

# PHYS 1441 – Section 501

## Lecture #12

*Monday, July 12, 2004*

*Dr. Jaehoon Yu*

- Center of Mass & Center of Gravity
- Motion of a group of particles
- Fundamentals on Rotation
- Rotational Kinematics
- Relationships between linear and angular quantities
- Rolling Motion

Remember the second term exam, Monday, July 19!!



# Announcements

- 2<sup>nd</sup> Term:
  - Date: next Monday, July 19
  - Time: 6:00 – 7:50pm
  - Location: Class room, Sh125
  - Coverage: Ch. 5.6 – Ch. 8.5
  - Mixture of multiple choice and free-style
  - **MUST NOT Miss the exam**



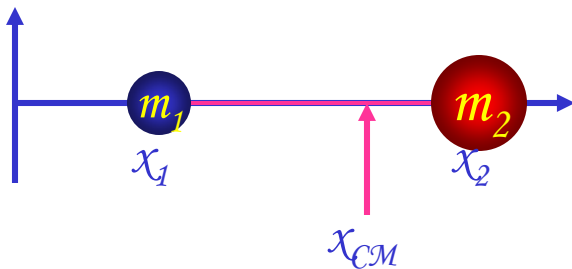
# Center of Mass

*We've been solving physical problems treating objects as sizeless points with masses, but in realistic situation objects have shapes with masses distributed throughout the body.*

Center of mass of a system is the average position of the system's mass and represents the motion of the system as if all the mass is on the point.

*What does above statement tell you concerning forces being exerted on the system?*

*The total external force exerted on the system of total mass  $M$  causes the center of mass to move at an acceleration given by  $\vec{a} = \sum \vec{F} / M$  as if all the mass of the system is concentrated on the center of mass.*



Consider a massless rod with two balls attached at either end.

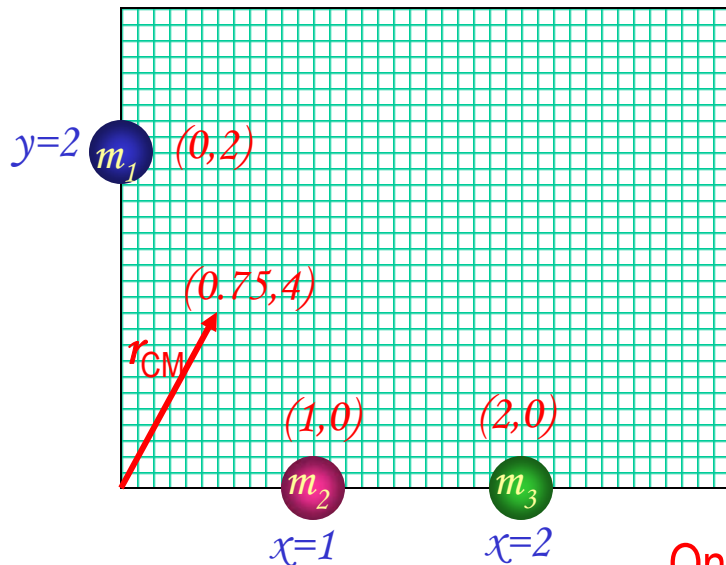
*The position of the center of mass of this system is the mass averaged position of the system*

$$x_{CM} \equiv \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

CM is closer to the heavier object

# Example for Center of Mass in 2-D

A system consists of three particles as shown in the figure. Find the position of the center of mass of this system.



Using the formula for CM for each position vector component

$$x_{CM} = \frac{\sum_i m_i x_i}{\sum_i m_i} \quad y_{CM} = \frac{\sum_i m_i y_i}{\sum_i m_i}$$

One obtains  $\vec{r}_{CM} = x_{CM} \vec{i} + y_{CM} \vec{j} = \frac{(m_2 + 2m_3)\vec{i} + 2m_1\vec{j}}{m_1 + m_2 + m_3}$

$$x_{CM} = \frac{\sum_i m_i x_i}{\sum_i m_i} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3} = \frac{m_2 + 2m_3}{m_1 + m_2 + m_3}$$

