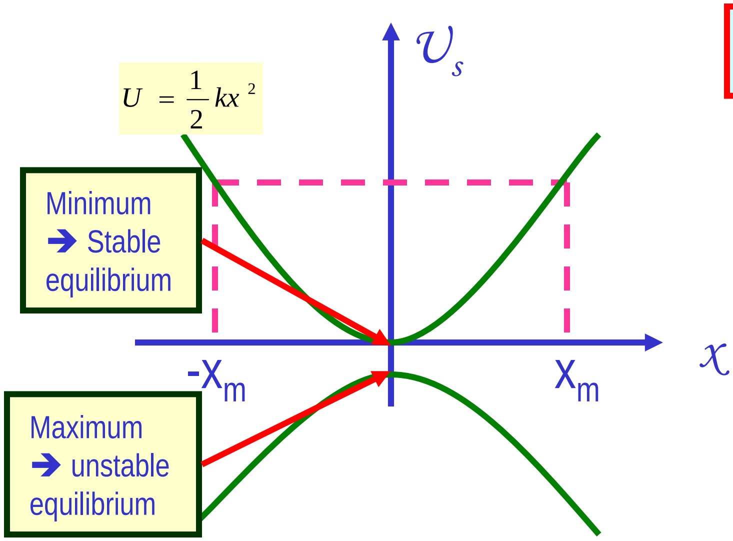
One can draw potential energy as a function of position ➔ *Energy Diagram*

Let's consider potential energy of a spring-ball system

$$U_s = \frac{1}{2}kx^2$$

What shape would this diagram be?

A Parabola



What does this energy diagram tell you?

1. *Potential energy for this system is the same independent of the sign of the position.*
2. *The force is 0 when the slope of the potential energy curve is 0 at the position.*
3. *x=0 is the stable equilibrium of this system where the potential energy is minimum.*

Position of a stable equilibrium corresponds to points where potential energy is at a minimum.

Position of an unstable equilibrium corresponds to points where potential energy is a maximum.

General Energy Conservation and Mass-Energy Equivalence

General Principle of Energy Conservation

The total energy of an isolated system is conserved as long as all forms of energy are taken into account.

What about friction?

Friction is a non-conservative force and causes mechanical energy to change to other irreversible forms of energy.

However, if you add the new forms of energy altogether, the system as a whole did not lose any energy, as long as it is self-contained or isolated.

In the grand scale of the universe, no energy can be destroyed or created but just transformed or transferred from one place to another. ***Total energy of universe is constant.***

Principle of Conservation of Mass

In any physical or chemical process, mass is neither created nor destroyed. Mass before a process is identical to the mass after the process.

Einstein's Mass-Energy equality.

$$\boxed{E_R = mc^2}$$

How many joules does your body correspond to?

Power

- Rate at which work is done
  - What is the difference for the same car with two different engines (4 cylinder and 8 cylinder) climbing the same hill? ➔ 8 cylinder car climbs up faster

Is the total amount of work done by the engines different? NO

Then what is different? The rate at which the same amount of work performed is higher for 8 cylinder than 4.

Average power $\boxed{\overline{P} = \frac{\Delta W}{\Delta t}}$

Instantaneous power $P \equiv \lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t} = \lim_{\Delta t \to 0} \left|\sum \vec{F}\right| \left|\frac{\Delta \vec{s}}{\Delta t}\right| \cos\theta = \left|\sum \vec{F}\right| \left|\vec{v}\right| \cos\theta$

Unit? $\boxed{J/s = Watts}$ $\boxed{1HP = 746\,Watts}$

What do power companies sell? $1kWH = 1000Watts \times 3600s = 3.6 \times 10^6 J$

$\boxed{\text{Energy}}$

Energy Loss in Automobile

Automobile uses only at 13% of its fuel to propel the vehicle.

Why?

67% in the engine:

1. *Incomplete burning*
2. *Heat*
3. *Sound*

16% in friction in mechanical parts

4% in operating other crucial parts such as oil and fuel pumps, etc

13% used for balancing energy loss related to moving vehicle, like air resistance and road friction to tire, etc

Two frictional forces involved in moving vehicles

$$m_{car} = 1450kg \quad Weight = mg = 14200N$$

Coefficient of Rolling Friction; μ=0.016

$$\mu n = \mu mg = 227N$$

Air Drag

$$f_a = \frac{1}{2} D\rho A v^2 = \frac{1}{2} \times 0.5 \times 1.293 \times 2v^2 = 0.647v^2$$

Total Resistance

$$f_t = f_r + f_a$$

Total power to keep speed v=26.8m/s=60mi/h

$$P = f_t v = (691N) \cdot 26.8 = 18.5kW$$

Power to overcome each component of resistance

$$P_r = f_r v = (227) \cdot 26.8 = 6.08kW$$

$$P_a = f_a v = (464.7) \cdot 26.8 = 12.5kW$$

Linear Momentum

The principle of energy conservation can be used to solve problems that are harder to solve just using Newton's laws. It is used to describe motion of an object or a system of objects.

A new concept of linear momentum can also be used to solve physical problems, especially the problems involving collisions of objects.

Linear momentum of an object whose mass is m and is moving at a velocity of **v** *is defined as*

$$\boxed{\vec{p} = m\vec{v}}$$

What can you tell from this definition about momentum?

1. *Momentum is a vector quantity.*
2. *The heavier the object the higher the momentum*
3. *The higher the velocity the higher the momentum*
4. *Its unit is kg.m/s*

What else can use see from the definition? Do you see force?

The change of momentum in a given time interval

$$\frac{\Delta\vec{p}}{\Delta t} = \frac{m\vec{v} - m\vec{v}_0}{\Delta t} = \frac{m\left(\vec{v} - \vec{v}_0\right)}{\Delta t} = m\frac{\Delta\vec{v}}{\Delta t} = m\vec{a} = \sum\vec{F}$$

Linear Momentum and Forces

$$\sum \vec{F} = \frac{\Delta \vec{p}}{\Delta t}$$

What can we learn from this Force-momentum relationship?

