PHYS 1444 – Section 003
Lecture #24

Monday, May 1, 2006
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• Gauss’ Law of Magnetism
• Maxwell’s Equations
• Production of Electromagnetic Waves
• EM Waves from Maxwell’s Equations
• Speed of EM Waves
• Energy in EM Waves
• Energy Transport
Announcements

• No class this Wednesday, May 3

• Reading assignments
  – Ch 32 – 6, 32 – 7, 32 – 8 and 32 – 9

• Final term exam
  – Time: 5:30pm – 7:00pm, Monday May. 8
  – Location: SH103
  – Covers: CH 29 – CH32
  – Please do not miss the exam
  – Two best of the three exams will be used for your grades
Displacement Current

- Maxwell interpreted the second term in the generalized Ampere’s law equivalent to an electric current
  - He called this term as the displacement current, $I_D$
  - While the other term is called as the conduction current, $I$

- Ampere’s law then can be written as

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \left( I + I_D \right)$$

- Where

$$I_D = \varepsilon_0 \frac{d\Phi_E}{dt}$$

- While it is in effect equivalent to an electric current, a flow of electric charge, this actually does not have anything to do with the flow itself
Gauss’ Law for Magnetism

- If there is symmetry between electricity and magnetism, there must be an equivalent law in magnetism as the Gauss’ Law in electricity.
- For a magnetic field \( \mathbf{B} \), the magnetic flux \( \Phi_B \) through the surface is defined as
  \[
  \Phi_B = \int \mathbf{B} \cdot d\mathbf{A}
  \]
  - Where the integration is over the area of either an open or a closed surface.
- The magnetic flux through a closed surface which completely encloses a volume is
  \[
  \Phi_B = \oint \mathbf{B} \cdot d\mathbf{A}
  \]
- What was the Gauss’ law in the electric case?
  - The electric flux through a closed surface is equal to the total net charge \( Q \) enclosed by the surface divided by \( \varepsilon_0 \).
    \[
    \oint \mathbf{E} \cdot d\mathbf{A} = \frac{Q_{encl}}{\varepsilon_0}
    \]
    Gauss’ Law for electricity
- Similarly, we can write Gauss’ law for magnetism as
  \[
  \oint \mathbf{B} \cdot d\mathbf{A} = 0
  \]
  Gauss’ Law for magnetism
- Why is result of the integral zero?
  - There is no isolated magnetic poles, the magnetic equivalent of single electric charges.
Gauss’ Law for Magnetism

• What does the Gauss’ law in magnetism mean physically?

\[ \oint \mathbf{B} \cdot d\mathbf{A} = 0 \]

– There are as many magnetic flux lines that enter the enclosed volume as leave it
– If magnetic monopole does not exist, there is no starting or stopping point of the flux lines
  • Electricity do have the source and the sink
– Magnetic field lines must be continuous
– Even for bar magnets, the field lines exist both insides and outside of the magnet
Maxwell’s Equations

• In the absence of dielectric or magnetic materials, the four equations developed by Maxwell are:

\[ \int \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\varepsilon_0} \]

**Gauss’ Law for electricity**
A generalized form of Coulomb’s law relating electric field to its sources, the electric charge

\[ \int \vec{B} \cdot d\vec{A} = 0 \]

**Gauss’ Law for magnetism**
A magnetic equivalent of Coulomb's law, relating magnetic field to its sources. This says there are no magnetic monopoles.

\[ \int \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt} \]

**Faraday’s Law**
An electric field is produced by a changing magnetic field

\[ \int \vec{B} \cdot d\vec{l} = \mu_0 I_{encl} + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} \]

**Ampére’s Law**
A magnetic field is produced by an electric current or by a changing electric field
Maxwell’s Amazing Leap of Faith