If  $m_1 = 2\text{kg}; m_2 = m_3 = 1\text{kg}$

$$y_{CM} = \frac{\sum_i m_i y_i}{\sum_i m_i} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{m_1 + m_2 + m_3} = \frac{2m_1}{m_1 + m_2 + m_3}$$

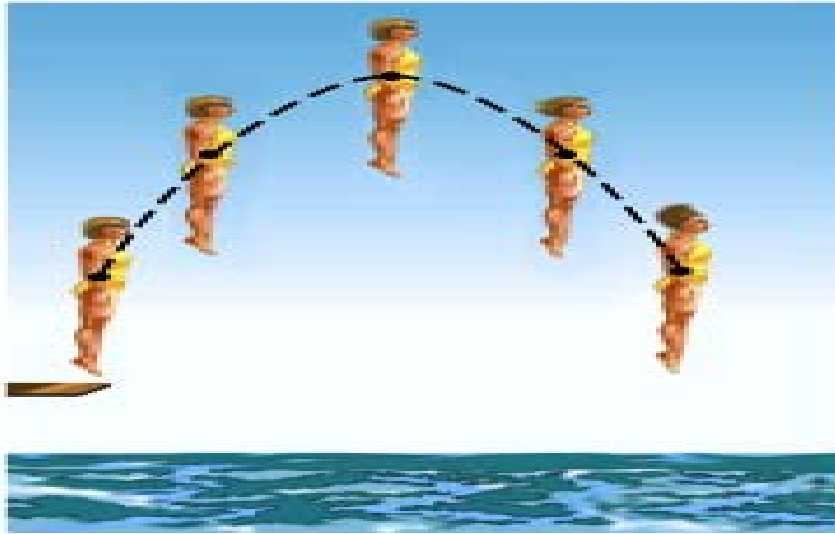
$$\vec{r}_{CM} = \frac{3\vec{i} + 4\vec{j}}{4} = 0.75\vec{i} + \vec{j}$$

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# Motion of a Diver and the Center of Mass



(a)

Diver performs a simple dive.  
The motion of the center of mass follows a parabola since it is a projectile motion.



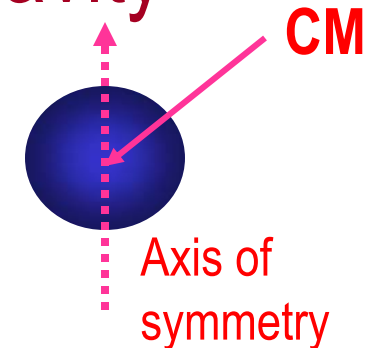
(b)

Diver performs a complicated dive.  
The motion of the center of mass still follows the same parabola since it still is a projectile motion.

**The motion of the center of mass of the diver is always the same.**

# Center of Mass and Center of Gravity

*The center of mass of any symmetric object lies on an axis of symmetry and on any plane of symmetry, if object's mass is evenly distributed throughout the body.*

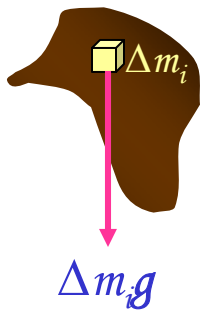


How do you think you can determine the CM of objects that are not symmetric?

One can use gravity to locate CM.

1. Hang the object by one point and draw a vertical line following a plum-bob.
2. Hang the object by another point and do the same.
3. The point where the two lines meet is the CM.

## Center of Gravity



What does this equation tell you?

Since a rigid object can be considered as **collection of small masses**, one can see the total gravitational force exerted on the object as

$$\vec{F}_g = \sum_i \vec{F}_i = \sum_i \Delta m_i \vec{g} = M \vec{g}$$

The net effect of these small gravitational forces is equivalent to a single force acting on a point (**Center of Gravity**) with mass M.