- The rate of the change of particle's momentum is the same as the net force exerted on it.
- When net force is 0, the particle's linear momentum is constant as a function of time.
- If a particle is isolated, the particle experiences no net force, therefore its momentum does not change and is conserved.

Something else we can do with this relationship. What do you think it is?

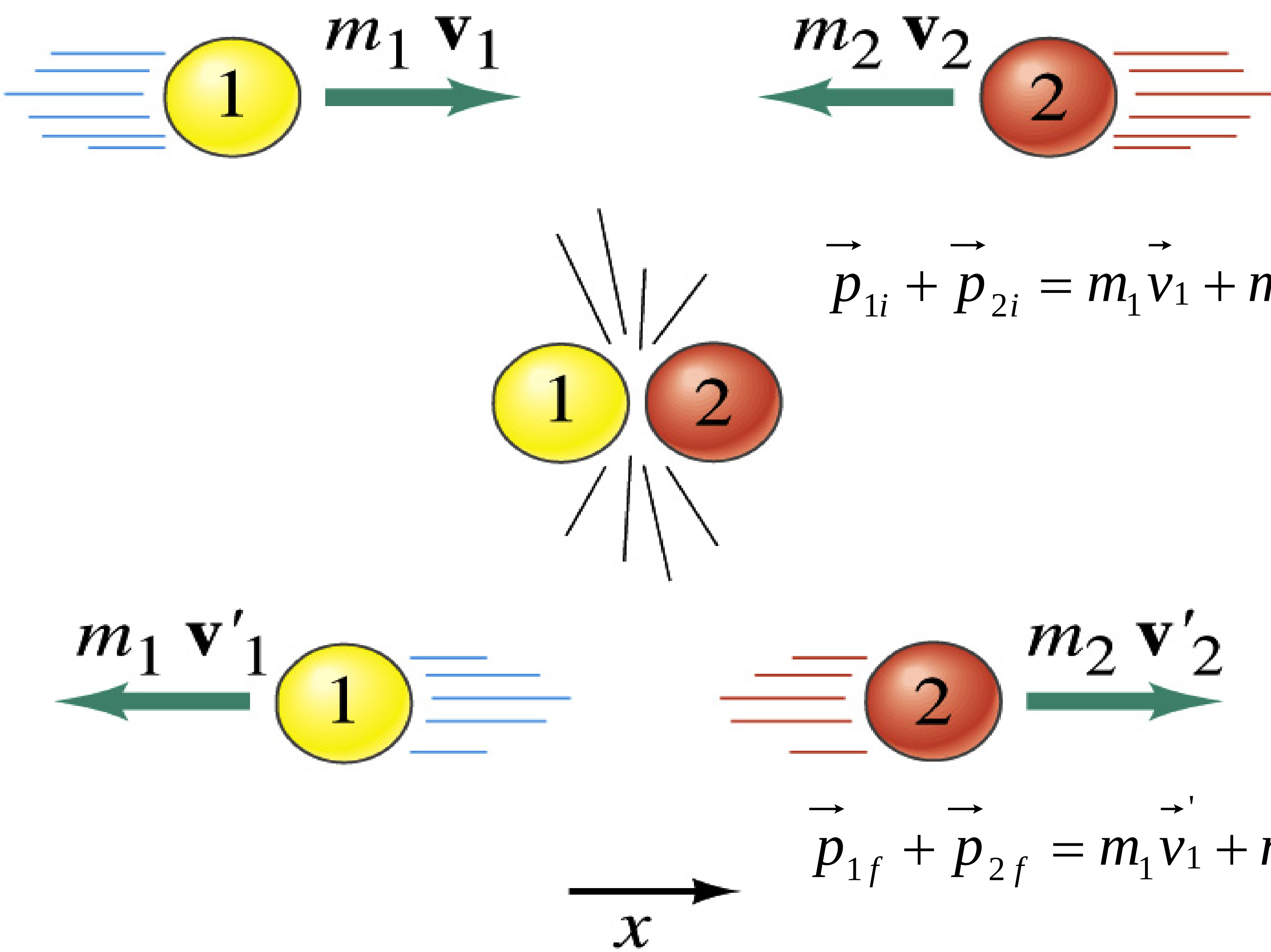
The relationship can be used to study the case where the mass changes as a function of time.

Can you think of a few cases like this?

Motion of a meteorite

Motion of a rocket

Linear Momentum Conservation



$$\vec{p}_{1i} + \vec{p}_{2i} = m_1\vec{v}_1 + m_2\vec{v}_2$$

$$\vec{p}_{1f} + \vec{p}_{2f} = m_1\vec{v}_1^{\,\prime} + m_2\vec{v}_2^{\,\prime}$$

Conservation of Linear Momentum in a Two Particle System

Consider a system with two particles that does not have any external forces exerting on it. What is the impact of Newton's 3rd Law?

If particle #1 exerts force on particle #2, there must be another force that the particle #2 exerts on #1 as the reaction force. Both the forces are internal forces and the net force in the SYSTEM is still 0.

Now how would the momenta of these particles look like?

Let say that the particle #1 has momentum $\mathbf{p}_1$ *and #2 has* $\mathbf{p}_2$ *at some point of time.*

Using momentum-force relationship

$$\vec{F}_{21} = \frac{\Delta\vec{p}_1}{\Delta t}$$ and $$\vec{F}_{12} = \frac{\Delta\vec{p}_2}{\Delta t}$$

And since net force of this system is 0

$$\sum\vec{F} = \vec{F}_{12} + \vec{F}_{21} = \frac{\Delta\vec{p}_2}{\Delta t} + \frac{\Delta\vec{p}_1}{\Delta t} = \frac{\Delta}{\Delta t}\left(\vec{p}_2 + \vec{p}_1\right) = 0$$

Therefore $\vec{p}_2 + \vec{p}_1 = const$ *The total linear momentum of the system is conserved!!!*

More on Conservation of Linear Momentum in a Two Particle System

From the previous slide we've learned that the total momentum of the system is conserved if no external forces are exerted on the system.

$$\sum \vec{p} = \vec{p}_2 + \vec{p}_1 = const$$

What does this mean?