- According to Maxwell, a magnetic field will be produced even in an empty space if there is a changing electric field
  - He then took this concept one step further and concluded that
    - If a changing magnetic field produces an electric field, the electric field is also changing in time.
    - This changing electric field in turn produces the magnetic field that also changes
    - This changing magnetic field then in turn produces the electric field that changes
    - This process continues
  - With the manipulation of the equations, Maxwell found that the net result of this interacting changing fields is a wave of electric and magnetic fields that can actually propagate (travel) through the space
Production of EM Waves

- Consider two conducting rods that will serve as an antenna are connected to a DC power source
  - What do you think will happen when the switch is closed?
    - The rod connected to the positive terminal is charged positive and the other negative
    - Then the electric field will be generated between the two rods
    - Since there is current that flows through the rods, a magnetic field around them will be generated

- How far would the electric and magnetic fields extend?
  - In static case, the field extends indefinitely
  - When the switch is closed, the fields are formed near the rods quickly but
  - The stored energy in the fields won’t propagate w/ infinite speed
Production of EM Waves

- What happens if the antenna is connected to an ac power source?
  - When the connection was initially made, the rods are charging up quickly with the current flowing in one direction as shown in the figure:
    - The field lines form as in the dc case
    - The field lines propagate away from the antenna
  - Then the direction of the voltage reverses
    - New field lines in the opposite direction forms
    - While the original field lines still propagates away from the rod reaching out far
      - Since the original field propagates through an empty space, the field lines must form a closed loop (no charge exist)
    - Since changing electric and magnetic fields produce changing magnetic and electric fields, the fields moving outward is self supporting and do not need antenna with flowing charge
  - The fields far from the antenna is called the **radiation field**
  - Both electric and magnetic fields form closed loops perpendicular to each other
Properties of Radiation Fields

- The fields travel on the other side of the antenna as well.
- The field strength are the greatest in the direction perpendicular to the oscillating charge while along the parallel direction is 0.
- The magnitude of \( E \) and \( B \) in the radiation field decrease with distance as \( 1/r \).
- The energy carried by the EM wave is proportional to the square of the amplitude, \( E^2 \) or \( B^2 \).
  - So the intensity of wave decreases as \( 1/r^2 \).
Properties of Radiation Fields

• The electric and magnetic fields at any point are perpendicular to each other and to the direction of motion

• The fields alternate in direction
  – The field strengths vary from maximum in one direction, to 0 and to maximum in the opposite direction

• The electric and magnetic fields are in phase

• Very far from the antenna, the field lines are pretty flat over a reasonably large area
  – Called plane waves
EM Waves

• If the voltage of the source varies sinusoidally, the field strengths of the radiation field vary sinusoidally

• We call these waves EM waves
• They are transverse waves
• EM waves are always waves of fields
  – Since these are fields, they can propagate through an empty space
• In general accelerating electric charges give rise to electromagnetic waves
• This prediction from Maxwell’s equations was experimentally proven by Heinrich Hertz through the discovery of radio waves
EM Waves and Their Speeds

• Let’s consider a region of free space. What’s a free space?
  – An area of space where there is no charges or conduction currents
  – In other words, far from emf sources so that the wave fronts are essentially flat or not distorted over a reasonable area
  – What are these flat waves called?
    • Plane waves
    • At any instance $\mathbf{E}$ and $\mathbf{B}$ are uniform over a large plane perpendicular to the direction of propagation
  – So we can also assume that the wave is traveling in the x-direction w/ velocity, $\mathbf{v}=\mathbf{v}_i$, and that $\mathbf{E}$ is parallel to y axis and $\mathbf{B}$ is parallel to z axis
Maxwell’s Equations w/ Q=I=0

- In this region of free space, Q=0 and I=0, thus the four Maxwell’s equations become

\[ \oint \mathbf{E} \cdot d\mathbf{A} = \frac{Q_{\text{encl}}}{\varepsilon_0} \]

\[ \oint \mathbf{B} \cdot d\mathbf{A} = 0 \]

\[ \oint \mathbf{E} \cdot d\mathbf{l} = -\frac{d\Phi_B}{dt} \]

\[ \oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_{\text{encl}} + \mu_0\varepsilon_0 \frac{d\Phi_E}{dt} \]

One can observe the symmetry between electricity and magnetism.