**M The CoG is the point in an object as if all the gravitational force is acting on!**

# Motion of a Group of Particles

*We've learned that the CM of a system can represent the motion of a system. Therefore, for an isolated system of many particles in which the total mass  $M$  is preserved, the velocity, total momentum, acceleration of the system are*

Velocity of the system

$$\vec{v}_{CM} = \frac{\Delta \vec{r}_{CM}}{\Delta t} = \frac{\Delta}{\Delta t} \left( \frac{1}{M} \sum m_i \vec{r}_i \right) = \frac{1}{M} \sum m_i \frac{\Delta \vec{r}_i}{\Delta t} = \frac{\sum m_i \vec{v}_i}{M}$$

Total Momentum of the system

$$\vec{p}_{CM} = M \vec{v}_{CM} = M \frac{\sum m_i \vec{v}_i}{M} = \sum m_i \vec{v}_i = \sum \vec{p}_i = \vec{p}_{tot}$$

Acceleration of the system

$$\vec{a}_{CM} = \frac{\Delta \vec{v}_{CM}}{\Delta t} = \frac{\Delta}{\Delta t} \left( \frac{1}{M} \sum m_i \vec{v}_i \right) = \frac{1}{M} \sum m_i \frac{\Delta \vec{v}_i}{\Delta t} = \frac{\sum m_i \vec{a}_i}{M}$$

External force exerting on the system

$$\sum \vec{F}_{ext} = M \vec{a}_{CM} = \sum m_i \vec{a}_i = \frac{\Delta \vec{p}_{tot}}{\Delta t}$$

*What about the internal forces?*

If net external force is 0

$$\sum \vec{F}_{ext} = 0 = \frac{\Delta \vec{p}_{tot}}{\Delta t}$$

$$\vec{p}_{tot} = \text{const}$$

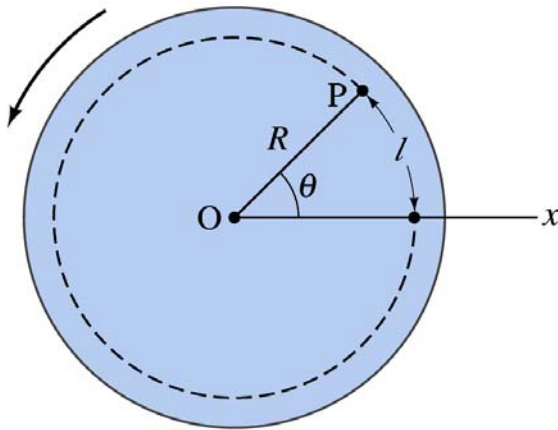
System's momentum is conserved.

# Fundamentals on Rotation

*Linear motions can be described as the motion of the center of mass with all the mass of the object concentrated on it.*

**Is this still true for rotational motions?**

*No, because different parts of the object have different linear velocities and accelerations.*



*Consider a motion of a rigid body – an object that does not change its shape – rotating about the axis protruding out of the slide.*

*The arc length, or sergita, is  $l = R\theta$*

*Therefore the angle,  $\theta$ , is  $\theta = \frac{l}{R}$ . And the unit of the angle is in radian.*

**One radian is the angle swept by an arc length equal to the radius of the arc.**

*Since the circumference of a circle is  $2\pi r$ ,  $360^\circ = 2\pi r / r = 2\pi$*

*The relationship between radian and degrees is  $1 \text{ rad} = 360^\circ / 2\pi = 180^\circ / \pi$*

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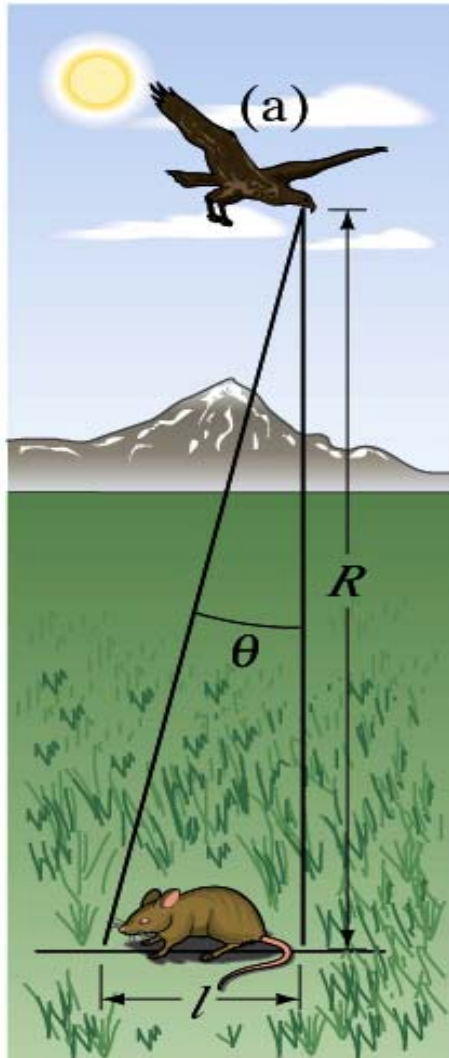