As in the case of energy conservation, this means that the total vector sum of all momenta in the system is the same before and after any interaction

Mathematically this statement can be written as

$$\vec{p}_{2i} + \vec{p}_{1_i} = \vec{p}_{2f} + \vec{p}_{1f}$$

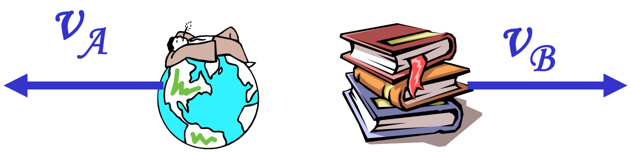
$$\sum_{system} P_{xi} = \sum_{system} P_{xf} \qquad \sum_{system} P_{yi} = \sum_{system} P_{yf} \qquad \sum_{system} P_{zi} = \sum_{system} P_{zf}$$

This can be generalized into conservation of linear momentum in many particle systems.

Whenever two or more particles in an isolated system interact, the total momentum of the system remains constant.

Example for Linear Momentum Conservation

Estimate an astronaut's resulting velocity after he throws his book to a direction in the space to move to a direction.



From momentum conservation, we can write

$$\vec{p}_i = 0 = \vec{p}_f = m_A \vec{v}_A + m_B \vec{v}_B$$

Assuming the astronaut's mass if 70kg, and the book's mass is 1kg and using linear momentum conservation

$$\vec{v}_A = -\frac{m_B \vec{v}_B}{m_A} = -\frac{1}{70}\vec{v}_B$$

Now if the book gained a velocity of 20 m/s in +x-direction, the Astronaut's velocity is

$$\vec{v}_A = -\frac{1}{70}\left(20\vec{i}\right) = -0.3\,\vec{i}\,(m/s)$$

Impulse and Linear Momentum

Net force causes change of momentum ➔ *Newton's second law*

$$\vec{F} = \frac{\Delta \vec{p}}{\Delta t} \qquad \Delta \vec{p} = \vec{F}\Delta t$$

By summing the above equation in a time interval t_i to t_f, one can obtain impulse I.

$$\Delta\vec{p} = \vec{p}_f - \vec{p}_i \quad = \vec{F}\Delta t \qquad \boxed{\vec{I} \equiv \vec{F}\Delta t = \Delta\vec{p}}$$

So what do you think an impulse is?

Impulse of the force F acting on a particle over the time interval $\Delta t = t_f - t_i$ is equal to the change of the momentum of the particle caused by that force. Impulse is the degree of which an external force changes momentum.

The above statement is called the impulse-momentum theorem and is equivalent to Newton's second law.

What are the dimension and unit of Impulse? What is the direction of an impulse vector?

Defining a time-averaged force

$$\boxed{\overline{\vec{F}} \equiv \frac{1}{\Delta t}\sum_i \vec{F}_i \Delta t}$$

Impulse can be rewritten

$$\boxed{\vec{I} \equiv \overline{\vec{F}}\Delta t}$$

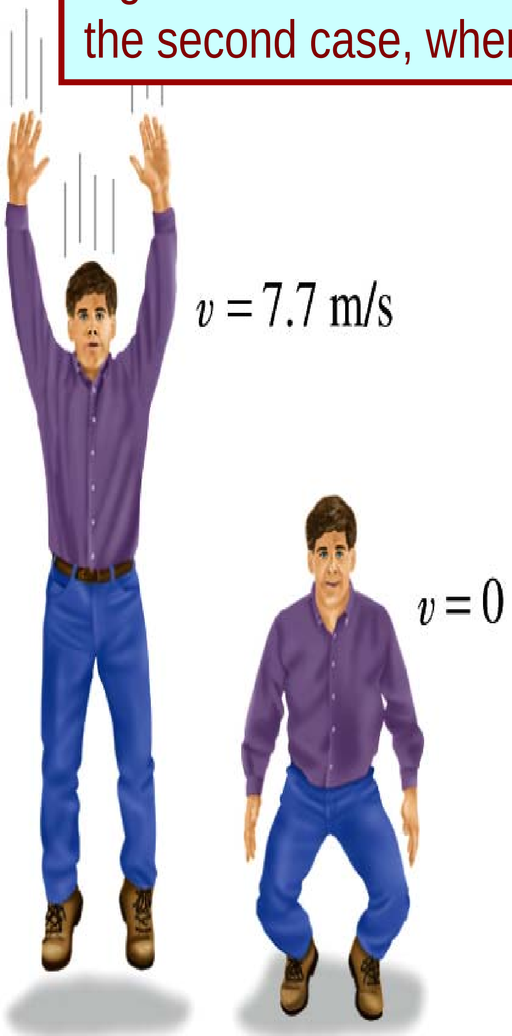
If force is constant

$$\boxed{\vec{I} \equiv \vec{F}\Delta t}$$

It is generally assumed that the impulse force acts on a short time but much greater than any other forces present.

Example 7-5

(a) Calculate the impulse experienced when a 70 kg person lands on firm ground after jumping from a height of 3.0 m. Then estimate the average force exerted on the person's feet by the ground, if the landing is (b) stiff-legged and (c) with bent legs. In the former case, assume the body moves 1.0cm during the impact, and in the second case, when the legs are bent, about 50 cm.



We don't know the force. How do we do this?