The last equation is the most important one for EM waves and their propagation!!
EM Waves from Maxwell’s Equations

• If the wave is sinusoidal with wavelength $\lambda$ and frequency $f$, such traveling wave can be written as

\[
E = E_y = E_0 \sin \left( kx - \omega t \right) \\
B = B_z = B_0 \sin \left( kx - \omega t \right)
\]

- Where

\[
k = \frac{2\pi}{\lambda} \quad \omega = 2\pi f \quad \therefore f \lambda = \frac{\omega}{k} = v
\]

- What is $v$?

  • It is the speed of the traveling wave

- What are $E_0$ and $B_0$?

  • The amplitudes of the EM wave. Maximum values of $E$ and $B$ field strengths.
From Faraday’s Law

• Let’s apply Faraday’s law

\[ \oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt} \]

– to the rectangular loop of height \( \Delta y \) and width \( dx \)

• \( \vec{E} \cdot d\vec{l} \) along the top and bottom of the loop is 0. Why?
  – Since \( \vec{E} \) is perpendicular to \( dl \).
  – So the result of the integral through the loop counterclockwise becomes

\[ \oint \vec{E} \cdot d\vec{l} = \vec{E} \cdot d\vec{x} + (\vec{E} + d\vec{E}) \cdot \Delta \vec{y} + \vec{E} \cdot d\vec{x}' + \vec{E} \cdot \Delta \vec{y}' = 0 + (E + dE) \Delta y - 0 - E \Delta y = dE \Delta y \]

– For the right-hand side of Faraday’s law, the magnetic flux through the loop changes as

\[ \frac{d\Phi_B}{dt} = \frac{dB}{dt} dx \Delta y \]

Thus

\[ dE \Delta y = -\frac{dB}{dt} dx \Delta y \]

\[ \frac{dE}{dx} = -\frac{dB}{dt} \]

Since \( E \) and \( B \) depend on \( x \) and \( t \)
From Modified Ampère’s Law

- Let’s apply Maxwell’s 4th equation
  \[ \oint \vec{B} \cdot d\vec{l} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} \]
  - to the rectangular loop of length \(\Delta z\) and width \(dx\)
- \(\vec{B} \cdot d\vec{l}\) along the x-axis of the loop is 0
  - Since \(\vec{B}\) is perpendicular to \(d\vec{l}\)
  - So the result of the integral through the loop counterclockwise becomes
  \[ \oint \vec{B} \cdot d\vec{l} = B\Delta Z - (B + dB)\Delta Z = -dB\Delta Z \]
  - For the right-hand side of the equation is
    \[ \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} = \mu_0 \varepsilon_0 \frac{dE}{dt} dx \Delta z \]
    \[ -dB \Delta z = \mu_0 \varepsilon_0 \frac{dE}{dt} dx \Delta z \]
    \[ \frac{dB}{dx} = -\mu_0 \varepsilon_0 \frac{dE}{dt} \]

Since \(E\) and \(B\) depend on \(x\) and \(t\)
Relationship between $E$, $B$ and $v$

- Let’s now use the relationship from Faraday’s law $\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t}$.

- Taking the derivatives of $E$ and $B$ as given their traveling wave form, we obtain

  \[ \frac{\partial E}{\partial x} = \frac{\partial}{\partial x} \left( E_0 \sin (kx - \omega t) \right) = kE_0 \cos (kx - \omega t) \]

  \[ \frac{\partial B}{\partial t} = \frac{\partial}{\partial t} \left( B_0 \sin (kx - \omega t) \right) = -\omega B_0 \cos (kx - \omega t) \]

  Since $\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t}$

  We obtain $kE_0 \cos (kx - \omega t) = \omega B_0 \cos (kx - \omega t)$

  Thus $\frac{E_0}{B_0} = \frac{\omega}{k} = v$

- Since $E$ and $B$ are in phase, we can write $E/B = v$

- This is valid at any point and time in space. What is $v$?