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$\cong 180^\circ / 3.14 \cong 57.3^\circ$  **8**

## Example 8-1

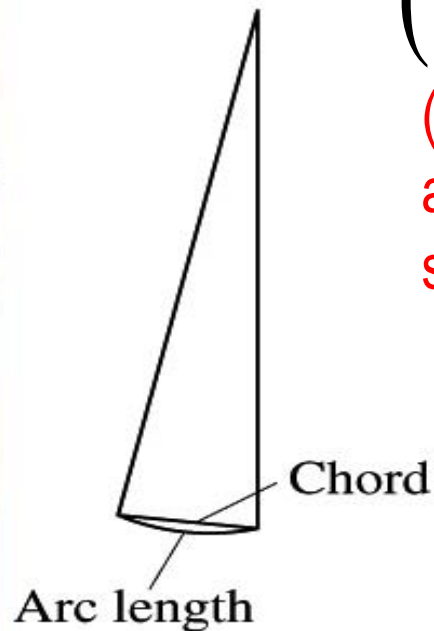
A particular bird's eyes can just distinguish objects that subtend an angle no smaller than about  $3 \times 10^{-4}$  rad. (a) How many degrees is this? (b) How small an object can the bird just distinguish when flying at a height of 100m?



(b) (a) One radian is  $360^\circ/2\pi$ . Thus

$$3 \times 10^{-4} \text{ rad} = \left( 3 \times 10^{-4} \text{ rad} \right) \times \left( 360^\circ / 2\pi \text{ rad} \right) = 0.017^\circ$$

(b) Since  $l = r\theta$  and for small angle arc length is approximately the same as the chord length.



$$l = r\theta = 100\text{m} \times 3 \times 10^{-4} \text{ rad} = 3 \times 10^{-2} \text{ m} = 3\text{cm}$$

# Angular Displacement, Velocity, and Acceleration

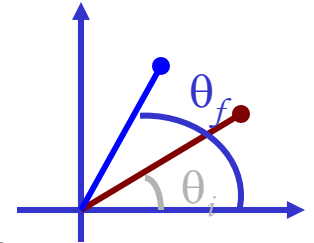
Using what we have learned in the previous slide, how would you define the angular displacement?

$$\Delta\theta = \theta_f - \theta_i$$

How about the average angular speed?

Unit? rad/s

$$\bar{\omega} \equiv \frac{\theta_f - \theta_i}{t_f - t_i} = \frac{\Delta\theta}{\Delta t}$$



And the instantaneous angular speed?

Unit? rad/s

$$\omega \equiv \lim_{\Delta t \rightarrow 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

By the same token, the average angular acceleration

Unit? rad/s<sup>2</sup>

$$\bar{\alpha} \equiv \frac{\omega_f - \omega_i}{t_f - t_i} = \frac{\Delta\omega}{\Delta t}$$

And the instantaneous angular acceleration? Unit? rad/s<sup>2</sup>

$$\alpha \equiv \lim_{\Delta t \rightarrow 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt}$$

When rotating about a fixed axis, every particle on a rigid object rotates through the same angle and has the same angular speed and angular acceleration.

# Rotational Kinematics

The first type of motion we have learned in linear kinematics was under a constant acceleration. We will learn about the rotational motion under constant angular acceleration about a fixed rotational axis, because these are the simplest motions in both cases.