Obtain velocity of the person before striking the ground.

$$KE = -\Delta PE \qquad \frac{1}{2}mv^2 = -mg\left(y - y_i\right) = mgy_i$$

Solving the above for velocity v, we obtain

$$v = \sqrt{2gy_i} = \sqrt{2\cdot 9.8\cdot 3} = 7.7m/s$$

Then as the person strikes the ground, the momentum becomes 0 quickly giving the impulse

$$I = \overline{F}\Delta t = \Delta p = p_f - p_i = 0 - mv =$$

$$= -70kg\cdot 7.7m/s = -540N\cdot s$$

Example 7-5 cont'd

In coming to rest, the body decelerates from 7.7m/s to 0m/s in a distance d=1.0cm=0.01m.

The average speed during this period is $$\bar{v} = \frac{0 + v_i}{2} = \frac{7.7}{2} = 3.8m/s$$

The time period the collision lasts is $$\Delta t = \frac{d}{\bar{v}} = \frac{0.01m}{3.8m/s} = 2.6 \times 10^{-3}s$$

Since the magnitude of impulse is $$I = \bar{F}\Delta t = \quad 540N \cdot s$$

The average force on the feet during this landing is $$\bar{F} = \frac{I}{\Delta t} = \frac{540}{2.6 \times 10^{-3}} = 2.1 \times 10^5 N$$

How large is this average force? $$Weight = 70kg \cdot 9.8m/s^2 = 6.9 \times 10^2 N$$

$$\bar{F} = 2.1 \times 10^5 N = 304 \times 6.9 \times 10^2 N = \quad 304 \times Weight$$

If landed in stiff legged, the feet must sustain 300 times the body weight. The person will likely break his leg.

Example for Impulse

In a crash test, an automobile of mass 1500kg collides with a wall. The initial and final velocities of the automobile are v_i=-15.0i m/s and v_f=2.60i m/s. If the collision lasts for 0.150 seconds, what would be the impulse caused by the collision and the average force exerted on the automobile?

Let's assume that the force involved in the collision is a lot larger than any other forces in the system during the collision. From the problem, the initial and final momentum of the automobile before and after the collision is

$$\vec{p}_i = m\vec{v}_i = 1500 \times (-15.0)\vec{i} = -22500\,\vec{i}\ kg \cdot m/s$$

$$\vec{p}_f = m\vec{v}_f = 1500 \times (2.60)\vec{i} = 3900\,\vec{i}\ kg \cdot m/s$$

Therefore the impulse on the automobile due to the collision is

$$\vec{I} = \Delta\vec{p} = \vec{p}_f - \vec{p}_i = (3900 + 22500)\vec{i}\ kg \cdot m/s$$
$$= 26400\,\vec{i}\ kg \cdot m/s = 2.64 \times 10^4\,\vec{i}\ kg \cdot m/s$$

The average force exerted on the automobile during the collision is

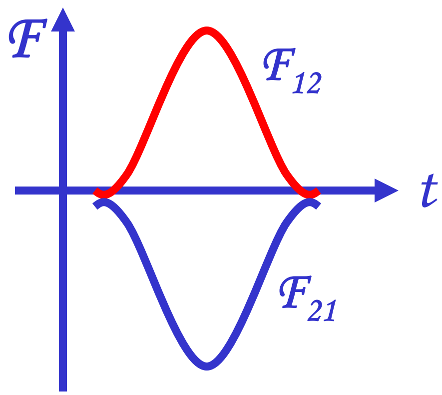
$$\overline{\vec{F}} = \frac{\Delta\vec{p}}{\Delta t} = \frac{2.64 \times 10^4}{0.150}$$
$$= 1.76 \times 10^5\,\vec{i}\ kg \cdot m/s^2 = 1.76 \times 10^5\,\vec{i}\ \mathrm{N}$$

Collisions

Generalized collisions must cover not only the physical contact but also the collisions without physical contact such as that of electromagnetic ones in a microscopic scale.

Consider a case of a collision between a proton on a helium ion.

The collisions of these ions never involves a physical contact because the electromagnetic repulsive force between these two become great as they get closer causing a collision.



Assuming no external forces, the force exerted on particle 1 by particle 2, $\mathcal{F}_{21}$, changes the momentum of particle 1 by

$$\Delta\vec{p}_1 = \vec{F}_{21}\Delta t$$

Likewise for particle 2 by particle 1

$$\Delta\vec{p}_2 = \vec{F}_{12}\Delta t$$

Using Newton's 3rd law we obtain

$$\Delta\vec{p}_2 = \vec{F}_{12}\Delta t = -\vec{F}_{21}\Delta t = -\Delta\vec{p}_1$$

So the momentum change of the system in the collision is 0 and the momentum is conserved

$$\Delta\vec{p} = \Delta\vec{p}_1 + \Delta\vec{p}_2 = 0$$

$$\vec{p}_{system} = \vec{p}_1 + \vec{p}_2 = \text{constant}$$

Elastic and Inelastic Collisions

Momentum is conserved in any collisions as long as external forces negligible.

Collisions are classified as elastic or inelastic by the conservation of kinetic energy before and after the collisions.

Elastic Collision	*A collision in which the total kinetic energy and momentum are the same before and after the collision.*
Inelastic Collision	*A collision in which the total kinetic energy is not the same before and after the collision, but momentum is.*

Two types of inelastic collisions:Perfectly inelastic and inelastic

Perfectly Inelastic: *Two objects stick together after the collision moving at a certain velocity together.*

Inelastic: *Colliding objects do not stick together after the collision but some kinetic energy is lost.*

Note: Momentum is constant in all collisions but kinetic energy is only in elastic collisions.

