1. Conservation of Mechanical Energy
2. Work done by non-conservative forces
3. How are conservative forces and potential energy related?
4. Equilibrium of a system
5. General Energy Conservation
7. Linear momentum, Impulse and Collisions

Today’s homework is homework #10, due 12:00pm, next Monday!!
Reminder

• If your term exam score is less than 40, come talk to me before the next exam
• If you still have not subscribed to the class e-mail list, please do so soon.
Conservative Forces and Potential Energy

The work done on an object by a conservative force is equal to the decrease in the potential energy of the system.

\[ W_c = \int_{x_i}^{x_f} F_x \, dx = -\Delta U \]

What else does this statement tell you?

The work done by a conservative force is equal to the negative of the change of the potential energy associated with that force.

Only the changes in potential energy of a system is physically meaningful!!

We can rewrite the above equation in terms of potential energy \( U \).

\[ \Delta U = U_f - U_i = -\int_{x_i}^{x_f} F_x \, dx \]

So the potential energy associated with a conservative force at any given position becomes

\[ U_f(x) = -\int_{x_i}^{x_f} F_x \, dx + U_i \]

Potential energy function

What can you tell from the potential energy function above?

Since \( U_i \) is a constant, it only shifts the resulting \( U_f(x) \) by a constant amount. One can always change the initial potential so that \( U_i \) can be 0.
Conservation of Mechanical Energy

Total mechanical energy is the sum of kinetic and potential energies:

\[ E \equiv K + U \]

Let's consider a brick of mass \( m \) at a height \( h \) from the ground.

\[ U_g = mgh \]

What is its potential energy?

What happens to the energy as the brick falls to the ground?

The brick gains speed.

\[ v = gt \]

By how much?

The brick's kinetic energy increased.

\[ K = \frac{1}{2}mv^2 = \frac{1}{2}mg^2t^2 \]

And?

The lost potential energy converted to kinetic energy.

What does this mean?

The total mechanical energy of a system remains constant in any isolated system of objects that interacts only through conservative forces:

**Principle of mechanical energy conservation**

\[ E_i = E_f \]

\[ K_i + \sum U_i = K_f + \sum U_f \]
Example 8.2

A ball of mass $m$ is dropped from a height $h$ above the ground. Neglecting air resistance determine the speed of the ball when it is at a height $y$ above the ground.

**Using the principle of mechanical energy conservation**

\[
K_i + U_i = K_f + U_f
\]

\[
0 + mgh = \frac{1}{2}mv^2 + mgy
\]

\[
\frac{1}{2}mv^2 = mg(h - y)
\]

\[
\therefore v = \sqrt{2g(h - y)}
\]

**b) Determine the speed of the ball at $y$ if it had initial speed $v_i$ at the time of release at the original height $h$.**

Again using the principle of mechanical energy conservation but with non-zero initial kinetic energy!!!

\[
K_i + U_i = K_f + U_f
\]

\[
\frac{1}{2}mv_i^2 + mgh = \frac{1}{2}mv_f^2 + mgy
\]

\[
\frac{1}{2}m(v_f^2 - v_i^2) = mg(h - y)
\]

\[
\therefore v_f = \sqrt{v_i^2 + 2g(h - y)}
\]

This result look very similar to a kinematic expression, doesn't it? Which one is it?
Example 8.3

A ball of mass \( m \) is attached to a light cord of length \( L \), making up a pendulum. The ball is released from rest when the cord makes an angle \( \theta_A \) with the vertical, and the pivoting point \( P \) is frictionless. Find the speed of the ball when it is at the lowest point, \( B \).