  - The velocity of the wave
Speed of EM Waves

• Let’s now use the relationship from Apmere’s law $\frac{\partial B}{\partial x} = -\varepsilon_0 \mu_0 \frac{\partial E}{\partial t}$

• Taking the derivatives of $E$ and $B$ as given their traveling wave form, we obtain

$$\frac{\partial B}{\partial x} = \frac{\partial}{\partial x} \left( B_0 \sin(kx - \omega t) \right) = kB_0 \cos(kx - \omega t)$$

$$\frac{\partial E}{\partial t} = \frac{\partial}{\partial t} \left( E_0 \sin(kx - \omega t) \right) = -\omega E_0 \cos(kx - \omega t)$$

Since $\frac{\partial B}{\partial x} = -\varepsilon_0 \mu_0 \frac{\partial E}{\partial t}$ We obtain $kB_0 \cos(kx - \omega t) = \varepsilon_0 \mu_0 \omega E_0 \cos(kx - \omega t)$

Thus $\frac{B_0}{E_0} = \frac{\varepsilon_0 \mu_0 \omega}{k} = \varepsilon_0 \mu_0 v$

– However, from the previous page we obtain $\frac{E_0}{B_0} = v = \frac{1}{\varepsilon_0 \mu_0 v}$

– Thus $v^2 = \frac{\varepsilon_0 \mu_0}{\varepsilon_0 \mu_0} 

\[ v = \frac{\varepsilon_0 \mu_0}{\sqrt{\varepsilon_0 \mu_0}} = \frac{1}{\sqrt{8.85 \times 10^{-12} \frac{C^2}{N \cdot m^2} \cdot (4\pi \times 10^{-7} \frac{T \cdot m}{A})}} = 3.00 \times 10^8 \text{ m/s} \]

The speed of EM waves is the same as the speed of light. EM waves behaves like the light.
Speed of Light w/o Sinusoidal Wave Forms

• Taking the time derivative on the relationship from Ampere’s laws, we obtain
  \[ \frac{\partial^2 B}{\partial x \partial t} = -\varepsilon_0 \mu_0 \frac{\partial^2 E}{\partial t^2} \]

• By the same token, we take position derivative on the relationship from Faraday’s law
  \[ \frac{\partial^2 E}{\partial x^2} = -\frac{\partial^2 B}{\partial x \partial t} \]

• From these, we obtain
  \[
  \frac{\partial^2 E}{\partial t^2} = \frac{1}{\varepsilon_0 \mu_0} \frac{\partial^2 E}{\partial x^2} \quad \text{and} \quad 
  \frac{\partial^2 B}{\partial t^2} = \frac{1}{\varepsilon_0 \mu_0} \frac{\partial^2 B}{\partial x^2}
  \]

• Since the equation for traveling wave is
  \[ \frac{\partial^2 x}{\partial t^2} = v^2 \frac{\partial^2 x}{\partial x^2} \]

• By correspondence, we obtain
  \[ v^2 = \frac{1}{\varepsilon_0 \mu_0} \]

• A natural outcome of Maxwell’s equations is that \( E \) and \( B \) obey the wave equation for waves traveling w/ speed \( v = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} \)

  – Maxwell predicted the existence of EM waves based on this
Light as EM Wave

• People knew some 60 years before Maxwell that light behaves like a wave, but …
  – They did not know what kind of waves they are.
    • Most importantly what is it that oscillates in light?