Elastic and Perfectly Inelastic Collisions

In perfectly Inelastic collisions, the objects stick together after the collision, moving together. Momentum is conserved in this collision, so the final velocity of the stuck system is

$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = (m_1 + m_2)\vec{v}_f$$

$$\boxed{\vec{v}_f = \frac{m_1\vec{v}_{1i} + m_2\vec{v}_{2i}}{(m_1 + m_2)}}$$

How about elastic collisions?

$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$$

In elastic collisions, both the momentum and the kinetic energy are conserved. Therefore, the final speeds in an elastic collision can be obtained in terms of initial speeds as

$$\frac{1}{2}m_1v_{1i}^2 + \frac{1}{2}m_2v_{2i}^2 = \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2$$

$$m_1\left(v_{1i}^2 - v_{1f}^2\right) = m_2\left(v_{2i}^2 - v_{2f}^2\right)$$

$$m_1\left(v_{1i} - v_{1f}\right)\left(v_{1i} + v_{1f}\right) = m_2\left(v_{2i} - v_{2f}\right)\left(v_{2i} + v_{2f}\right)$$

From momentum conservation above

$$m_1\left(v_{1i} - v_{1f}\right) = m_2\left(v_{2i} - v_{2f}\right)$$

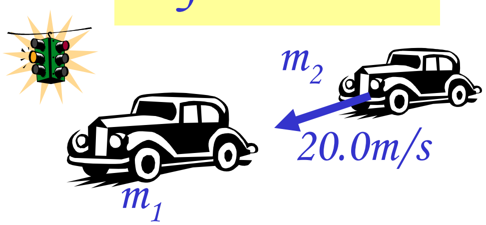
$$\boxed{v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_{1i} + \left(\frac{2m_2}{m_1 + m_2}\right)v_{2i}}$$

$$\boxed{v_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right)v_{1i} + \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_{2i}}$$

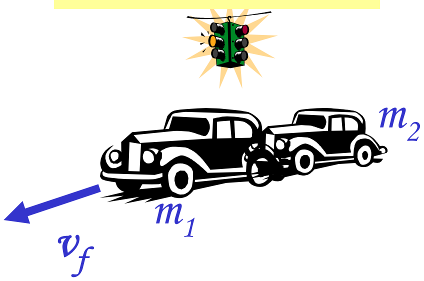
Example for Collisions

A car of mass 1800kg stopped at a traffic light is rear-ended by a 900kg car, and the two become entangled. If the lighter car was moving at 20.0m/s before the collision what is the velocity of the entangled cars after the collision?

Before collision



After collision



The momenta before and after the collision are

$$\vec{p}_i = m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = 0 + m_2\vec{v}_{2i}$$

$$\vec{p}_f = m_1\vec{v}_{1f} + m_2\vec{v}_{2f} = (m_1 + m_2)\vec{v}_f$$

Since momentum of the system must be conserved

$$\vec{p}_i = \vec{p}_f \Rightarrow (m_1 + m_2)\vec{v}_f = m_2\vec{v}_{2i}$$

$$\vec{v}_f = \frac{m_2\vec{v}_{2i}}{(m_1 + m_2)} = \frac{900 \times 20.0\vec{i}}{900 + 1800} = 6.67\vec{i}\ m/s$$

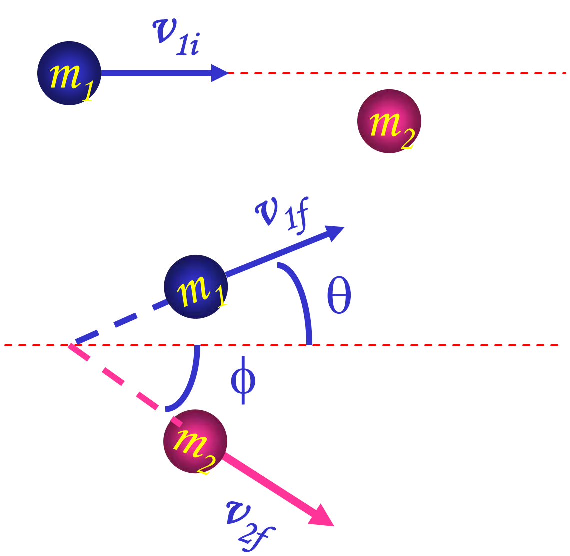
What can we learn from these equations on the direction and magnitude of the velocity before and after the collision?

The cars are moving in the same direction as the lighter car's original direction to conserve momentum.

The magnitude is inversely proportional to its own mass.

Two dimensional Collisions

In two dimension, one can use components of momentum to apply momentum conservation to solve physical problems.



$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$$

x-comp. $$m_1 v_{1ix} + m_2 v_{2ix} = m_1 v_{1fx} + m_2 v_{2fx}$$

y-comp. $$m_1 v_{1iy} + m_2 v_{2iy} = m_1 v_{1fy} + m_2 v_{2fy}$$

Consider a system of two particle collisions and scatters in two dimension as shown in the picture. (This is the case at fixed target accelerator experiments.) The momentum conservation tells us:

$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1i}$$

$$m_1 v_{1ix} = m_1 v_{1fx} + m_2 v_{2fx} = m_1 v_{1f}\cos\theta + m_2 v_{2f}\cos\phi$$

$$m_1 v_{1iy} = 0 = m_1 v_{1fy} + m_2 v_{2fy} = m_1 v_{1f}\sin\theta - m_2 v_{2f}\sin\phi$$

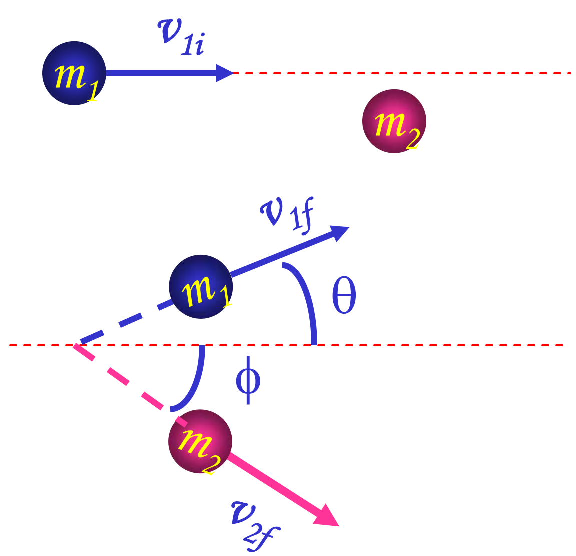
And for the elastic conservation, the kinetic energy is conserved:

$$\frac{1}{2}m_1 v_{1i}^2 = \frac{1}{2}m_1 v_{1f}^2 + \frac{1}{2}m_2 v_{2f}^2$$

What do you think we can learn from these relationships?