Just like the case in linear motion, one can obtain

Angular Speed under constant angular acceleration:

$$\omega_f = \omega_i + \alpha t$$

Angular displacement under constant angular acceleration:

$$\theta_f = \theta_i + \omega_i t + \frac{1}{2} \alpha t^2$$

One can also obtain

$$\omega_f^2 = \omega_i^2 + 2\alpha(\theta_f - \theta_i)$$



# Example for Rotational Kinematics

A wheel rotates with a constant angular acceleration of  $3.50 \text{ rad/s}^2$ . If the angular speed of the wheel is  $2.00 \text{ rad/s}$  at  $t_i=0$ , a) through what angle does the wheel rotate in  $2.00\text{s}$ ?

Using the angular displacement formula in the previous slide, one gets

$$\begin{aligned}\theta_f - \theta_i &= \omega t + \frac{1}{2} \alpha t^2 \\ &= 2.00 \times 2.00 + \frac{1}{2} 3.50 \times (2.00)^2 = 11.0 \text{ rad} \\ &= \frac{11.0}{2\pi} \text{ rev.} = 1.75 \text{ rev.}\end{aligned}$$



# Example for Rotational Kinematics cnt'd

What is the angular speed at  $t=2.00\text{s}$ ?

Using the angular speed and acceleration relationship

$$\omega_f = \omega_i + \alpha t = 2.00 + 3.50 \times 2.00 = 9.00 \text{ rad/s}$$

Find the angle through which the wheel rotates between  $t=2.00\text{s}$  and  $t=3.00\text{s}$ .

Using the angular kinematic formula

$$\theta_f - \theta_i = \omega t + \frac{1}{2} \alpha t^2$$

At  $t=2.00\text{s}$

$$\theta_{t=2} = 2.00 \times 2.00 + \frac{1}{2} 3.50 \times 2.00^2 = 11.0 \text{ rad}$$

At  $t=3.00\text{s}$

$$\theta_{t=3} = 2.00 \times 3.00 + \frac{1}{2} 3.50 \times (3.00)^2 = 21.8 \text{ rad}$$

Angular displacement

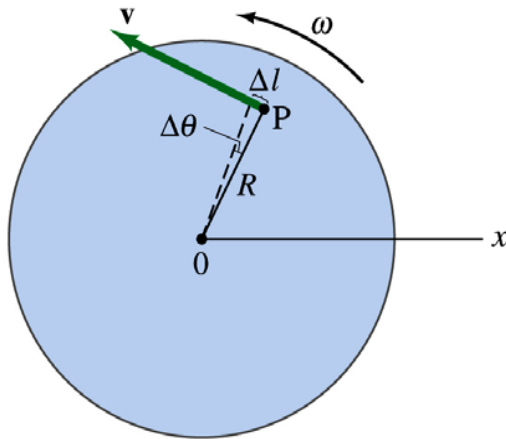
$$\Delta \theta = \theta_3 - \theta_2 = 10.8 \text{ rad} = \frac{10.8}{2\pi} \text{ rev.} = 1.72 \text{ rev.}$$



# Relationship Between Angular and Linear Quantities

What do we know about a rigid object that rotates about a fixed axis of rotation?

Every particle (or masslet) in the object moves in a circle centered at the axis of rotation.



When a point rotates, it has both the linear and angular motion components in its motion.

What is the linear component of the motion you see?

**Linear velocity along the tangential direction.**

How do we relate this linear component of the motion with angular component?

The direction of  $\omega$  follows a right-hand rule.

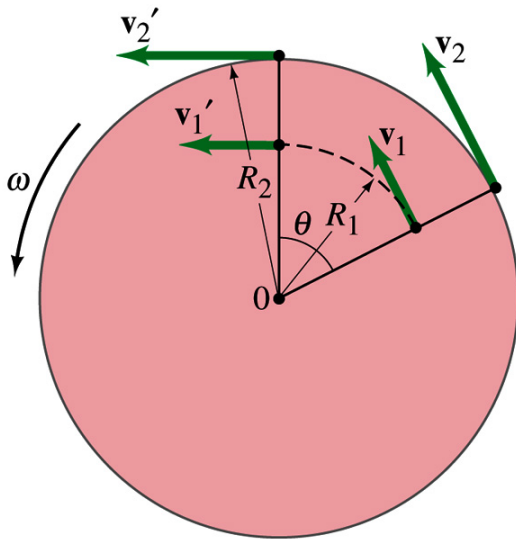
The arc-length is  $l = R\theta$  So the tangential speed  $v$  is 
$$v = \frac{\Delta l}{\Delta t} = \frac{\Delta}{\Delta t}(r\theta) = r \frac{\Delta \theta}{\Delta t} = r\omega$$

What does this relationship tell you about the tangential speed of the points in the object and their angular speed?:

Although every particle in the object has the same angular speed, its tangential speed differs proportional to its distance from the axis of rotation.