Compute the potential energy at the maximum height, \( h \). Remember where the 0 is.

\[
h = L - L \cos \theta_A = L(1 - \cos \theta_A)
\]

Using the principle of mechanical energy conservation

\[
K_i + U_i = K_f + U_f
\]

\[
0 + mgh = mgL(1 - \cos \theta_A) = \frac{1}{2}mv^2
\]

\[
v^2 = 2gL(1 - \cos \theta_A)
\]

\[
\therefore v = \sqrt{2gL(1 - \cos \theta_A)}
\]

b) Determine tension \( T \) at the point \( B \).

Using Newton’s 2nd law of motion and recalling the centripetal acceleration of a circular motion

\[
\sum F_r = T - mg = ma_r = m \frac{v^2}{r} = m \frac{v^2}{L}
\]

\[
T = mg + m \frac{v^2}{L} = m \left( g + \frac{v^2}{L} \right) = m \left( g + \frac{2gL(1 - \cos \theta_A)}{L} \right)
\]

\[
= mL + 2gL(1 - \cos \theta_A)
\]

\[
\therefore T = mg(3 - 2\cos \theta_A)
\]

Cross check the result in a simple situation. What happens when the initial angle \( \theta_A \) is 0? \( T = mg \).
Work Done by Non-conserved Forces

Mechanical energy of a system is not conserved when any one of the forces in the system is a non-conservative force.

Two kinds of non-conservative forces:

Applied forces: Forces that are external to the system. These forces can take away or add energy to the system. So the mechanical energy of the system is no longer conserved.

If you were to carry around a ball, the force you apply to the ball is external to the system of ball and the Earth. Therefore, you add kinetic energy to the ball-Earth system.

Kinetic friction: Internal non-conservative force that causes irreversible transformation of energy. The friction force causes the kinetic and potential energy to transfer to internal energy.

\[ W_{\text{you}} + W_g = \Delta K; \quad W_g = -\Delta U \]
\[ W_{\text{you}} = W_{\text{app}} = \Delta K + \Delta U \]

\[ W_{\text{friction}} = \Delta K_{\text{friction}} = -f_k d \]
\[ \Delta E = E_f - E_i = \Delta K + \Delta U = -f_k d \]
Example 8.6

A skier starts from rest at the top of frictionless hill whose vertical height is 20.0 m and the inclination angle is 20°. Determine how far the skier can get on the snow at the bottom of the hill with a coefficient of kinetic friction between the ski and the snow is 0.210.

\[ ME = mgh = \frac{1}{2}mv^2 \]

\[ v = \sqrt{2gh} \]

\[ v = \sqrt{2 \times 9.8 \times 20.0} = 19.8 \text{ m/s} \]

The change of kinetic energy is the same as the work done by kinetic friction.

\[ \Delta K = K_f - K_i = -f_k d \]

Compute the speed at the bottom of the hill, using the mechanical energy conservation on the hill before friction starts working at the bottom.

What does this mean in this problem?

Since we are interested in the distance the skier can get before stopping, the friction must do as much work as the available kinetic energy.

Since \( K_f = 0 \)

\[ -K_i = -f_k d; \quad f_k d = K_i \]

\[ f_k = \mu_k n = \mu_k mg \]

\[ d = \frac{K_i}{\mu_k mg} = \frac{1}{2} \frac{mv^2}{\mu_k mg} = \frac{v^2}{2\mu_k g} = \frac{(19.8)^2}{2 \times 0.210 \times 9.8} = 95.2 \text{ m} \]

Don't we need to know mass?

Well, it turns out we don't need to know mass.

What does this mean?

No matter how heavy the skier is he will get as far as anyone else has gotten.
How are Conservative Forces Related to Potential Energy?

Work done by a force component on an object through a displacement $\Delta x$ is

$$W = F_x \Delta x = -\Delta U$$

For an infinitesimal displacement $\Delta x$

$$dU = -F_x dx$$

This relationship says that any conservative forces acting on an object within a given system is the same as the negative derivative of the potential energy of the system with respect to position.