Example of Two Dimensional Collisions

Proton #1 with a speed 3.50×10^5 m/s collides elastically with proton #2 initially at rest. After the collision, proton #1 moves at an angle of 37^o to the horizontal axis and proton #2 deflects at an angle ϕ to the same axis. Find the final speeds of the two protons and the scattering angle of proton #2, ϕ.



Since both the particles are protons $m_1=m_2=m_p$.

Using momentum conservation, one obtains

x-comp. $m_p v_{1i} = m_p v_{1f} \cos\theta + m_p v_{2f} \cos\phi$

y-comp. $m_p v_{1f} \sin\theta - m_p v_{2f} \sin\phi = 0$

Canceling m_p and put in all known quantities, one obtains

$$v_{1f}\cos 37^o + v_{2f}\cos\phi = 3.50\times10^5 \quad (1)$$

$$v_{1f}\sin 37^o = v_{2f}\sin\phi \quad (2)$$

From kinetic energy conservation:

$$\left(3.50\times10^5\right)^2 = v_{1f}^2 + v_{2f}^2 \quad (3)$$

Solving Eqs. 1-3 equations, one gets

$$v_{1f} = 2.80\times10^5\ m/s$$
$$v_{2f} = 2.11\times10^5\ m/s$$
$$\phi = 53.0^o$$

Do this at home☺

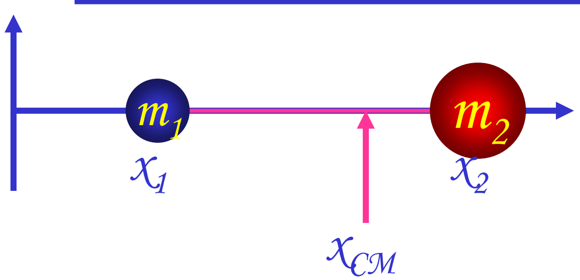
Center of Mass

We've been solving physical problems treating objects as sizeless points with masses, but in realistic situation objects have shapes with masses distributed throughout the body.

Center of mass of a system is the average position of the system's mass and represents the motion of the system as if all the mass is on the point.

What does above statement tell you concerning forces being exerted on the system?

The total external force exerted on the system of total mass M causes the center of mass to move at an acceleration given by $\vec{a} = \sum \vec{F} / M$ *as if all the mass of the system is concentrated on the center of mass.*



Consider a massless rod with two balls attached at either end.

The position of the center of mass of this system is the mass averaged position of the system

$$x_{CM} \equiv \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

CM is closer to the heavier object

Center of Mass of a Rigid Object

The formula for CM can be expanded to Rigid Object or a system of many particles

$$x_{CM}=\frac{m_1x_1+m_2x_2+\cdots+m_nx_n}{m_1+m_2+\cdots+m_n}=\frac{\sum_i m_ix_i}{\sum_i m_i}$$

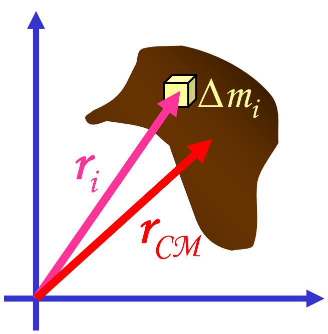
$$y_{CM}=\frac{\sum_i m_iy_i}{\sum_i m_i}$$

$$z_{CM}=\frac{\sum_i m_iz_i}{\sum_i m_i}$$

The position vector of the center of mass of a many particle system is

$$\vec{r}_{CM}=x_{CM}\vec{i}+y_{CM}\vec{j}+z_{CM}\vec{k}=\frac{\sum_i m_ix_i\vec{i}+\sum_i m_iy_i\vec{j}+\sum_i m_iz_i\vec{k}}{\sum_i m_i}$$

$$\vec{r}_{CM}=\frac{\sum_i m_i\vec{r}_i}{M}$$



A rigid body – an object with shape and size with mass spread throughout the body, ordinary objects – can be considered as a group of particles with mass m_i densely spread throughout the given shape of the object