• Heinrich Hertz first generated and detected EM waves experimentally in 1887 using a spark gap apparatus
  – Charge was rushed back and forth in a short period of time, generating waves with frequency about $10^9$Hz (these are called radio waves)
  – He detected using a loop of wire in which an emf was produced when a changing magnetic field passed through
  – These waves were later shown to travel at the speed of light
Light as EM Wave

• The wavelengths of visible light were measured in the first decade of the 19th century
  – The visible light wave length were found to be between $4.0 \times 10^{-7}\text{m (400nm)}$ and $7.5 \times 10^{-7}\text{m (750nm)}$
  – The frequency of visible light is $f \lambda = c$
    • Where $f$ and $\lambda$ are the frequency and the wavelength of the wave
      – What is the range of visible light frequency?
        – $4.0 \times 10^{14}\text{Hz to 7.5} \times 10^{14}\text{Hz}$
    • $c$ is $3 \times 10^8\text{m/s}$, the speed of light

• EM Waves, or EM radiation, are categorized using EM spectrum
Electromagnetic Spectrum

- Low frequency waves, such as radio waves or microwaves can be easily produced using electronic devices.
- Higher frequency waves are produced naturally, such as emission from atoms, molecules or nuclei.
- Or they can be produced from the acceleration of charged particles.
- Infrared radiation (IR) is mainly responsible for the heating effect of the Sun.
  - The Sun emits visible lights, IR and UV.
    • The molecules of our skin resonate at infrared frequencies so IR is preferentially absorbed and thus warm up.

Monday, May 1, 2006

PHYS 1444-501, Spring 2006
Dr. Jaehoon Yu
Example 32 – 2

Wavelength of EM waves. Calculate the wavelength (a) of a 60-Hz EM wave, (b) of a 93.3-MHz FM radio wave and (c) of a beam of visible red light from a laser at frequency 4.74x10^{14}Hz.

What is the relationship between speed of light, frequency and the wavelength? \[ c = f \lambda \]

Thus, we obtain \[ \lambda = \frac{c}{f} \]

For \( f = 60 \text{Hz} \)
\[ \lambda = \frac{3 \times 10^8 \text{ m/s}}{60 \text{s}^{-1}} = 5 \times 10^6 \text{ m} \]

For \( f = 93.3 \text{MHz} \)
\[ \lambda = \frac{3 \times 10^8 \text{ m/s}}{93.3 \times 10^6 \text{s}^{-1}} = 3.22 \text{ m} \]

For \( f = 4.74 \times 10^{14} \text{Hz} \)
\[ \lambda = \frac{3 \times 10^8 \text{ m/s}}{4.74 \times 10^{14} \text{s}^{-1}} = 6.33 \times 10^{-7} \text{ m} \]
EM Wave in the Transmission Lines

• Can EM waves travel through a wire?
  – Can it not just travel through the empty space?
  – Nope. It sure can travel through a wire.

• When a source of emf is connected to a transmission line, the electric field within the wire does not set up immediately at all points along the line
  – When two wires are separated via air, the EM wave travel through the air at the speed of light, c.
  – However, through medium w/ permittivity $\varepsilon$ and permeability $\mu$, the speed of the EM wave is given $v = \frac{1}{\sqrt{\varepsilon \mu}} < c$

• Is this faster than c? **Nope! It is slower.**
Energy in EM Waves

• Since $B = \frac{E}{c}$ and $c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$, we can rewrite the energy density

$$u = u_E + u_B = \frac{1}{2} \varepsilon_0 E^2 + \frac{1}{2} \frac{\varepsilon_0 \mu_0 E^2}{\mu_0} = \varepsilon_0 E^2$$

  – Note that the energy density associated with $B$ field is the same as that associated with $E$
  – So each field contribute half to the total energy

• By rewriting in $B$ field only, we obtain

$$u = \frac{1}{2} \varepsilon_0 \frac{B^2}{\varepsilon_0 \mu_0} + \frac{1}{2} \frac{B^2}{\mu_0} = \frac{B^2}{\mu_0}$$

• We can also rewrite to contain both $E$ and $B$

$$u = \varepsilon_0 E^2 = \varepsilon_0 EcB = \frac{\varepsilon_0 EB}{\sqrt{\varepsilon_0 \mu_0}} = \sqrt{\frac{\varepsilon_0}{\mu_0}} EB$$
Energy Transport

What is the energy the wave transport per unit time per unit area?