For an infinitesimal displacement $\Delta x$

$$\lim_{\Delta x \to 0} \Delta U = -\lim_{\Delta x \to 0} F_x \Delta x$$

Results in the conservative force-potential relationship

$$F_x = -\frac{dU}{dx}$$

Does this statement make sense?

1. Spring-ball system:

$$F_s = -\frac{dU_s}{dx} = -\frac{d}{dx} \left( \frac{1}{2} kx^2 \right) = -kx$$

2. Earth-ball system:

$$F_g = -\frac{dU_g}{dy} = -\frac{d}{dy} (mg) = -mg$$

The relationship works in both the conservative force cases we have learned!!!
Energy Diagram and the Equilibrium of a System

One can draw potential energy as a function of position ➔ Energy Diagram

Let's consider potential energy of a spring-ball system

\[ U_s = \frac{1}{2} kx^2 \]

What shape would this diagram be? A Parabola

What does this energy diagram tell you?

1. Potential energy for this system is the same independent of the sign of the position.
2. The force is 0 when the slope of the potential energy curve is 0 with respect to position.
3. \( x=0 \) is one of the stable or equilibrium of this system when the potential energy is minimum.

Position of a stable equilibrium corresponds to points where potential energy is at a minimum.

Position of an unstable equilibrium corresponds to points where potential energy is a maximum.
General Energy Conservation and Mass-Energy Equivalence

**General Principle of Energy Conservation**
The total energy of an isolated system is conserved as long as all forms of energy are taken into account.

**Friction**
Friction is a non-conservative force and causes mechanical energy to change to other forms of energy.

However, if you add the new form of energy altogether the system as a whole did not lose any energy, as long as it is self-contained or isolated.

In the grand scale of the universe, no energy can be destroyed or created but just transformed or transferred from one place to another. **Total energy of universe is constant.**

**Principle of Conservation of Mass**
In any physical or chemical process, mass is neither created nor destroyed. Mass before a process is identical to the mass after the process.

**Einstein’s Mass-Energy Equality**

\[ E_R = mc^2 \]

How many joules does your body correspond to?

Monday, Oct. 14, 2002

Dr. Jaehoon Yu
Example 8.12

The sun converts $4.19 \times 10^9$ kg of mass into energy per second. What is the power output of the sun?

Using Einstein's mass-energy equivalence

\[ E_R = mc^2 \]

\[ = 4.19 \times 10^9 \times (3 \times 10^8)^2 \]

\[ = 3.77 \times 10^{25} \text{ J} \]

Since the sun gives out this amount of energy per second the power is simply

\[ P = 3.77 \times 10^{25} \text{ W} \]

How many 60 W bulbs does this correspond to? If the cost for electricity is 9c/kWh, how much does an 8 hour worth of sun’s energy cost?

\[ N_{60W} = \frac{P}{60} = 6.28 \times 10^{24} \text{ W} \]

\[ E = P \times t \]

\[ = 3.77 \times 10^{25} \times 8 \]

\[ = 3.02 \times 10^{27} \text{ kWh} \]

\[ \text{Cost} = 3.02 \times 10^{27} \times 0.09 = 2.72 \times 10^{26} \]
Linear Momentum

The principle of energy conservation can be used to solve problems that are harder to solve just using Newton’s laws. It is used to describe motion of an object or a system of objects.

A new concept of linear momentum can also be used to solve physical problems, especially the problems involving collisions of objects.

Linear momentum of an object whose mass is $m$ and is moving at a velocity of $v$ is defined as

$$\vec{p} = m\vec{v}$$

What can you tell from this definition about momentum?

1. Momentum is a vector quantity.
2. The heavier the object the higher the momentum
3. The higher the velocity the higher the momentum
4. Its unit is kg.m/s

What else can you see from the definition? Do you see force?

The change of momentum in a given time interval

$$\frac{d\vec{p}}{dt} = \frac{d}{dt}(m\vec{v}) = m\frac{d\vec{v}}{dt} = m\vec{a} = \vec{F}$$