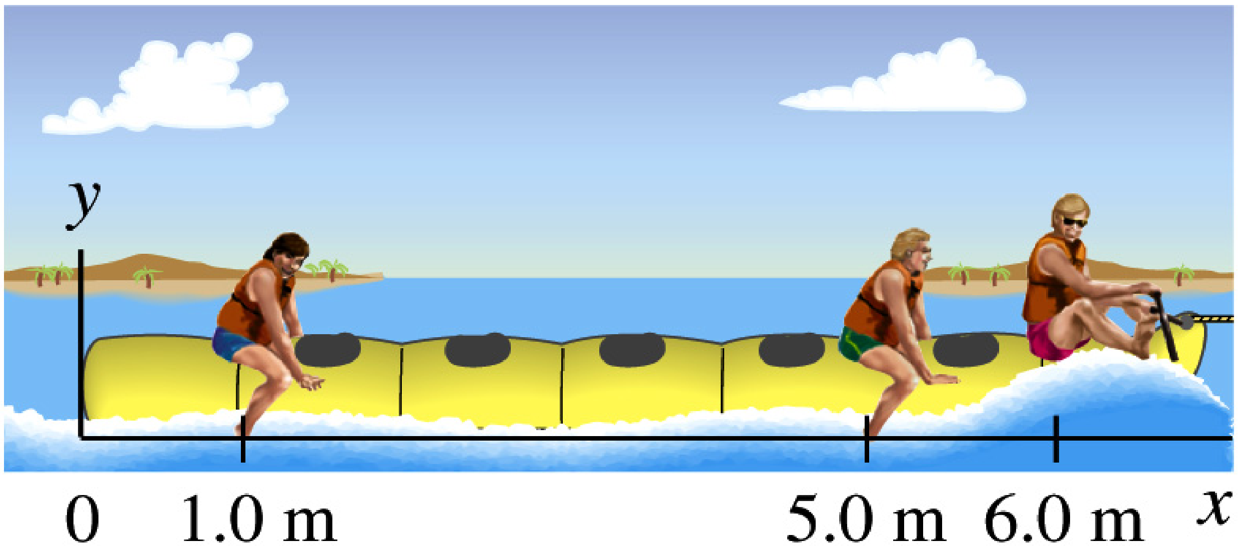
$$x_{CM}\approx\frac{\sum_i \Delta m_ix_i}{M}$$

$$x_{CM}=\lim_{\Delta m_i\to 0}\frac{\sum_i \Delta m_ix_i}{M}=\frac{1}{M}\int x\,dm$$

$$\vec{r}_{CM}=\frac{1}{M}\int \vec{r}\,dm$$

Example 7-11

Thee people of roughly equivalent mass M on a lightweight (air-filled) banana boat sit along the x axis at positions x_1=1.0m, x_2=5.0m, and x_3=6.0m. Find the position of CM.



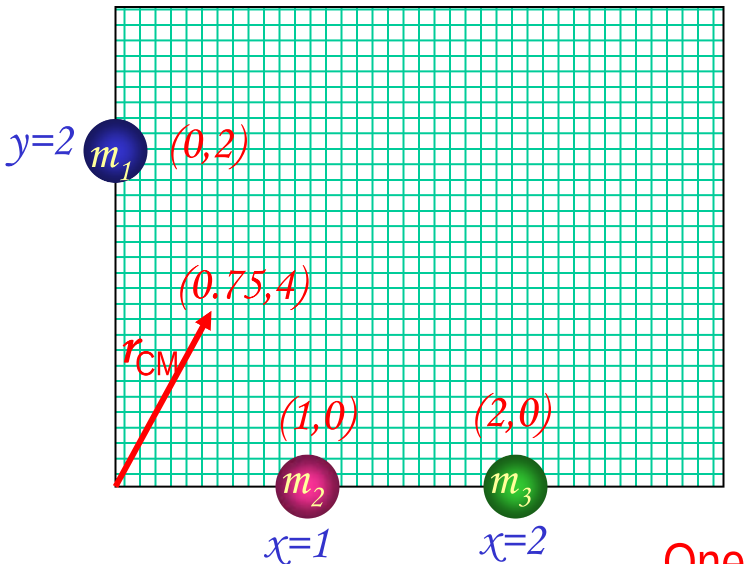
Using the formula for CM

$$x_{CM} = \frac{\sum_i m_i x_i}{\sum_i m_i}$$

$$= \frac{M \cdot 1.0 + M \cdot 5.0 + M \cdot 6.0}{M + M + M} = \frac{12.0M}{3M} = 4.0(m)$$

Example for Center of Mass in 2-D

A system consists of three particles as shown in the figure. Find the position of the center of mass of this system.



Using the formula for CM for each position vector component

$$x_{CM} = \frac{\sum_i m_i x_i}{\sum_i m_i} \qquad y_{CM} = \frac{\sum_i m_i y_i}{\sum_i m_i}$$

One obtains $\vec{r}_{CM} = x_{CM}\vec{i} + y_{CM}\vec{j} = \frac{(m_2 + 2m_3)\vec{i} + 2m_1\vec{j}}{m_1 + m_2 + m_3}$

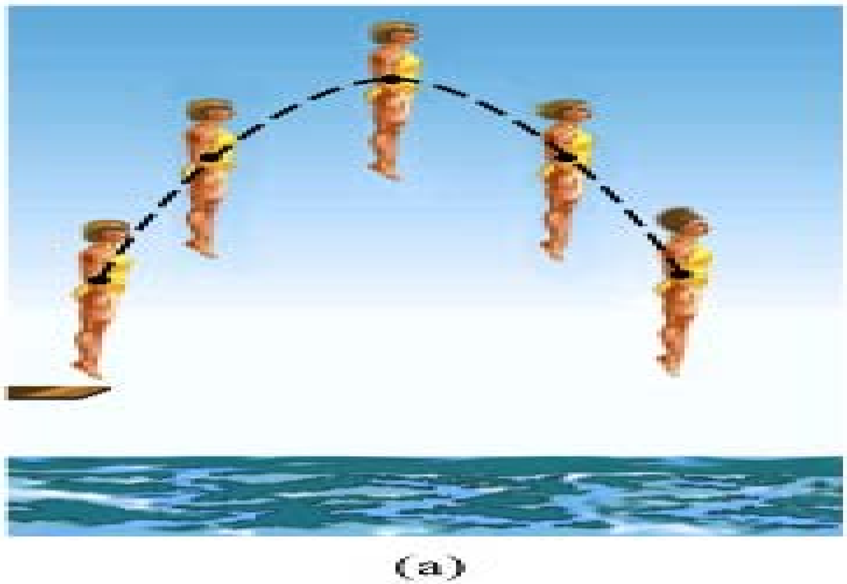
$$x_{CM} = \frac{\sum_i m_i x_i}{\sum_i m_i} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3} = \frac{m_2 + 2m_3}{m_1 + m_2 + m_3}$$

$$y_{CM} = \frac{\sum_i m_i y_i}{\sum_i m_i} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{m_1 + m_2 + m_3} = \frac{2m_1}{m_1 + m_2 + m_3}$$

