PHYS 1444 – Section 501
Lecture #22

Monday, Apr. 24, 2006
Dr. Jaehoon Yu

- AC Circuit w/ Resistance only
- AC Circuit w/ Inductance only
- AC Circuit w/ Capacitance only
- AC Circuit w/ LRC
Announcements

• Colloquium at 4pm this Wednesday, Apr. 26
  – Dr. Ian Hinchliffe from LBL
  – Title: “Early Physics with ATLAS at LHC”
  – Location: Planetarium

• Reading assignments
  – CH. 31 – 6, 31 – 7 and 31 – 8

• Final term exam
  – Time: 5:30pm – 7:00pm, Monday May. 8
  – Location: SH103
  – Covers: CH 29 – whichever chapter we finish Monday, May 1
  – Please do not miss the exam
  – Two best of the three exams will be used for your grades
Why do we care about circuits on AC?

The circuits we’ve learned so far contain resistors, capacitors and inductors and have been connected to a DC source or a fully charged capacitor

- What? This does not make sense.
- The inductor does not work as an impedance unless the current is changing. So an inductor in a circuit with DC source does not make sense.
- Well, actually it does. When does it impede the current flow?
  - Immediately after the circuit is connected to the source so the current is still changing.
  - So what?
    - It causes the change of magnetic flux.
- Now does it make sense?

Anyhow, learning the responses of resistors, capacitors and inductors in a circuit connected to an AC emf source is important. Why is this?

- Since most the generators produce sinusoidal current
- Any voltage that varies over time can be expressed in the superposition of sine and cosine functions
AC Circuits – the preamble

• Do you remember how the rms and peak values for current and voltage are related?

\[ V_{rms} = \frac{V_0}{\sqrt{2}} \quad I_{rms} = \frac{I_0}{\sqrt{2}} \]

• The symbol for an AC power source is

• We assume that the voltage gives rise to current

\[ I = I_0 \sin 2\pi ft = I_0 \sin \omega t \]

– where \( \omega = 2\pi f \)
AC Circuit w/ Resistance only

- What do you think will happen when an ac source is connected to a resistor?
- From Kirchhoff's loop rule, we obtain
  \[ V - IR = 0 \]
- Thus
  \[ V = I_0 R \sin \omega t = V_0 \sin \omega t \]
  - where \( V_0 = I_0 R \)
- What does this mean?
  - Current is 0 when voltage is 0 and current is in its peak when voltage is in its peak.
  - Current and voltage are “in phase”
- Energy is lost via the transformation into heat at an average rate
  \[ \overline{P} = \overline{I} \overline{V} = I_{rms}^2 R = V_{rms}^2 / R \]
AC Circuit w/ Inductance only

- From Kirchhoff's loop rule, we obtain
  \[ V - L \frac{dI}{dt} = 0 \]

- Thus
  \[ V = L \frac{dI}{dt} = L \frac{d(I_0 \sin \omega t)}{dt} = \omega LI_0 \cos \omega t \]
  - Using the identity \( \cos \theta = \sin(\theta + 90^\circ) \)

- \[ V = \omega LI_0 \sin(\omega t + 90^\circ) = V_0 \sin(\omega t + 90^\circ) \]
  - where \( V_0 = \omega LI_0 \)

- What does this mean?
  - Current and voltage are “out of phase by \( \pi/2 \) or 90°”.
  - In other words the current reaches its peak \( 1/4 \) cycle after the voltage

- What happens to the energy?
  - No energy is dissipated
  - The average power is 0 at all times
  - The energy is stored temporarily in the magnetic field
  - Then released back to the source

\[ V_0 = V_0 \cos \omega t = V_0 \sin(\omega t + 90^\circ) \]
AC Circuit w/ Inductance only

• How are the resistor and inductor different in terms of energy?
  – Inductor: Stores the energy temporarily in the magnetic field and then releases it back to the emf source
  – Resistor: Does not store energy but transforms it to thermal energy, getting it lost to the environment

• How are they the same?
  – They both impede the flow of charge
  – For a resistance $R$, the peak voltage and current are related to $V_0 = I_0 R$
  – Similarly, for an inductor we can write $V_0 = I_0 X_L$
    – Where $X_L$ is the inductive reactance of the inductor
    – What do you think is the unit of the reactance? $\Omega$
    – The relationship $V_0 = I_0 X_L$ is not valid at a particular instance. Why not?
      – Since $V_0$ and $I_0$ do not occur at the same time

\[
\text{X_L} = \omega L \quad \text{0 when } \omega = 0.
\]

\[
V_{\text{rms}} = I_{\text{rms}} X_L \quad \text{is valid!}
\]
Example 31 – 1

Reactance of a coil. A coil has a resistance $R = 1.00 \Omega$ and an inductance of $0.300 \text{H}$. Determine the current in the coil if (a) 120 V dc is applied to it; (b) 120 V ac (rms) at 60.0 Hz is applied.