- This is given by the vector $\mathbf{S}$, the Poynting vector
  - The unit of $\mathbf{S}$ is $\text{W/m}^2$.
  - The direction of $\mathbf{S}$ is the direction in which the energy is transported. Which direction is this?
    - The direction the wave is moving

Let’s consider a wave passing through an area $A$ perpendicular to the $x$-axis, the axis of propagation

- How much does the wave move in time $dt$?
  - $dx=cdt$
  - The energy that passes through $A$ in time $dt$ is the energy that occupies the volume $dV$,
    $$dV = A dx = Acdt$$
  - Since the energy density is $u=\varepsilon_0 E^2$, the total energy, $dU$, contained in the volume $V$ is
    $$dU = u dV = \varepsilon_0 E^2 Acdt$$
Energy Transport

• Thus, the energy crossing the area A per time dt is

\[ S = \frac{1}{A} \frac{dU}{dt} = \varepsilon_0 cE^2 \]

• Since \( E = cB \) and \( c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} \), we can also rewrite

\[ S = \varepsilon_0 cE^2 = \frac{cB^2}{\mu_0} = \frac{EB}{\mu_0} \]

• Since the direction of \( S \) is along \( \mathbf{v} \), perpendicular to \( \mathbf{E} \) and \( \mathbf{B} \), the Poynting vector \( \mathbf{S} \) can be written

\[ \mathbf{S} = \frac{1}{\mu_0} (\mathbf{E} \times \mathbf{B}) \]

– This gives the energy transported per unit area per unit time at any instant
Average Energy Transport

- The average energy transport in an extended period of time since the frequency is so high we do not detect the rapid variation with respect to time.

- If E and B are sinusoidal, \( \overline{E^2} = \frac{E_0^2}{2} \)

- Thus we can write the magnitude of the average Poynting vector as

\[
\overline{S} = \frac{1}{2} \epsilon_0 c E_0^2 = \frac{1}{2} \frac{c}{\mu_0} B_0^2 = \frac{E_0 B_0}{2 \mu_0}
\]

- This time averaged value of \( S \) is the intensity, defined as the average power transferred across unit area. \( E_0 \) and \( B_0 \) are maximum values.

- We can also write

\[
\overline{S} = \frac{E_{\text{rms}} B_{\text{rms}}}{\mu_0}
\]

- Where \( E_{\text{rms}} \) and \( B_{\text{rms}} \) are the rms values ( \( E_{\text{rms}} = \sqrt{\overline{E^2}} \), \( B_{\text{rms}} = \sqrt{\overline{B^2}} \) )
Example 32 – 4

E and B from the Sun. Radiation from the Sun reaches the Earth (above the atmosphere) at a rate of about 1350W/m². Assume that this is a single EM wave and calculate the maximum values of E and B.

What is given in the problem? The average S!!

\[
\bar{S} = \frac{1}{2} \varepsilon_0 c E_0^2 = \frac{1}{2} \frac{c}{\mu_0} B_0^2 = \frac{E_0 B_0}{2\mu_0}
\]

For \( E_0 \),

\[
E_0 = \sqrt{\frac{2\bar{S}}{\varepsilon_0 c}} = \sqrt{\frac{2 \cdot 1350 \text{W/m}^2}{(8.85 \times 10^{-12} \text{C}^2/\text{N} \cdot \text{m}^2) \cdot (3.00 \times 10^8 \text{m/s})}} = 1.01 \times 10^3 \text{V/m}
\]

For \( B_0 \)

\[
B_0 = \frac{E_0}{c} = \frac{1.01 \times 10^3 \text{V/m}}{3 \times 10^8 \text{m/s}} = 3.37 \times 10^{-6} \text{T}
\]