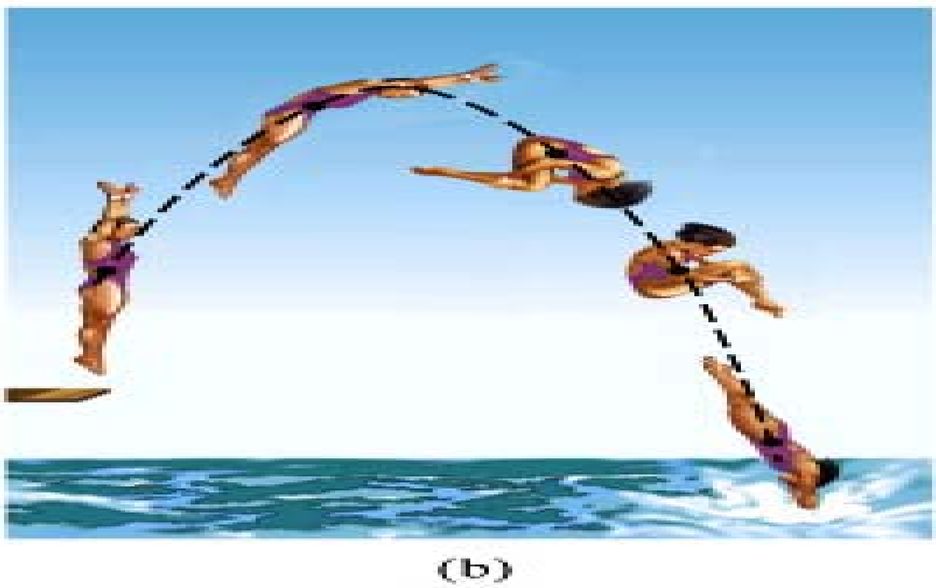
If $m_1 = 2kg; m_2 = m_3 = 1kg$

$$\vec{r}_{CM} = \frac{3\vec{i} + 4\vec{j}}{4} = 0.75\vec{i} + \vec{j}$$

Motion of a Diver and the Center of Mass



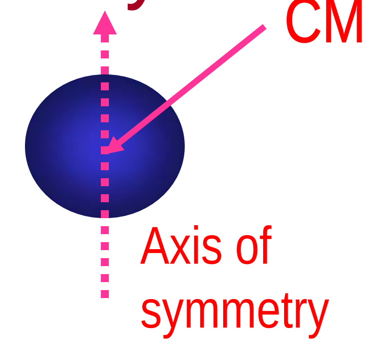
Diver performs a simple dive.
The motion of the center of mass follows a parabola since it is a projectile motion.



Diver performs a complicated dive.
The motion of the center of mass still follows the same parabola since it still is a projectile motion.

The motion of the center of mass of the diver is always the same.

Center of Mass and Center of Gravity



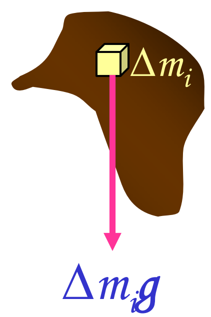
The center of mass of any symmetric object lies on an axis of symmetry and on any plane of symmetry, if object's mass is evenly distributed throughout the body.

How do you think you can determine the CM of objects that are not symmetric?

One can use gravity to locate CM.

1. Hang the object by one point and draw a vertical line following a plum-bob.
2. Hang the object by another point and do the same.
3. The point where the two lines meet is the CM.

Center of Gravity



Since a rigid object can be considered as collection of small masses, one can see the total gravitational force exerted on the object as

$$\vec{F}_g = \sum_i \vec{F}_i = \sum_i \Delta m_i \vec{g} = M\vec{g}$$

What does this equation tell you?

The net effect of these small gravitational forces is equivalent to a single force acting on a point (Center of Gravity) with mass M.

M

The CoG is the point in an object as if all the gravitational force is acting on!

Motion of a Group of Particles

We've learned that the CM of a system can represent the motion of a system. Therefore, for an isolated system of many particles in which the total mass M is preserved, the velocity, total momentum, acceleration of the system are

Velocity of the system

$$\vec{v}_{CM} = \frac{\Delta \vec{r}_{CM}}{\Delta t} = \frac{\Delta}{\Delta t}\left(\frac{1}{M}\sum m_i \vec{r}_i\right) = \frac{1}{M}\sum m_i \frac{\Delta \vec{r}_i}{\Delta t} = \frac{\sum m_i \vec{v}_i}{M}$$

Total Momentum of the system

$$\vec{p}_{CM} = M\vec{v}_{CM} = M\frac{\sum m_i \vec{v}_i}{M} = \sum m_i \vec{v}_i = \sum \vec{p}_i = \vec{p}_{tot}$$

Acceleration of the system

$$\vec{a}_{CM} = \frac{\Delta \vec{v}_{CM}}{\Delta t} = \frac{\Delta}{\Delta t}\left(\frac{1}{M}\sum m_i \vec{v}_i\right) = \frac{1}{M}\sum m_i \frac{\Delta \vec{v}_i}{\Delta t} = \frac{\sum m_i \vec{a}_i}{M}$$

External force exerting on the system

$$\sum \vec{F}_{ext} = M\vec{a}_{CM} = \sum m_i \vec{a}_i = \frac{\Delta \vec{p}_{tot}}{\Delta t}$$

What about the internal forces?

If net external force is 0

$$\sum \vec{F}_{ext} = 0 = \frac{\Delta \vec{p}_{tot}}{\Delta t}$$

$$\boxed{\vec{p}_{tot} = \text{const}}$$

System's momentum is conserved.