Is there a reactance for dc? 

Nope. Why not? Since $\omega = 0$, $X_L = \omega L = 0$

So for dc power, the current is from Kirchhoff's rule $V - IR = 0$

\[
I_0 = \frac{V_0}{R} = \frac{120V}{1.00\Omega} = 120 \text{A}
\]

For an ac power with $f = 60 \text{Hz}$, the reactance is

\[
X_L = \omega L = 2\pi fL = 2\pi \cdot \left(60.0 \text{s}^{-1}\right) \cdot 0.300 \text{H} = 113 \Omega
\]

Since the resistance can be ignored compared to the reactance, the rms current is

\[
I_{rms} \approx \frac{V_{rms}}{X_L} = \frac{120V}{113\Omega} = 1.06 \text{A}
\]
AC Circuit w/ Capacitance only

• What happens when a capacitor is connected to a dc power source?
  – The capacitor quickly charges up.
  – There is no steady current flow in the circuit
    • Since a capacitor prevents the flow of a dc current

• What do you think will happen if it is connected to an ac power source?
  – The current flows continuously. Why?
  – When the ac power turns on, charge begins to flow one direction, charging up the plates
  – When the direction of the power reverses, the charge flows in the opposite direction
AC Circuit w/ Capacitance only

• From Kirchhoff's loop rule, we obtain

\[ V = \frac{Q}{C} \]

• Current at any instance is

\[ I = \frac{dQ}{dt} = I_0 \sin \omega t \]

• Thus, the charge \( Q \) on the plate at any instance is

\[ Q = \int_{Q=0}^{Q} dQ = \int_{t=0}^{t} I_0 \sin \omega t dt = -\frac{I_0}{\omega} \cos \omega t \]

• The voltage across the capacitor is

\[ V = \frac{Q}{C} = -I_0 \frac{1}{\omega C} \cos \omega t \]

– Using the identity \( \cos \theta = -\sin(\theta - 90^\circ) \)

\[ V = I_0 \frac{1}{\omega C} \sin(\omega t - 90^\circ) = V_0 \sin(\omega t - 90^\circ) \]

– Where

\[ V_0 = \frac{I_0}{\omega C} \]
AC Circuit w/ Capacitance only

• So the voltage is $V = V_0 \sin(\omega t - 90^\circ)$

• What does this mean?
  – Current and voltage are “out of phase by $\pi/2$ or $90^\circ$” but in this case, the voltage reaches its peak $1/4$ cycle after the current

• What happens to the energy?
  – No energy is dissipated
  – The average power is 0 at all times
  – The energy is stored temporarily in the electric field
  – Then released back to the source

• Relationship between the peak voltage and the peak current in the capacitor can be written as

$$V_0 = I_0 X_C$$

– Where the capacitance reactance $X_C$ is defined as
– Again, this relationship is only valid for rms quantities

$$X_C = \frac{1}{\omega C}$$

$V_{rms} = I_{rms} X_C$
Example 31 – 2

**Capacitor reactance.** What are the peak and rms current in the circuit in the figure if \( C = 1.0 \mu F \) and \( V_{rms} = 120V \)? Calculate for (a) \( f = 60Hz \), and then for (b) \( f = 6.0 \times 10^5Hz \).