# Is the lion faster than the horse?

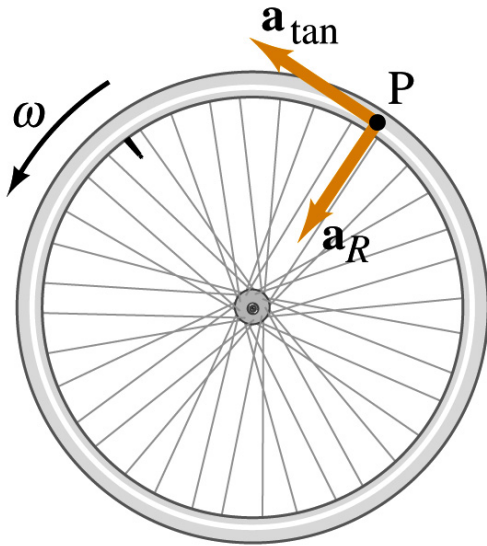
A rotating carousel has one child sitting on a horse near the outer edge and another child on a lion halfway out from the center. (a) Which child has the greater linear speed? (b) Which child has the greater angular speed?



(a) Linear speed is the distance traveled divided by the time interval. So the child sitting at the outer edge travels more distance within the given time than the child sitting closer to the center. Thus, the horse is faster than the lion.

(b) Angular speed is the angle traveled divided by the time interval. The angle both the child travel in the given time interval is the same. Thus, both the horse and the lion has the same angular speed.

# How about the acceleration?



How many different linear accelerations do you see in a circular motion and what are they? **Two**

**Tangential,  $a_t$ , and the radial acceleration,  $a_r$**

Since the tangential speed  $v$  is  $v = r\omega$

The magnitude of tangential acceleration  $a_t$  is  $a_t = \frac{\Delta v}{\Delta t} = \frac{\Delta}{\Delta t}(r\omega) = r \frac{\Delta \omega}{\Delta t} = r\alpha$

What does this relationship tell you?

Although every particle in the object has the same angular acceleration, its tangential acceleration differs proportional to its distance from the axis of rotation.

The radial or centripetal acceleration  $a_r$  is  $a_r = \frac{v^2}{r} = \frac{(r\omega)^2}{r} = r\omega^2$

What does this tell you?

The farther away the particle is from the rotation axis, the more radial acceleration it receives. In other words, it receives more centripetal force.

Total linear acceleration is  $a = \sqrt{a_t^2 + a_r^2} = \sqrt{(r\alpha)^2 + (r\omega^2)^2} = r\sqrt{\alpha^2 + \omega^4}$

## Example 8-3

(a) What is the linear speed of a child seated 1.2m from the center of a steadily rotating merry-go-around that makes one complete revolution in 4.0s? (b) What is her total linear acceleration?

First, figure out what the angular speed of the merry-go-around is.

$$\omega = \frac{1 \text{ rev}}{4.0 \text{ s}} = \frac{2\pi \text{ rad}}{4.0 \text{ s}} = 1.6 \text{ rad / s}$$

Using the formula for linear speed

$$v = r\omega = 1.2 \text{ m} \times 1.6 \text{ rad / s} = 1.9 \text{ m / s}$$

Since the angular speed is constant, there is no angular acceleration.

Tangential acceleration is

$$a_t = r\alpha = 1.2 \text{ m} \times 0 \text{ rad / s}^2 = 0 \text{ m / s}^2$$

Radial acceleration is

$$a_r = r\omega^2 = 1.2 \text{ m} \times (1.6 \text{ rad / s})^2 = 3.1 \text{ m / s}^2$$

Thus the total acceleration is

$$a = \sqrt{a_t^2 + a_r^2} = \sqrt{0 + (3.1)^2} = 3.1 \text{ m / s}^2$$

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