The peak voltage is \( V_0 = \sqrt{2} V_{rms} = 120V \cdot \sqrt{2} = 170V \)

The capacitance reactance is

\[
X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} = \frac{1}{2\pi \cdot (60s^{-1}) \cdot 1.0 \times 10^{-6} F} = 2.7k\Omega
\]

Thus the peak current is

\[
I_0 = \frac{V_0}{X_C} = \frac{170V}{2.7k\Omega} = 63mA
\]

The rms current is

\[
I_{rms} = \frac{V_{rms}}{X_C} = \frac{120V}{2.7k\Omega} = 44mA
\]
AC Circuit w/ LRC

- The voltage across each element is
  - $V_R$ is in phase with the current
  - $V_L$ leads the current by $90^\circ$
  - $V_C$ lags the current by $90^\circ$
- From Kirchhoff’s loop rule
- $V = V_R + V_L + V_C$
  - However since they do not reach the peak voltage at the same time, the peak voltage of the source $V_0$ will not equal $V_{R0} + V_{L0} + V_{C0}$
  - The rms voltage also will not be the simple sum of the three
- Let’s try to find the total impedance, peak current $I_0$ and the phase difference between $I_0$ and $V_0$. 
AC Circuit w/ LRC

• The current at any instance is the same at all point in the circuit
  – The currents in each elements are in phase
  – Why?
    • Since the elements are in series
  – How about the voltage?
    • They are not in phase.

• The current at any given time is
  \[ I = I_0 \sin \omega t \]

• The analysis of LRC circuit is done using the “phasor” diagram in which arrows are drawn in an xy plane to represent the amplitude of each voltage, just like vectors
  – The lengths of the arrows represent the magnitudes of the peak voltages across each element; \( V_{R0} = I_0R \), \( V_{L0} = I_0X_L \) and \( V_{C0} = I_0X_C \)
  – The angle of each arrow represents the phase of each voltage relative to the current, and the arrows rotate at the angular frequency \( \omega \) to take into account the time dependence.
    • The projection of each arrow on y axis represents voltage across each element at any given time
Phasor Diagrams

• At t=0, I=0.
  – Thus $V_{R0}=0$, $V_{L0}=I_0X_L$, $V_{C0}=I_0X_C$

• At t=t, $I = I_0 \sin \omega t$

• Thus, the voltages (y-projections) are

\[
V_R = V_{R0} \sin \omega t \\
V_L = V_{L0} \sin (\omega t + 90^\circ) \\
V_C = V_{C0} \sin (\omega t - 90^\circ)
\]
AC Circuit w/ LRC

- Since the sum of the projections of the three vectors on the y axis is equal to the projection of their sum,
  - The sum of the projections represents the instantaneous voltage across the whole circuit which is the source voltage.
  - So we can use the sum of all vectors as the representation of the peak source voltage \( V_0 \).

- \( V_0 \) forms an angle \( \phi \) to \( V_{R0} \) and rotates together with the other vectors as a function of time, \( V = V_0 \sin(\omega t + \phi) \).

- We determine the total impedance \( Z \) of the circuit defined by the relationship \( V_{rms} = I_{rms} Z \) or \( V_0 = I_0 Z \).

- From Pythagorean theorem, we obtain
  \[
  V_0 = \sqrt{V_{R0}^2 + (V_{L0} - V_{C0})^2} = I_0 \sqrt{R^2 + (X_L - X_C)^2} = I_0 Z
  \]

- Thus the total impedance is
  \[
  Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + \left(\frac{\omega L - \frac{1}{\omega C}}{2}ight)^2}
  \]
AC Circuit w/ LRC

• The phase angle is
  \[ \tan \phi = \frac{V_{L0} - V_{C0}}{V_{R0}} = \frac{I_0 (X_L - X_C)}{I_0 R} = \frac{(X_L - X_C)}{R} \]

• Or
  \[ \cos \phi = \frac{V_{R0}}{V_0} = \frac{I_0 R}{I_0 Z} = \frac{R}{Z} \]

• What is the power dissipated in the circuit?
  – Which element dissipates the power?
  – Only the resistor

• The average power is
  \[ \bar{P} = I_{rms}^2 R \]
  – Since \( R = Z \cos \phi \)
  – We obtain
    \[ \bar{P} = I_{rms}^2 Z \cos \phi = I_{rms} V_{rms} \cos \phi \]
  – The factor \( \cos \phi \) is referred as the power factor of the circuit
  – For a pure resistor, \( \cos \phi = 1 \) and \( \bar{P} = I_{rms} V_{rms} \)
  – For a capacitor or inductor alone \( \phi = -90^\circ \) or \( +90^\circ \), so \( \cos \phi = 0 \) and \( \bar{P} = 0 \).